7.

PUMP AND PIPELINE CHARACTERISTICS

In the previous chapters the flow of mixture in pipelines was discussed in details. The following chapters will deal with the effects of an interaction between a pipeline and pumps on flow of mixture through a dredging installation. Study of these chapters assumes knowledge of a basic theory of slurry pipeline flows (handled in Chapters 1 to 6) and of a basic theory of centrifugal dredge pumps and pump drives as given in professor Vlasblom's course.

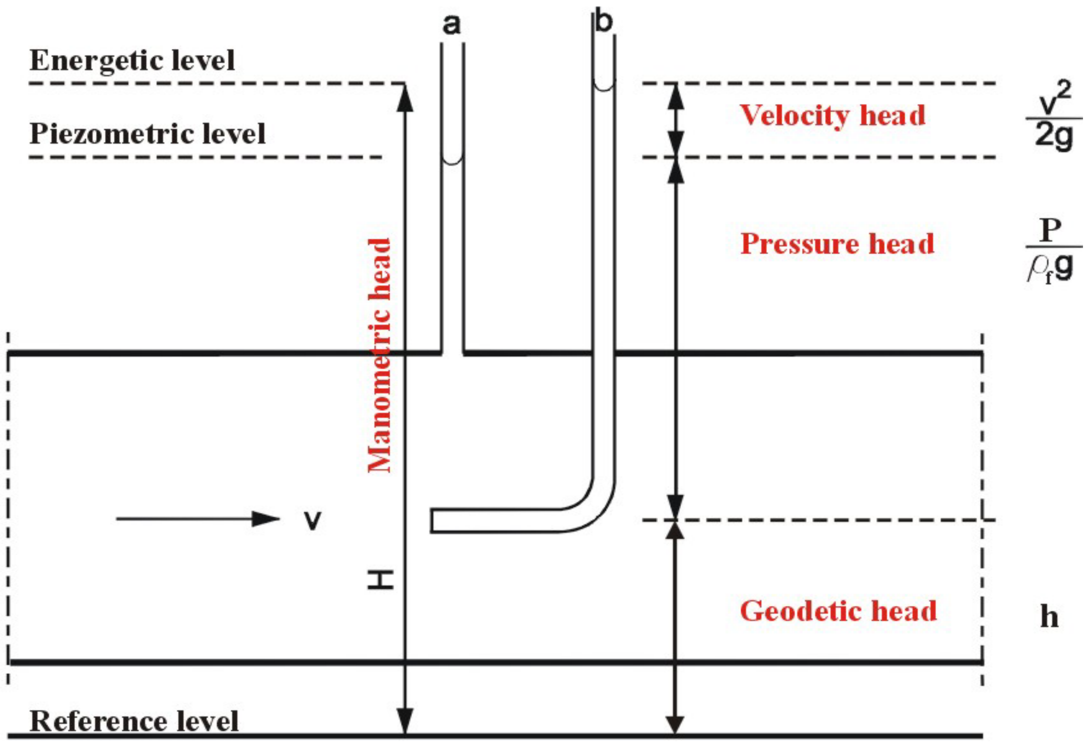
7.1 THE BERNOULLI EQUATION

The amount of mechanical energy available in a pipeline flow of fluid is quantified in the Bernoulli equation. This equation is obtained by integrating the Euler's equation of motion along a streamline. If flow is steady, frictionless and incompressible then at an arbitrary location along a streamline

$$h + \frac{p}{\rho_f g} + \frac{v_f^2}{2g} = \text{const.} \qquad (7.1)$$

h	geodetic position of a location, elevation above datum	[m]
p	pressure at a location on a stream line	[Pa]
v_f	velocity of fluid at a location on a streamline	[m/s].
ρ_f	density of flowing fluid	[kg/m^3]
g	gravitational acceleration	[m/s^2].

Each term of the above equation represents a head (Dutch: opvoerhoogte) with unit [m]. The head can be interpreted as energy per unit gravity force (see Chapter 3). The first term is *potential energy* of fluid control volume per unit gravity force, the second term *flow energy* (or flow work) and the third term *kinetic energy* (see Fig. 7.1). Thus for two fluid control volumes at two different locations along a streamline the sum of these three energy terms is constant. However, the proportion of the values of the particular energy terms changes when flow conditions change during a motion of a fluid particle from one location to another.



$$h + \frac{P}{\rho_f g} + \frac{V_m^2}{2g} = Level \quad of \quad Mech. \quad Energy$$

Figure 7.1. Different forms of head in Bernoulli equation.

Consider a water flow through a pipe section. Flow is steady (flow rate is constant) and incompressible (density is constant). If the pipe section is horizontal and a pipe diameter at the beginning of a pipe section is smaller than at the end of the pipe section than the pressure at the section begin is lower than that at the section end (see Fig. 7.2: a horizontal pipe section in front of a pump). This is because the velocity at the inlet is higher than at the outlet. A portion of kinetic energy is transformed to flow energy in a pipe section. If a pipe remains of a constant diameter but the pipe section is inclined the pressure at the top of the pipe section is smaller than that at the bottom of the pipe section (see Fig. 7.2: an inclined suction pipe). Work had to be done (flow energy lost) to lift water particles from the bottom to the top of a pipe section. Lifted particles gained potential energy.

In practice, transported media of our interest (water or mixture) are considered incompressible but the flow of these media can not be considered frictionless. If flowing through a pipe, water or mixture dissipates a portion of their mechanical energy. They transform a portion of their mechanical energy into thermal energy (heat). The mechanical energy loss along a pipe section (between cross sections 1 and 2) must be incorporated to the Bernoulli equation so that

$$h_1 + \frac{P_1}{\rho_f g} + \frac{V_1^2}{2g} = h_2 + \frac{P_2}{\rho_f g} + \frac{V_2^2}{2g} + H_{totalloss} \qquad (7.2)$$

$H_{totalloss}$	total head loss due to mechanical energy dissipation between pipe cross sections 1 and 2	[m]
P	mean absolute pressure in a pipe cross section	[Pa]
h	geodetic height of a pipe cross section	[m]
V	mean velocity in a pipe cross section	[m/s]
ρ_f	density of fluid	[kg/m^3]
g	gravitational acceleration	[m/s^2].

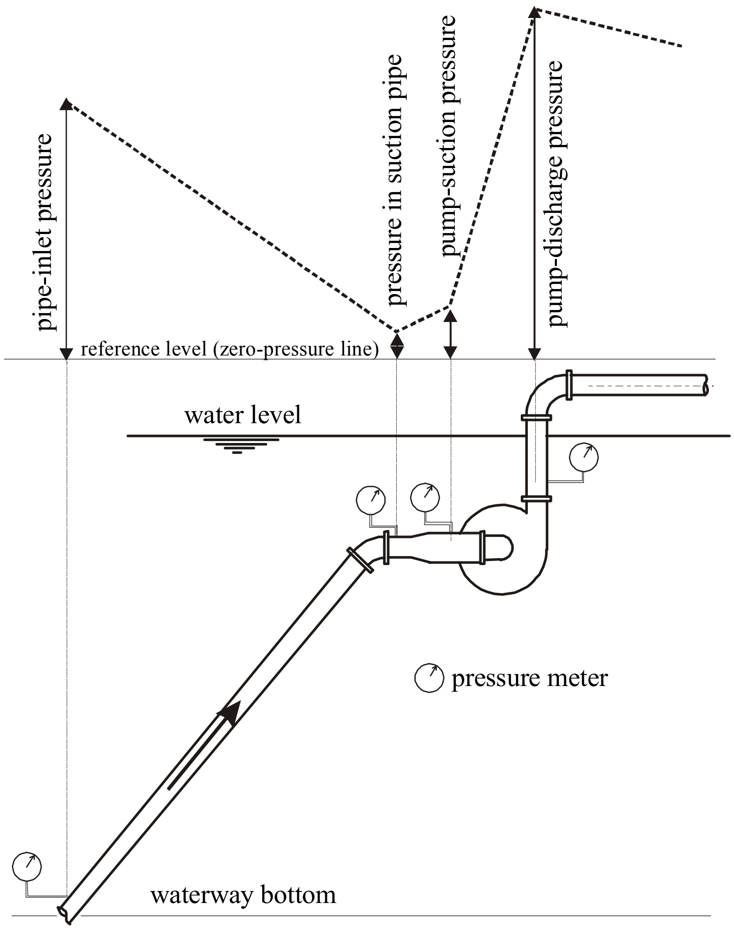


Figure 7.2. Pressure variation along a pipeline connected with a pump (schematic).

7.2 H - Q CURVE OF A CENTRIFUGAL PUMP

A rotating impeller of a centrifugal pump adds mechanical energy to the medium flowing through a pump. As a result of an energy addition a pressure differential occurs in the pumped medium between the inlet and the outlet of a pump (see the difference between the pump-suction pressure and the pump-discharge pressure in Fig. 7.2). The pressure, or the energy head, added to the medium depends on the speed (revolutions per minute, r.p.m.) of an impeller and on the flow rate of medium through a pump. A relationship between the head, H, the flow rate (called also capacity), Q, and the revolutions per minute of the impeller, n, is given by a set of H-Q curves (Fig. 7.3). A course of these curves is specific for each particular pump. The course of the curves is sensitive to the geometry of a pump housing and of an impeller and thus to flow conditions within a pump. The curves are determined by a pump test. Usually, a pump manufacturer delivers the curves (called pump characteristics) with a pump. For dredge pumps the pump characteristics may change in time because the flow conditions within a pump are influenced by a wear of an impeller and pump housing.

As discussed in Chapter 3, the *head* H is a measure of the mechanical energy of a flowing liquid per unit gravity force. It is expressed as the height of the column of liquid of ρ_f exerting the pressure differential ΔP, so that $H = \frac{\Delta P}{\rho_f g}$.

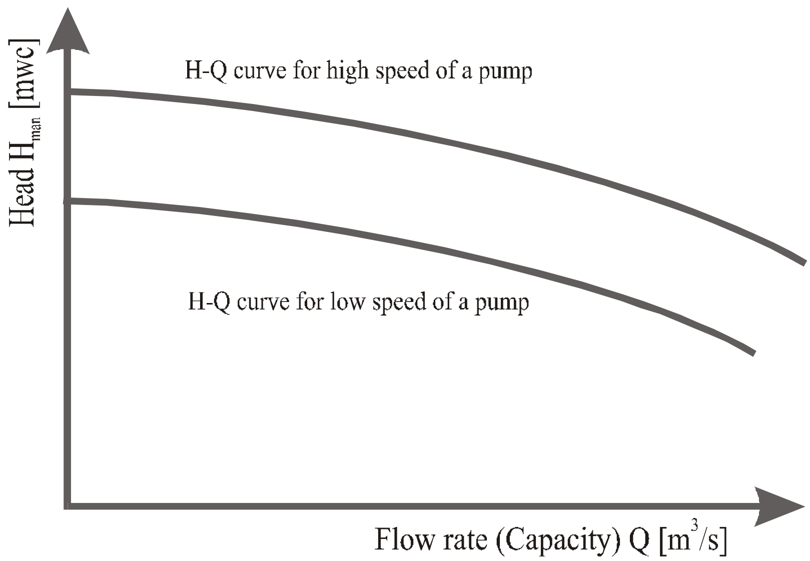


Figure 7.3. Schematic Q-H curves of a centrifugal pump.

The head due to pressure differential generated by a pump is called the manometric head, H_{man}, and it has a unit meter water column [mwc] (see Fig. 7.3).

The H_{man}-Q curve of a pump gives an amount of energy that a pump provides to a pump-pipeline system for a certain r.p.m. (speed) of a pump impeller and a flow rate (Q) through a pump.

The manometric head that is delivered by a pump to medium is determined by a parameter called the manometric pressure and it is given (see also Fig. 7.4) as

$$P_{man} = P_p - P_s + \rho_m g\left(h_p + h_s\right) + \frac{\rho_m\left(V_p^2 - V_s^2\right)}{2} \qquad (7.3)$$

P_{man}	absolute manometric pressure	[Pa]
P_p	absolute pressure at the discharge outlet of a pump	[Pa]
P_s	absolute pressure at the suction inlet of a pump	[Pa]
h_p	vertical distance between the pump axis and the discharge outlet of a pump	[m]
h_s	vertical distance between the pump axis and the suction inlet of a pump	[m]
V_p	mean mixture velocity at the discharge outlet of a pump	[m/s]
V_s	mean mixture velocity at the suction inlet of a pump	[m/s]
ρ_m	density of pumped medium	[kg/m^3]
g	gravitational acceleration	[m/s^2].

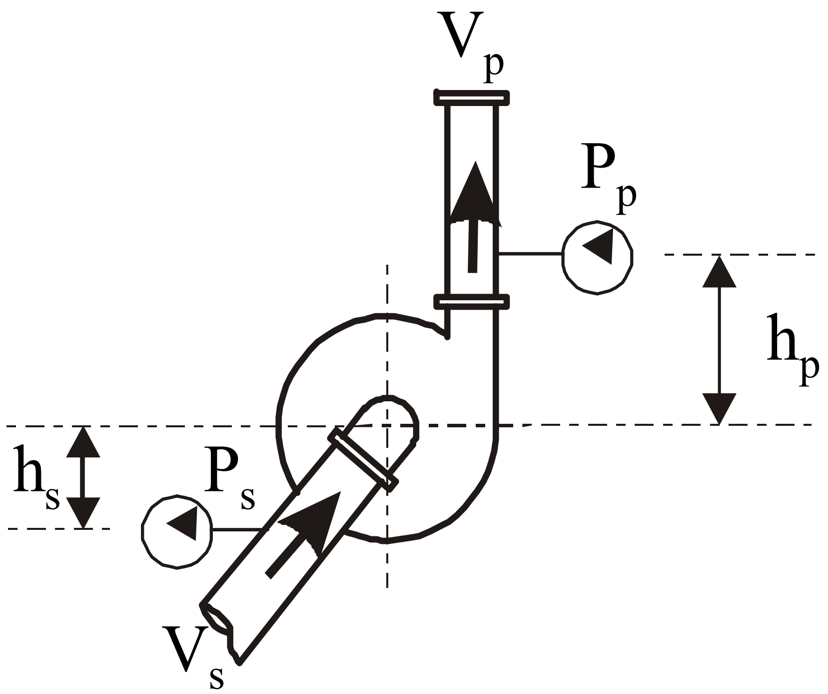


Figure 7.4. Conditions at inlet and outlet of a pump.

7.2.1 Affinity laws for pump characteristics

Pump characteristic curves give H_{man}-Q, W_{out} -Q (W_{out} is pump output power) and η-Q (η is pump efficiency) relationships for certain constant speed n [rpm].

The affinity laws

$$\frac{Q_{m,n1}}{Q_{m,n2}} = \frac{n_1}{n_2}, \quad \frac{H_{man,n1}}{H_{man,n2}} = \left(\frac{n_1}{n_2}\right)^2, \quad \frac{W_{out,n1}}{W_{out,n2}} = \left(\frac{n_1}{n_2}\right)^3, \quad \frac{\eta_{f,n1}}{\eta_{f,n2}} = 1$$

enable to produce the pump curves for different constant speeds, n, of the pump (for more details see Vlasblom's lecture notes referred in Chapter 11). An application of the affinity laws is shown in Case study 7 at the end of this chapter (Fig. C7.2).

7.2.2 Different regions of pump operation (an interaction between pump and drive)

A pump-drive system operates in a region of

- the constant speed or
- the constant torque

(for details see Vlasblom's lecture notes referred in Chapter 11). This affects a shape of an H_{man}-Q curve (see Fig. 7.5).

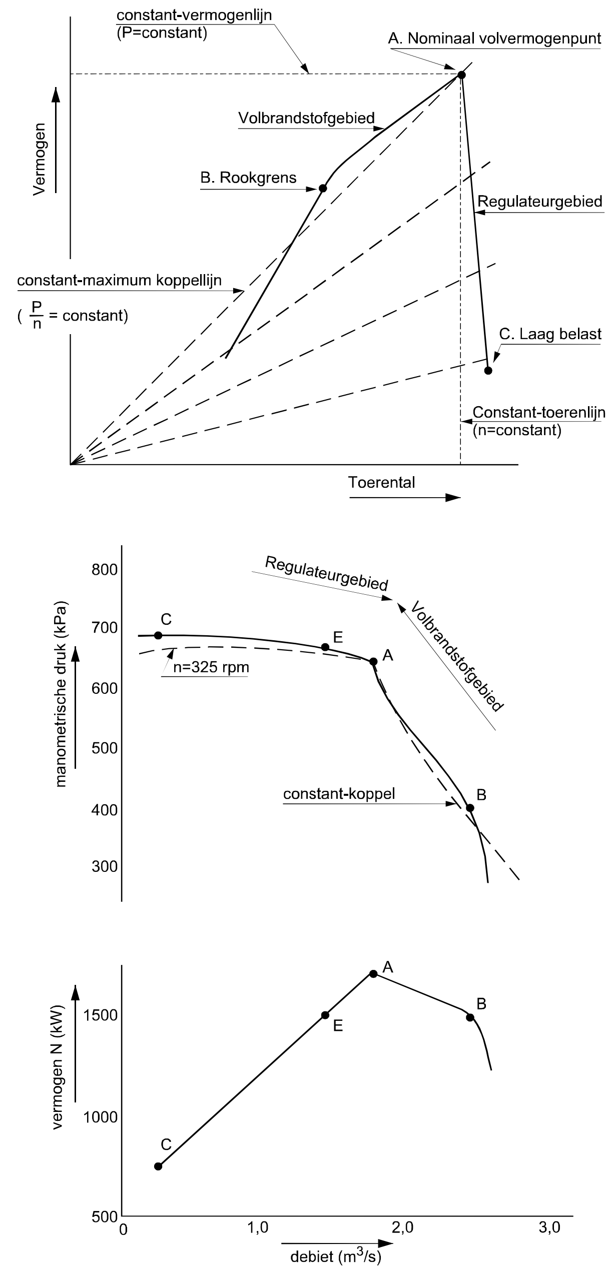


Figure 7.5. Operation regions of a pump-drive system.

7.2.3 Effect of solids on pump performance

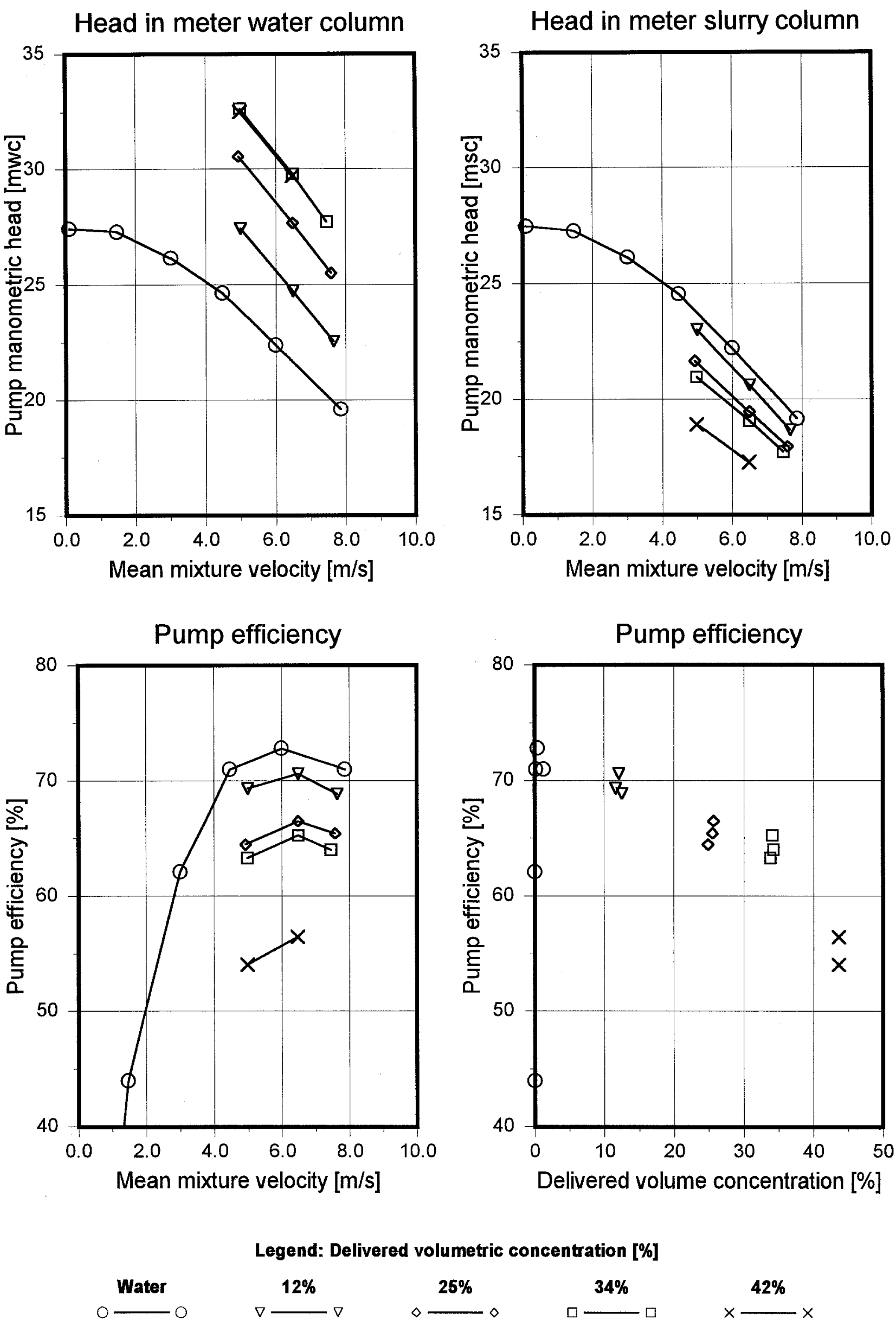


Figure 7.6. Effect of mixture density on pump characteristics. Tests of a 0.5-m-impeller pump connected with a 162 kW MAN diesel engine. Speed: 1000 rpm. Pumped material: 0.2 – 0.5 mm sand. (Data from Laboratory of Dredging Technology, TU Delft).

Solid particles of a pumped mixture diminish the efficiency of a dredge pump (see Fig. 7.6). The ratio of pump efficiencies when pumping mixture or water $f_c = \eta_m/\eta_f$ is also a measure of manometric pressure reduction and output power reduction

$$\frac{P_{man,m}}{P_{man,f}} = \frac{\rho_m}{\rho_f} f_c, \quad \frac{W_{in,m}}{W_{in,f}} = \frac{\rho_m}{\rho_f}, \quad \frac{W_{out,m}}{W_{out,f}} = \frac{\rho_m}{\rho_f} f_c, \quad \frac{\eta_m}{\eta_f} = f_c \qquad (7.4).$$

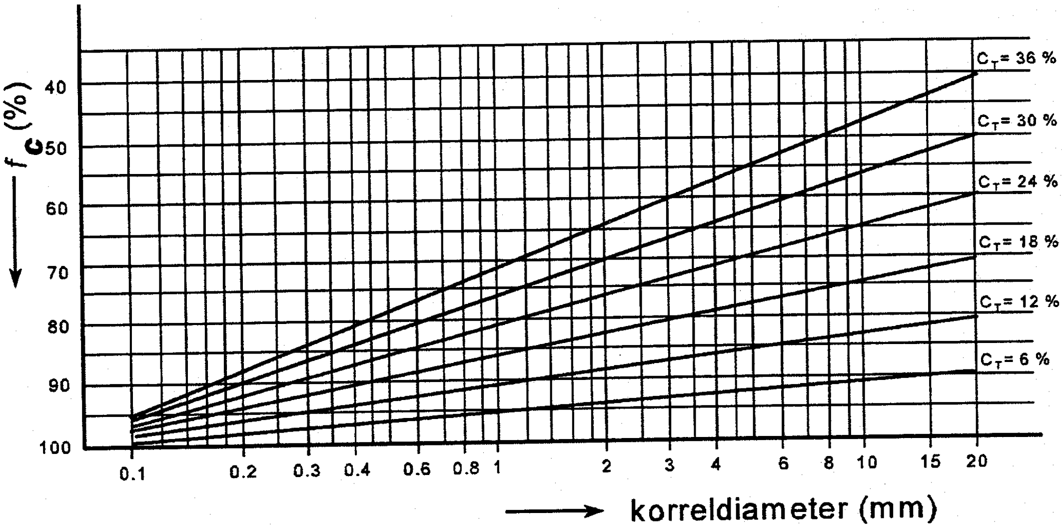


Figure 7.7. Parameter f_c giving reduction of the pump performance due to presence of sand/gravel particles in pumped liquid according to Eq. 7.5a.

The parameter f_c for sand and gravel mixtures is according to Stepanoff (1965) related to the particle size, d_{50}, and delivered concentration, C_{vd}, of solids in transported mixture by

$$f_c = 1 - C_{vd}\left(0.8 + 0.6 \log d_{50}\right) \qquad (7.5a).$$

In this equation d_{50} is in [mm] and C_{vd} in [-]. Reduction of pump efficiency and manometric head increases with a particle size and solids concentration. The reduction is relatively small for fine sand but it is very significant if mixtures of coarse sand or gravel are pumped (see Fig. 7.7).

The original Stepanoff equation (Eq. 7.5a) does not consider the effect of an impeller size. However, this effect may be of significant importance. The revised Stepanoff equation (e.g. Miedema, 1999) including the impeller diameter D_{impel} is

$$f_c = 1 - \frac{C_{vd}\left(0.466 + 0.4 \log d_{50}\right)}{D_{impel}} \qquad (7.5b).$$

In a pump-pipeline system the manometric pressure (or manometric head) of a dredge pump is required to overcome the total head loss in mixture transported in a pipeline connected to a dredge pump. The total head loss is composed of

- the major and minor losses due to flow friction in a suction pipeline,
- the loss due to the change in elevation of a suction pipeline,
- the major and minor losses due to flow friction in a discharge pipeline,
- the loss due to the change in elevation of a discharge pipeline,
- the losses due to mixture acceleration in a pipeline.

7.3 H-Q CURVE OF A PIPELINE

The H_{man}-Q curve of a pipeline gives an amount of energy that the pipeline requires maintaining a certain flow rate in a pump-pipeline system.

The required amount of mechanical energy is equal to the sum of the energy dissipated due to friction in a flow of mixture through a pipeline and the potential energy delivered to (or lost in) mixture to reach a pipeline outlet if this is at a higher (or lower) geodetic level than a pipeline inlet.
A head lost due to flow of mixture in a pipeline is regarded as sum of major losses due to internal friction in flow of mixture through straight pipeline sections and minor losses due to flow friction caused by pipeline fittings.

7.3.1 Head loss in straight pipelines (major loss)

A determination of the frictional head loss for flow of water or mixture in straight pipelines was a subject to discussion in Chapters 1, 4 and 5. The frictional head loss in *water flow* is determined using the Darcy-Weisbach equation (Chapter 1). This gives a parabolic H-Q curve (called a pipeline-resistance curve) described by the equation

$$H_{major,f} = \frac{\lambda_f L}{D}\frac{V_f^2}{2g} = \frac{\lambda_f L}{D}\frac{Q_f^2}{2gA^2} \qquad (7.6).$$

$H_{major,f}$	head loss due to friction of water in a straight pipe	[Pa]
λ_f	flow friction coefficient	[-]
L	length of a pipe	[m]
D	diameter of a pipe	[m]
V_f	mean mixture velocity in a pipe	[m/s]
Q_f	mixture flow rate through a pipe	[m^3/s]
A	area of a pipe cross section	[m^2]
g	gravitational acceleration	[m/s^2].

The course of a pipeline-resistance curve is more complicated for *mixture flows*. However, they are models available that are capable of predicting the resistance curves for a various mixtures flowing in a pipeline (see Chapters 4 and 5). The models predict the hydraulic gradient I_m and this is interpreted as the head lost along a pipeline of a length L using

$$H_{major,m} = I_m L \qquad (7.7)$$

$H_{major,m}$	head loss due to friction of mixture in a straight pipe	[Pa]
I_m	hydraulic gradient in mixture flow according to a suitable model	[-]
L	length of a pipe	[m].

7.3.2 Head loss in flow through fittings (minor loss)

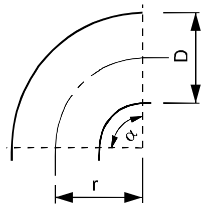
Fittings as bends, joint balls, expansions and contractions of a discharge area, valves and measuring instruments act as obstructions to the flow. The pipeline inlet and outlet are also sources of local losses. Obstructions cause flow separation and an induced mixing process in the separated zones dissipates mechanical energy. This energy dissipation is additional to that in flow through straight pipeline sections. A portion of energy dissipated due to a presence of fittings is usually considerably smaller than frictional losses in straight pipes. In long dredging pipelines behind a dredge the minor losses might be even considered negligible in comparison with straight-pipe losses.

The minor losses for *water flow* obey a quadratic relationship between local head loss and mean velocity through a fitting

$$H_{min\,or,f} = \xi \frac{V_f^2}{2g} \qquad (7.8)$$

$H_{minor,f}$	head loss due to friction of water flow in fittings	[Pa]
ξ	minor loss coefficient	[-].

Remark: ξ value for the pipeline outlet is 1.0 (ξ_{outlet}=1.0).



r/D	ξ-waarden: bocht α in ° 15°	22,5°	30°	45°	60°	90°
1.5	0.03	0.050	0.085	0.13	0.17	0.20
2	0.03	0.045	0.060	0.09	0.12	0.13
3	0.03	0.045	0.055	0.08	0.10	0.13
5	0.03	0.045	0.050	0.07	0.08	0.11
10	0.03	0.045	0.050	0.07	0.07	0.11

			Overeenkomstigeweerstandslengte in meters		
		ξ	D=0,30 m	D=0,60 m	D=0.90 m
Bocht 45°	r/D= 1,5	0,13	6,0	11	17
	r/D= 2,0	0,09	3,7	7,5	11
Bocht 90°	r/D= 1,5	0,20	7,5	15	22
	r/D= 2,0	0,13	5,0	10	15
Knik 30°		0,15	3,7	7,5	11
45°		0,3	7,5	15	22
Kogel		0,2-0.3	5,0-7,5	10-15	15-22
Zak, afsluiter		0.1	2,5	5	7
Wisselmet α	30°	0.6	15	30	45
		0.3	7,5	15	22

Figure 7.8. Coefficient of minor losses for some fittings.

Values of the coefficient ξ vary between zero and one for different fittings (e.g. Fig. 7.8). Experimentally determined values for ξ are available for water flow through various fittings and various fitting configurations in the literature.

A correct determination of minor losses for *mixture flows* is more complicated, particularly for stratified flows. In a dredging practice a simple assumption is often applied that mixture density alone sufficiently represents an effect of solids on the minor loss so that

$$H_{minor,m} = \xi \frac{V_f^2}{2g} \frac{\rho_m}{\rho_f} \quad (7.9).$$

This might be a suitable approach for fully-suspended flows in which slurry density might directly influence frictional losses through wall shear stresses. In stratified flows, however, an induced local turbulence of a carrying liquid might take a portion of particles from a bed to suspension and reduce frictional losses in a pipeline section of a certain length behind a fitting. This effect must be taken into account. If no extra suspension is assumed due to flow disturbances (in flow of coarse particles), an energy dissipation takes place through small turbulent eddies of a carrying liquid that decay to the viscosity of the carrier. The carrier viscosity is not affected by a presence of coarse solid particles. Therefore the value of a minor head should not be influenced either. Thus Eq. 7.9 might overestimate minor losses in stratified flows. However, very little is known about an effect of solids on the minor losses in pipelines yet. It is an interesting subject to further investigation.

7.3.3 Total frictional head loss

The total frictional head loss is a sum of head losses due to friction in a straight pipeline sections and in fittings mounted to a pipeline

$$H_{totalloss} = H_{major} + H_{minor} \quad (7.10).$$

A total frictional pressure drop over a pipeline of the length L is given for a *water flow* by the equation

$$\Delta P_{totalloss,f} = \left(\lambda_f \frac{L}{D} + \xi\right)\frac{1}{2}\rho_f V_f^2 \quad (7.11)$$

and for a *mixture flow* by the equation

$$\Delta P_{totalloss,m} = I_m \rho_f g L + \xi \frac{1}{2}\rho_m V_m^2 \quad (7.12).$$

A development of the total frictional pressure drop under the changing mean velocity in a pipeline is given schematically in Fig. 7.9.

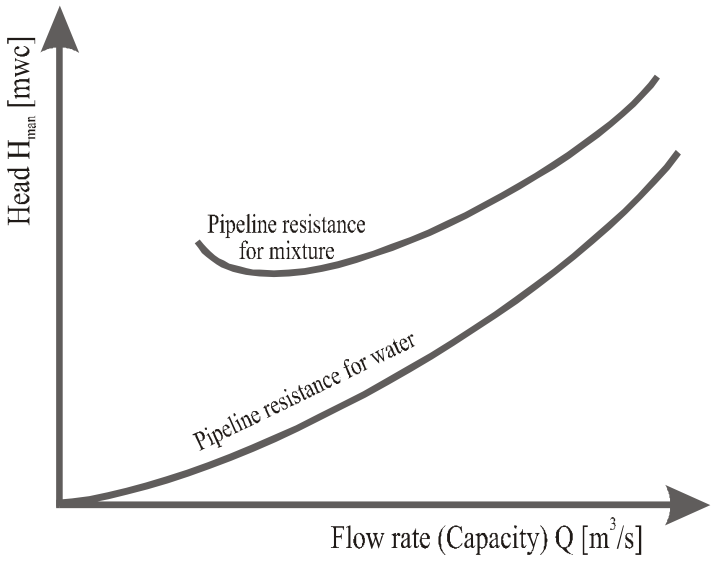


Figure 7.9. Schematic $H_{totalloss}$-Q curve of a horizontal pipeline.

7.3.4 Geodetic head

A geodetic head (called also static head) was discussed in detail in Chapter 6 concerning inclined flows,

$$H_{static} = S_m \Delta h \tag{7.13}$$

H_{static}	geodetic head	[m]
Δh	elevation change over a pipeline; difference in geodetic height between pipeline inlet and outlet	[m]
S_m	relative density of mixture	[-].

7.4 WORKING POINT OF A PUMP-PIPELINE SYSTEM

Velocity of transported medium in a pump-pipeline system is determined by a cross point of a pump H-Q curve and a pipeline H-Q curve (Fig. 7.10). The cross point gives the velocity at which a balance is found between the energy provided to a system by a pump and the energy required to overcome a flow resistance in a pipeline and a change in a geodetic height between the pipeline inlet and outlet.

Practically this means that if water is pumped through a pipeline of certain geometry (given by diameter, length, elevation and a number of fittings) the rpm installed on a pump determines directly the velocity of water in a pipeline. An increase in the rpm (i.e. a step to an another pump H-Q curve of constant rpm) increases the water velocity because a new working point (Dutch: werkpunt) is found on a pipeline resistance curve.

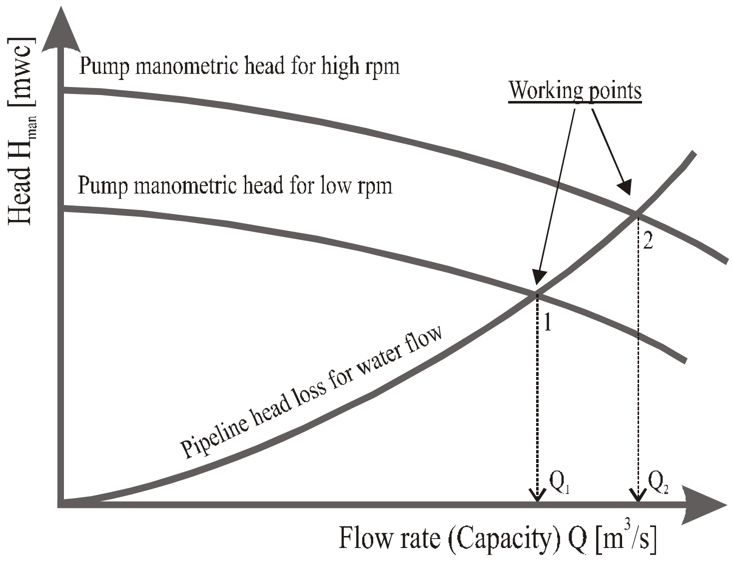


Figure 7.10. Working points of a pump-pipeline system when water is pumped (schematic).

The same rules are valid for mixture pumping if flow conditions are steady, i.e. mixture density and size of transported solids do not change in time.

For a dredging operation, however, a fluctuation of mixture density is typical. Even if other parameters (as pump rpm and size of transported solids) are constant, this density fluctuation might produce a fluctuation of the mean mixture velocity in a pipeline. The increasing density of mixture in a pipeline is a source of increasing flow resistance. Thus a balance between a provided energy head (that is constant if rpm of a pump does not change) and a required energy head (that increases with mixture density in a pipeline) is found at lower velocity. The velocity increases again if mean mixture density gradually drops in a pipeline due to lower density of mixture generated in a pipeline inlet.

7.5 WORKING RANGE OF A PUMP-PIPELINE SYSTEM

If a pipeline resistance is given by just one H-Q curve of a pipeline (a pipeline lay out, properties of soil and density of transported mixture are constant in time) there is just one working point at which an installation of a pipeline and a pump at a constant speed operates. If pipeline resistance changes (usually due to fluctuating mixture density in a pipeline) an installation operates within a working range (Dutch: werkgebied) instead of at a working point (Fig. 7.11).

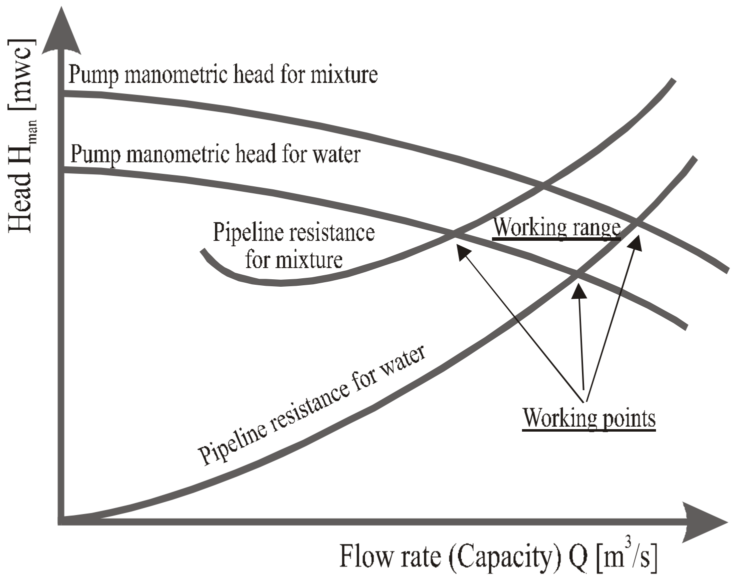


Figure 7.11. Working points and working range of a pump-pipeline system (schematic).

A mixture density fluctuates with a high frequency and amplitude within a dredging pipeline, particularly if a discharge pipeline is connected with a cutter suction dredge (see Fig. 7.12).

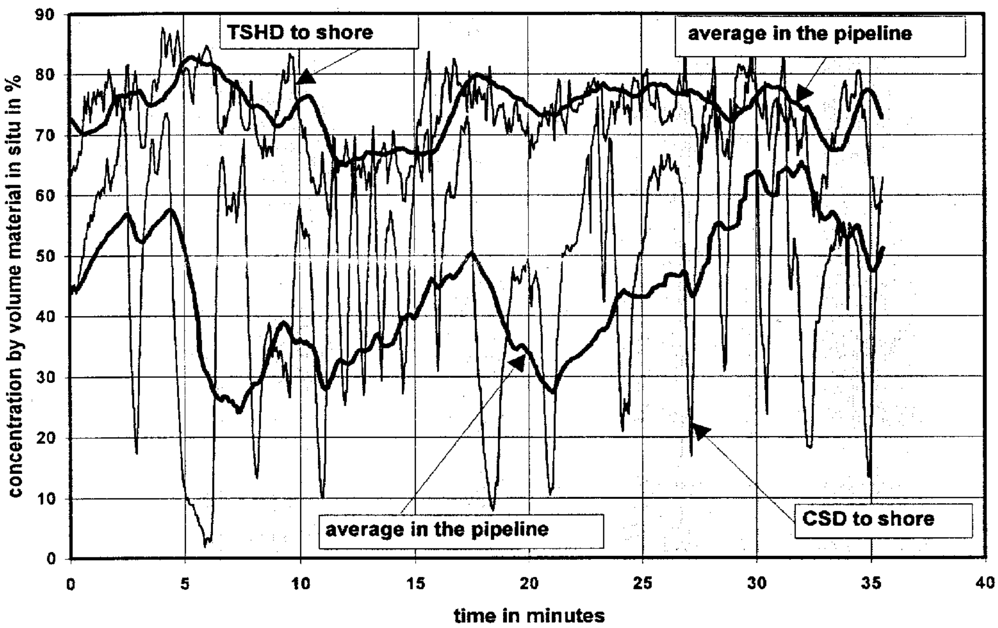


Figure 7.12. Mixture density fluctuations (here referred as C_{vsi} fluctuations) in a 500 meter long discharge pipeline connected with a trailing suction hopper dredge (TSHD) and a cutter suction dredge (CSD) (after v.d. Berg, 1998).

This causes fluctuation in a manometric pressure provided by a pump. However, a position of a working point of a pump-pipeline installation is influenced by the mean mixture density in an entire pipeline rather than by local density fluctuation in a pump.

Let's consider an installation composed of a pump, a suction pipe and a discharge pipe that is considerably longer than a suction pipe. A dredging operation is monitored from the beginning to the end of a transportation cycle. The short-time fluctuations of the density of transported mixture can be neglected. The following stages of a pump-pipeline operation are of importance:

1. the beginning of a cycle: only water flows through the suction pipe and the discharge pipe
2. the beginning of a soil excavation process: the suction pipe and the pump are filled with mixture, the discharge pipe is still filled with water only
3. the mixture transportation: both the suction and the discharge pipes are filled with mixture
4. the end of a cycle: the suction pipe and the pump are filled with water, the discharge pipe is filled with mixture.

7.5.1 Pump operation within a range of the constant speed

If a pump operates within a range of the constant speed during an entire transportation cycle a position of a working point varies for the four different stages (described above) in a way displayed on Fig. 7.13.

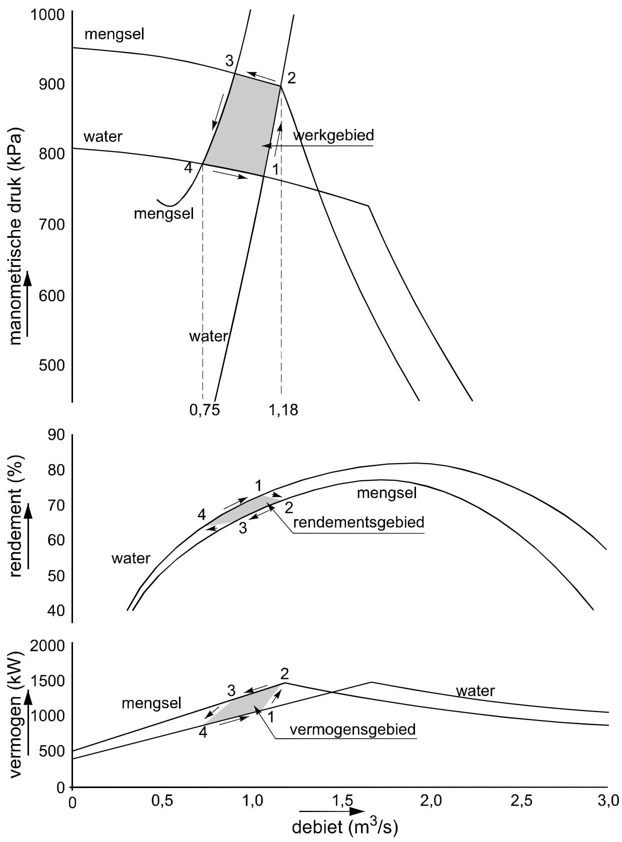


Figure 7.13. Working range within a constant speed region of a dredge pump.

An area defined by points 1,2,3 and 4 on Fig. 7.13 gives the working range.

7.5.2 Pump operation within a range of the constant torque

Shifting of a working point during a cycle described by steps 1 to 4 is different from that for a pump operating in a constant-speed regime (compare Figs. 7.13 and 7.14).

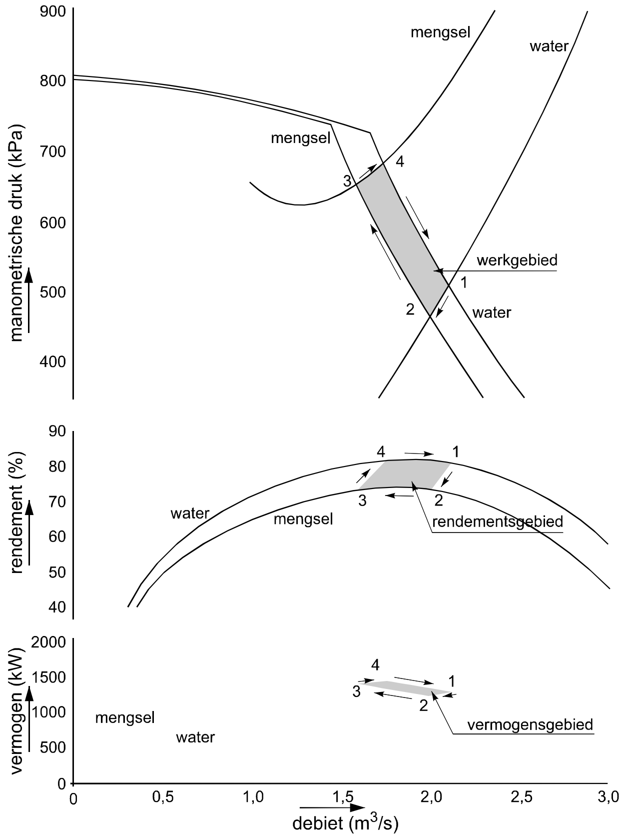


Figure 7.14. Working range within a constant torque region of a dredge pump.

7.5.3 Pump operation within a range around the nominal torque point

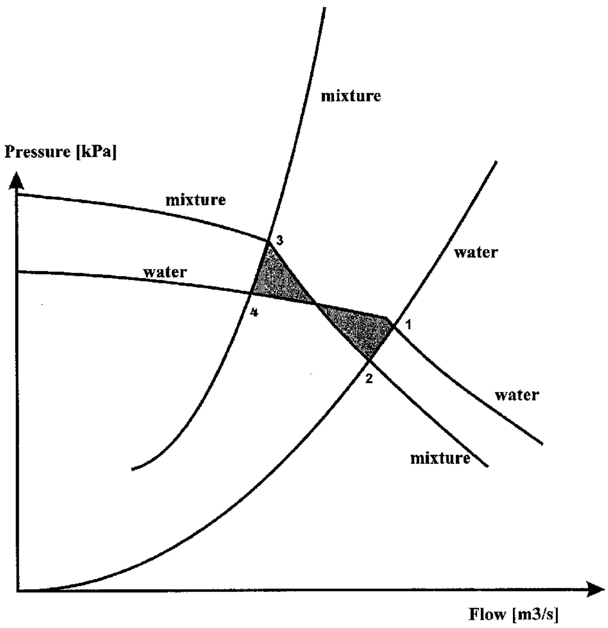


Figure 7.15. Working range round a nominal torque point.

Shifting of a working point during a cycle described by steps 1 to 4 is different from that for pumps operating either in the constant-speed regime or in the constant-torque regime (Fig. 7.15).

Let's compare operations of a certain installation (pumping only water for this particular example) if a pipeline length changes and other parameters remain constant (Fig. 7.16). The resistance curve R_2 is for a pipeline of an original length, R_1 for a longer pipeline and R_3 for a shorter pipeline.

The Fig. 7.16 shows that pumping through a longer pipeline (R1) is associated with the drop in the output power than pumping through a pipeline of an original length (R2) if operation realizes within a constant-speed region of a pump. However, a shortening of the pipeline (R3) can lead to similar drop in the output power the working point shifts to the constant torque line. In the constant-torque regime the engine speed decreases in order to avoid overloading of the motor.

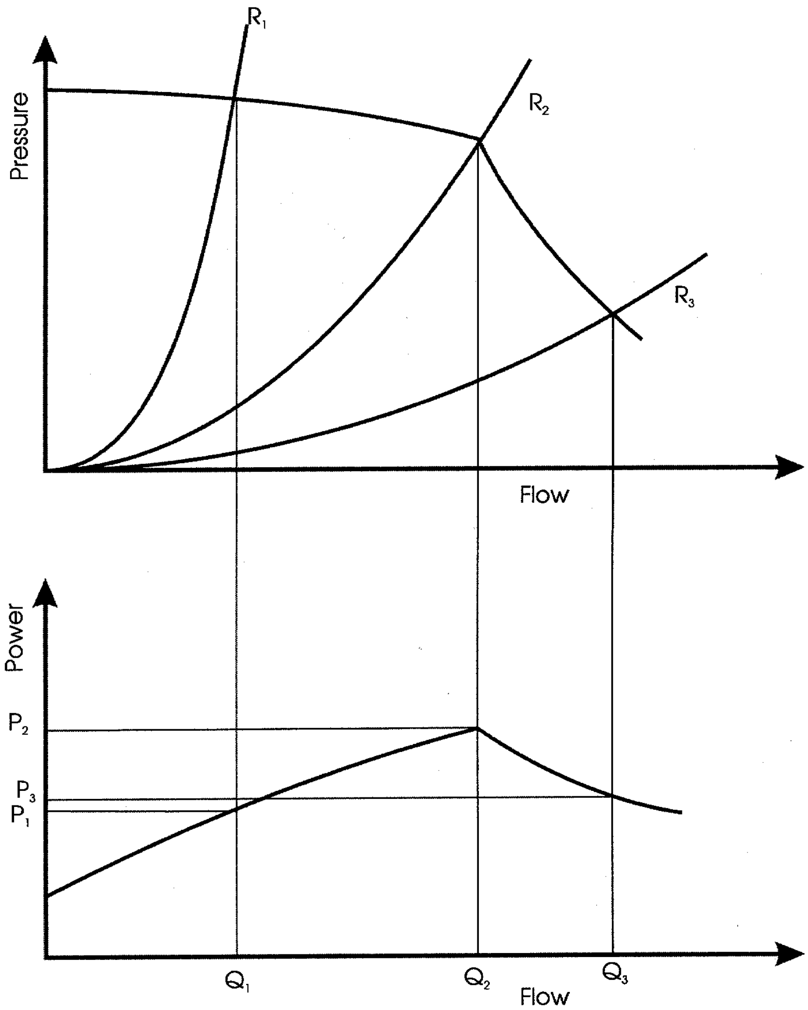


Figure 7.16. Working points for systems of different pipeline lengths. (*Legend*: R1 longer pipe, R2 original length, R3 shorter pipe)

7.6 OPERATION UNDER THE CONDITION OF CONTINUOUSLY FLUCTUATING DENSITY OF MIXTURE

A pump of a dredging installation reacts on short-time fluctuations of density of mixture passing through the pump (Figs. 7.17, 7.18). Thus the manometric head provided by the pump fluctuates in time as fluctuates the mixture density. The variation in the manometric head should lead to variation in mixture velocity in a pipeline connected with a pump. However, an effect of the discharge pressure fluctuation on the flow conditions in a discharge pipeline depends on a length of the pipeline. At each moment a working point of a pump-pipeline installation is determined by the mean flow conditions (average mixture density, see Fig. 7.17) in an entire pipeline rather than by local flow conditions in a pump.

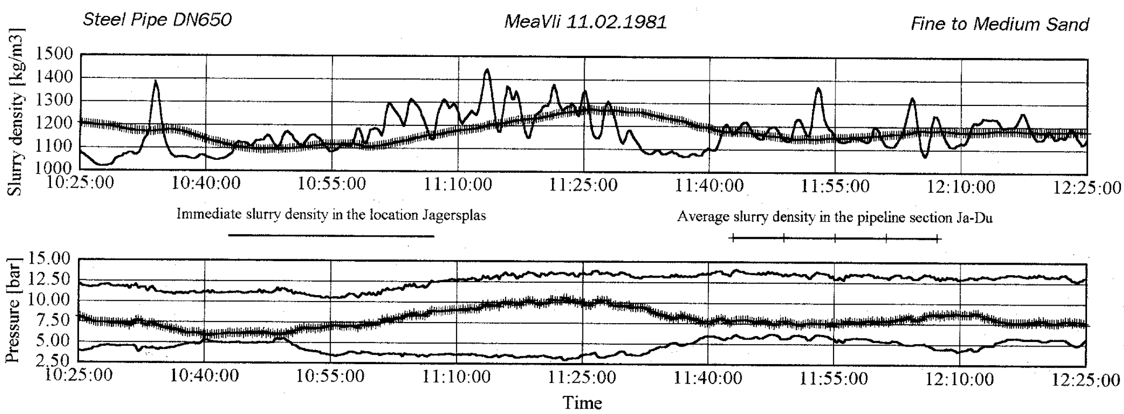


Figure. 7.17. Immediate slurry density in a pump (Ja), immediate pressure at the beginning (Ja) and at the end (Du) of a pipeline section (Ja-Du).
Average mixture density and pressure drop in the pipeline section (Ja-Du) [++++].

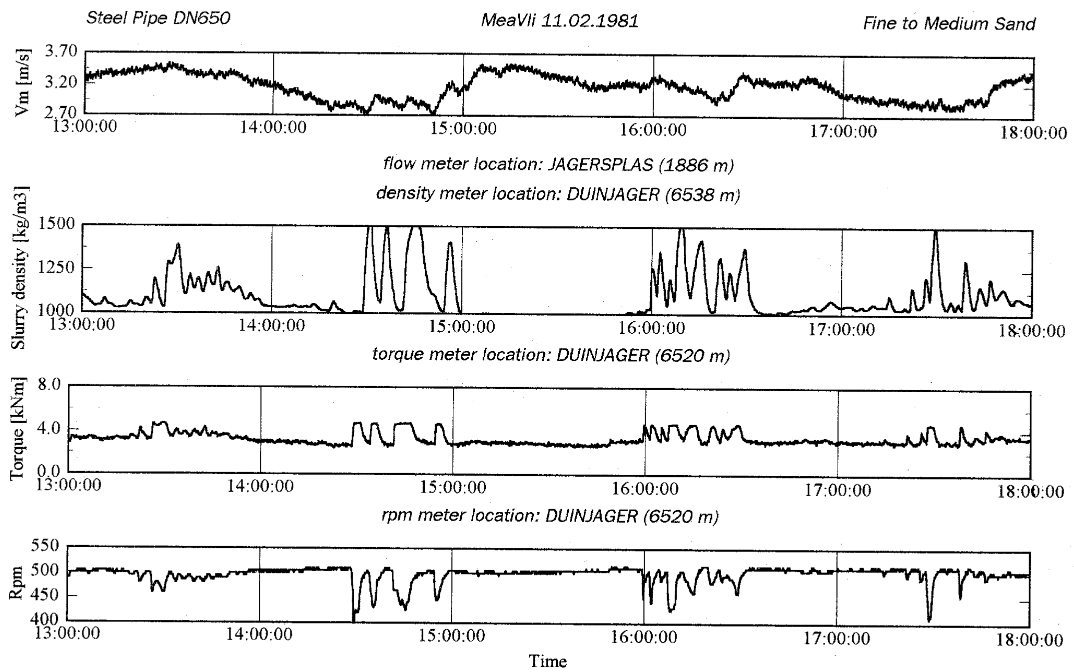


Figure. 7.18. Effect of mixture density fluctuation on torque and speed of a booster pump (Du) (after Matousek, 1997).

This contradiction between momentarily large density fluctuations in a pump and small changes in a mean density of mixture over an entire pipeline length (see Fig. 7.18, 7.19 or 7.12) makes a prediction and a control of processes in a dredging installation insecure. This is particularly the case if more pumps are installed in series in a transportation system.

7.7 OPERATION UNDER THE CONDITION OF FLUCTUATING DENSITY OF MIXTURE AND MEAN PARTICLE SIZE IN A PIPELINE

Fig. 7.19 shows a variation of a working-point position in a pump-pipeline system under the condition of changing mean mixture density (marked here as γ_{gem} [that is ρ_m]) and particle size (normaal materiaal/grover materiaal). The system operates within a region of a constant torque of a pump drive. The working-point variation is plotted in H_{man} [mwc, meter water column] versus Q [litre/second] co-ordinates.

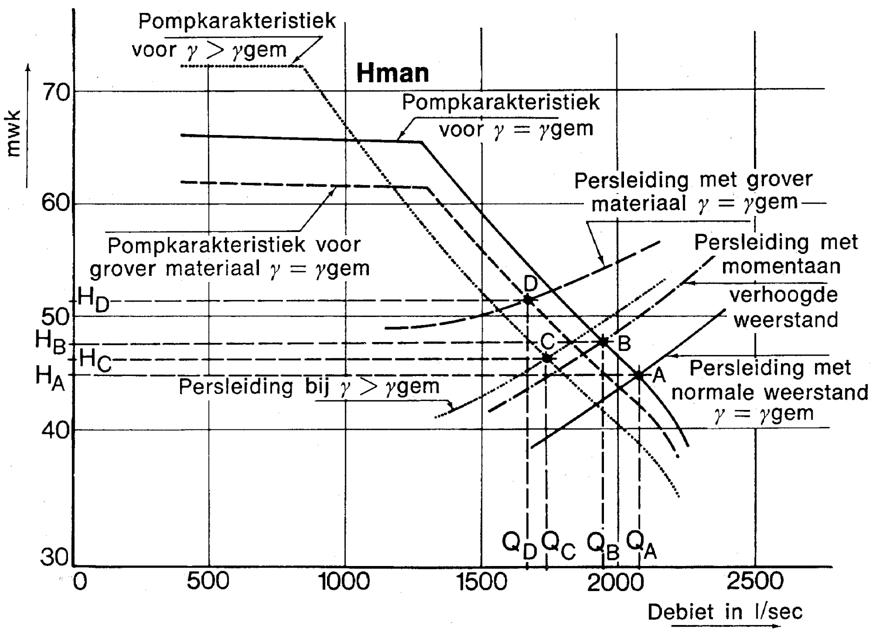


Figure 7.19. Variation of working point under various mixture flow conditions in a pump-pipeline system.

The flow rate through a system drops if mixture density or particle size of transported solids increases in a system.

7.8 EFFECT OF IMPELLER PARAMETERS ON WORKING POINT OF A SYSTEM

If a pipeline of a pump-pipeline system becomes very short the working point of the system reaches the smoke limit of a diesel drive and the drive collapses. This can be avoided by replacing the impeller of a pump if a pipeline becomes short. The use of a smaller impeller or of an impeller with fewer blades causes a shift (from A to B in Figs. 7.21 and 7.21) of a working point to a position far above the smoke-limit point.

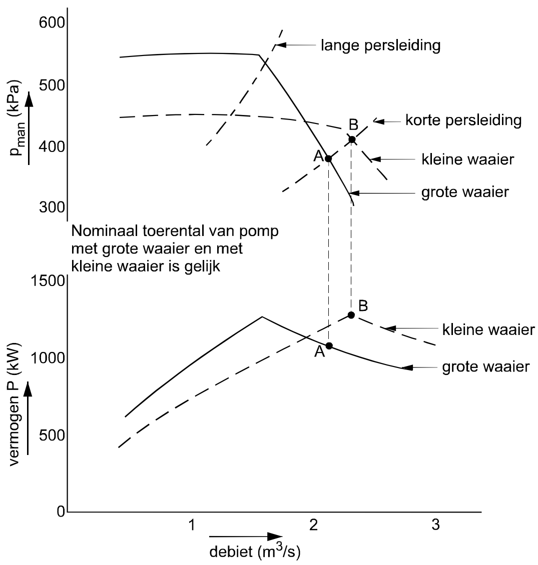


Figure 7.20. Effect of a use of smaller impeller in a system with a shorter pipeline.

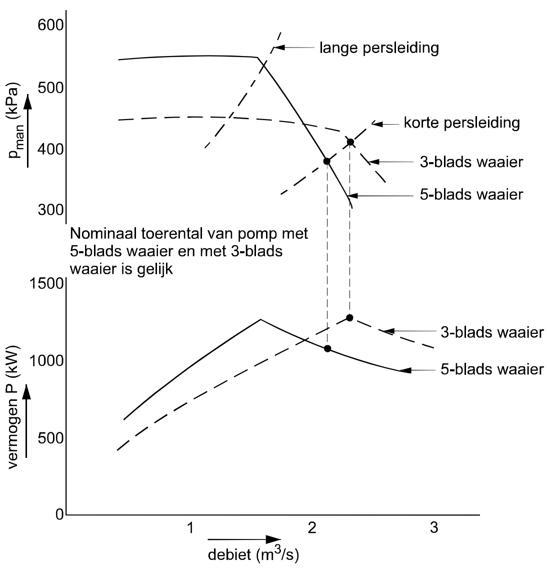


Figure 7.21. Effect of a use of impeller with fewer blades in a system with a shorter pipeline.

7.9 REFERENCES

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7.10 RECOMMENDED LITERATURE

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CASE STUDY 7.1

A deep dredge has a centrifugal pump on board. The heart of the pump is on the same geodetic height as the water level. The suction and the discharge pipes are mounted to the pump at the pump-heard level. The suction pipe of the dredge is vertical and the discharge pipe is horizontal. Both pipes have a diameter 500 mm. The dredge pump pumps the 0.2-mm sand from the bottom of the waterway that is 7 meter below the water level (thus the dredging depth is 7 meter). The density of a pumped sand-water mixture is 1400 kg/m^3. The discharge pipe is 750 meter long. The pump-pipeline installation is supposed to keep the production at 700 cubic meter of sand per hour.

1. Determine the manometric pressure (manometric head) that the pump must deliver to ensure the required production of the sand for mixture of the density 1400 kg/m^3.

2. What is the equivalent manometric pressure of the pump for water service at the same flow rate? This enables to place the working point to the eventually available pump characteristic H-Q for water service.

3. Assume that the plotting of the working point into the H-Q nomograph of the pump revealed that the working point corresponds with the pump speed 400 rpm (the point lays on the H-Q curve for 400 rpm). The maximum speed with which an engine can provide the pump is 450 rpm. What would be the flow rate and the manometric pressure if the maximum speed would be installed?

For the calculation consider the friction coefficient of the suction/discharge pipes λ = 0.011. The following minor losses must be considered:

- the inlet to the suction pipe: $\xi = 0.5$,
- the 90-deg bend in suction pipe: $\xi = 0.1$,
- several flanges in suction/discharge pipes: $\xi = 0.3$,
- the outlet from the discharge pipe: $\xi = 1.0$.

Additional inputs:

$\rho_f = 1000$ kg/m^3
$\rho_s = 2650$ kg/m^3

Inputs:

$\Delta h_{depth} = 7$ m
$L_{hor} = 750$ m
$D = 500$ mm
$d_{50} = 0.20$ mm
$\rho_s = 2650$ kg/m^3, $\rho_f = 1000$ kg/m^3, $\rho_m = 1400$ kg/m^3
$\lambda_f = 0.011$, $\Sigma\xi = 1.9$
$Q_s = 700$ m^3/hour = 0.194 m^3/s

Remark: To make a calculation simpler the effect of a pipeline roughness on frictional losses in a pipeline is considered to be represented by a constant value of the frictional coefficient λ_f, i.e. independent of variation of mean mixture velocity.

1. Determination of the manometric pressure for mixture service

Mean velocity of mixture in a pipeline, V_m:

$$C_{vd} = \frac{\rho_m - \rho_f}{\rho_s - \rho_f} = \frac{1400-1000}{2650-1000} = 0.2424 \quad [-],$$

$$Q_m = \frac{Q_s}{C_{vd}} = \frac{0.1944}{0.2424} = 0.802 \quad m^3/s,$$

$$V_m = \frac{4Q_m}{\pi D^2} = \frac{4 \ x \ 0.802}{3.1416 \ x \ 0.5^2} = 4.085 \quad m/s.$$

Manometric pressure of the pump, P_{man}:

There is a balance between
the pressure difference among the inlet and the outlet of the installation:
$P_{inlet} - P_{outlet}$ and the pressure drop over the total length of a pipeline:
$\Delta P_{static} + \Delta P_{totloss,m} - P_{man}$.

ΔP_{static} is the static pressure differential between the inlet and the outlet;
$\Delta P_{totloss,m}$ is the total pressure loss (both major and minor) over the length of a pipe;
P_{man} is the manometric pressure of the pump.

The balance reads
$P_{inlet} - P_{outlet} = \Delta P_{static} + \Delta P_{totloss,m} - P_{man}$,

in which $P_{inlet} = P_{atm} + \Delta h_{depth} \cdot \rho_f \cdot g$,
$\Delta P_{static} = \Delta h_{depth} \cdot \rho_m \cdot g$,
$P_{outlet} = P_{atm}$.

Thus $P_{man} = \Delta h_{depth}(\rho_m - \rho_f)g + \Delta P_{totloss,m}$.

The pressure drop due to losses:

$\Delta P_{totloss,m} = \Delta P_{major,m} + \Delta P_{minor,m}$.

Minor loss: $\Delta P_{minor,m} = \Sigma\xi \frac{V_m^2}{2} \rho_m = 1.9 \frac{4.09^2}{2} 1400 = 22.2$ kPa.

Major loss:

vertical pipe: the equivalent-liquid model: $\Delta P_{vert,m} = \Delta P_{vert,f} \, \rho_m$ [Pa].

The Darcy-Weisbach equation:

$$\Delta P_{vert,f} = \lambda_f \frac{\Delta h_{depth}}{D}\frac{V_m^2}{2}\rho_f = 0.011\frac{7}{0.5}\frac{4.09^2}{2}1000 = 12.9 \text{ kPa}$$

$\Delta P_{vert,m} = \Delta P_{vert,f}\,\rho_m =$ 12880 x 1400 = 18.0 kPa.

horizontal pipe: the Wilson model: $\Delta P_{hor,m} = fn(d, D, C_{vd}, \rho_s, \rho_f, \Delta P_{pipe,f})$ [Pa].

The Wilson model:

$$V_{50} \approx 3.93(d_{50})^{0.35}\left(\frac{S_s - 1}{1.65}\right)^{0.45} = 3.93(0.20)^{0.35}1 = 2.24 \text{ m/s.}$$

$$\frac{I_m - I_f}{C_{vd}(S_s - 1)} = 0.22\left(\frac{V_m}{V_{50}}\right)^{-1.7} \Rightarrow \frac{I_m - I_f}{C_{vd}(S_s - 1)} = 0.22\left(\frac{4.09}{2.24}\right)^{-1.7} = 0.079 \text{ [-].}$$

$\Delta P_{hor,m} = 0.079 x C_{vd} (S_s - 1) g \rho_f L_{hor} + \Delta P_{hor,f}$ [kPa].

$$\Delta P_{hor,f} = \lambda_f \frac{L_{hor}}{D}\frac{V_m^2}{2}\rho_f = 0.011\frac{750}{0.5}\frac{4.09^2}{2}1000 = 138.0 \text{ kPa}$$

$\Delta P_{hor,m}$ = 0.079 x 0.2424 x (1.65-1) x 9.81x1000 x 750 + 138 000 = 229.6 kPa.

$\Delta P_{totloss,m} = \Delta P_{hor,m} + \Delta P_{vert,m} + \Delta P_{minor,m}$ = 229.6 + 18.0 + 22.2 = 269.8 kPa

Thus
P_{man} $= \Delta h_{depth}(\rho_m - \rho_f)g + \Delta P_{totloss,m} =$ 7(1400-1000)9.81+269800 = **297.3 kPa**.

The manometric pressure that the pump must deliver to maintain the required production is 297 kPa, i.e. 3 bar. The pump provides this manometric pressure at the flow rate of mixture 0.802 m^3/s.

2. Determination of the manometric pressure for water service

Equivalent manometric pressure for water service, $P_{man,f}$:

$$P_{man,f} = \frac{P_{man,m}}{\frac{\rho_m}{\rho_f}\left[1 - C_{vd}\left(0.8 + 0.6\log d_{50}\right)\right]} = \frac{297}{\frac{1400}{1000}\left[1 - 0.2424\left(0.8 + 0.6\log 0.2\right)\right]},$$

$P_{man,f}$ = 233.7 kPa.

The working point for water service is:

$P_{man,f}$ = 233.7 kPa at Q_m = 0.802 m^3/s.

Further, it is assumed that this working point holds for pump operation at 400 rpm.

3. Determination of the working point for another speed of the pump

The affinity laws: $\frac{Q_{m,n1}}{Q_{m,n2}} = \frac{n_1}{n_2}$, $\frac{P_{man,n1}}{P_{man,n2}} = \left(\frac{n_1}{n_2}\right)^2$.

The working point at 450 rpm:

$\frac{0.802}{Q_{m,450}} = \frac{400}{450}$, thus $Q_{m,450} = 0.902$ m^3/s.

$\frac{233.7}{P_{man,f,n2}} = \left(\frac{400}{450}\right)^2$, thus $P_{man,f,450} = 295.7$ kPa.

The working point for water service at 450 rpm is:

$P_{man,f} = 295.7$ kPa at $Q_m = 0.902$ m^3/s.

$$P_{man,m,450} = P_{man,f,450} \frac{\rho_m}{\rho_f}\left[1 - C_{vd}\left(0.8 + 0.6 \log d_{50}\right)\right] =$$

$$= 295.7 \frac{1400}{1000}\left[1 - 0.24\left(0.8 + 0.6 \log 0.2\right)\right] = 375.8 \text{ kPa.}$$

The working point for mixture service at 450 rpm is:

$P_{man,m} = 375.8$ kPa at $Q_m = 0.902$ m^3/s.

CASE STUDY 7.2

A dredging installation transports a mixture of narrow-graded medium sand (d_{50} = 0.30 mm, ρ_S = 2650 kg/m^3) and lake water (ρ_f = 1000 kg/m^3) from a dredging pit to a construction site. The installation is composed of an on-board pump (a centrifugal pump IHC 125-27.5-50, see pump characteristics in Tab. C7.1) and a pipeline of the diameter 500 millimetre. Fig. C7.0 shows a lay-out of the installation. The dredging depth is 15 meter. A suction pipeline is inclined under the angle 45 deg (the pipe length 21 meter) and horizontal (a 2 meter long section in front of a pump suction mouth). The centre of the pump is at the water-level position. A discharge pipeline is horizontal and its geodetic position is considered identical with a water level along its entire length (an elevation of a discharge pipeline is zero). The discharge pipeline is composed of a 200 meter long floating pipeline and of an on-shore pipeline of a variable length. During a dredging operation the average density of pumped mixture is 1412.5 kg/ m^3.

The dredged material has to be delivered to a construction site of a quite large area. Therefore a length of a discharge pipeline will vary during an operation. Determine the maximum length of a pipeline attainable when pumping mixture of the above required density. What will be the flow rate through a pipeline of a maximum length?

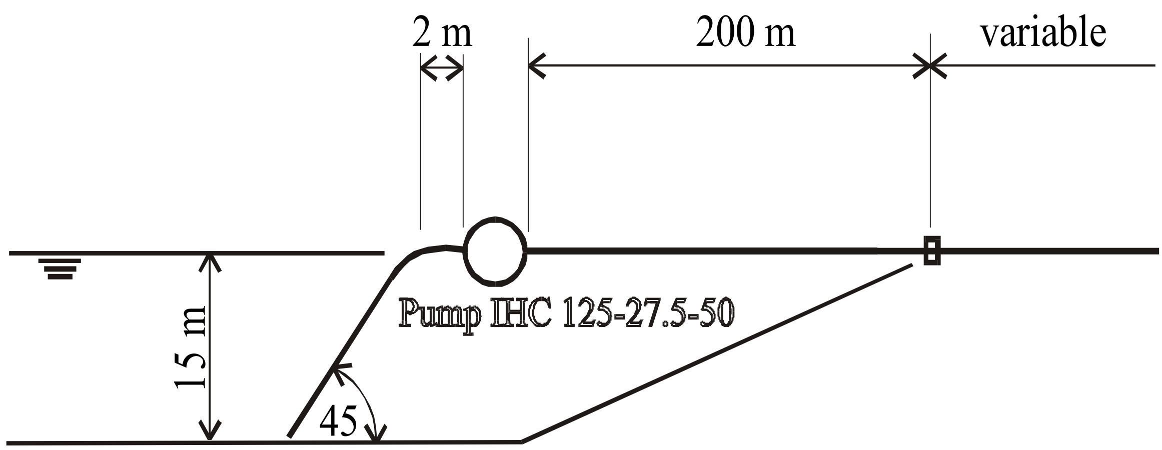

Figure C7.0. Schematic lay-out of a pump-pipeline system.

INPUTS:

Lay-out of a pump-pipeline system (see also Fig. C7.0):

Δh_{depth} = 15 m
ω = 45 deg
$L_{horiz,suction}$ = 2 m
$L_{horiz,floating}$ = 200 m
$L_{horiz,shore}$ = variable
D = 500 mm

Mixture flow characteristics:

d_{50} = 0.30 mm
ρ_s = 2650 kg/m^3, ρ_f = 1000 kg/m^3, ρ_m = 1412.5 kg/m^3 (i.e. C_{vd} = 0.25)
λ_f = 0.011

Remark: To make a calculation simpler the effect of a pipeline roughness on frictional losses in a pipeline is considered to be represented by a constant value of the frictional coefficient λ_f , i.e. independent of variation of mean mixture velocity.

Pump & drive parameters:

The dredging pump: type IHC 125-27.5-50
the 5-blades impeller of a diameter 1250 mm and a breadth 275 mm
the diameter of a pump inlet: 500 mm

Table C7.1. ***Input to Case study 7***
Characteristics of the pump IHC 125-27.5-50 if pumping water at the maximum speed (n_{max} = 475 rpm)

$Q_{m,nmax}$ [m^3/s]	$P_{man,f,nmax}$ [kPa]	$\eta_{f,nmax}$ [%]
0.45	679.9	55.7
0.50	676.7	58.8
0.55	673.4	61.6
0.60	670.1	64.0
0.65	666.7	66.1
0.70	663.2	67.9
0.75	659.7	69.5
0.80	656.1	70.9
0.85	652.5	72.1
0.90	648.8	73.1
0.95	645.0	74.0
1.00	641.0	74.8
1.05	637.0	75.4
1.10	632.9	76.0
1.15	628.7	76.5
1.20	624.4	76.9
1.25	619.9	77.2
1.30	615.3	77.5
1.35	610.6	77.7
1.40	605.7	77.9
1.45	600.7	78.0
1.50	595.6	78.1

Drive parameter:
The maximum power available at the pump shaft is 1000 kW (at the speed 475 rpm).

CALCULATION:

a. Pump characteristics

The $P_{man,f,nmax}$–$Q_{m,nmax}$ curve of the IHC pump pumping water at the maximum speed 475 rpm is a curve fitting the points given in Tab. C7.1. This *constant-speed curve* can be approximated (correlation coefficient Rxy = 1.00, Fig. C7.1) by the equation

$$P_{man,f,n\max} = 702.5 - 42.44Q_{m,n\max} - 19.06Q^{2}_{m,n\max} \text{ [kPa]} \quad \text{(C7.1)}.$$

The *efficiency curve* ($\eta_{f,nmax}$–$Q_{m,nmax}$) of the pump operating at the maximum speed is a curve fitting the points given in Tab. C7.1. This curve can be approximated (correlation coefficient Rxy = 1.00, Fig. C7.1) by the equation

$$\eta_{f,n\max} = 1.953Q_{m,n\max} - 2.0Q^{2}_{m,n\max} + 0.989Q^{3}_{m,n\max} - 0.195Q^{4}_{m,n\max} \text{ [-]} \quad \text{(C7.2)}.$$

Remark: Theoretically, the 3rd order polynomial is sufficient to relate the pump efficiency with the flow rate.

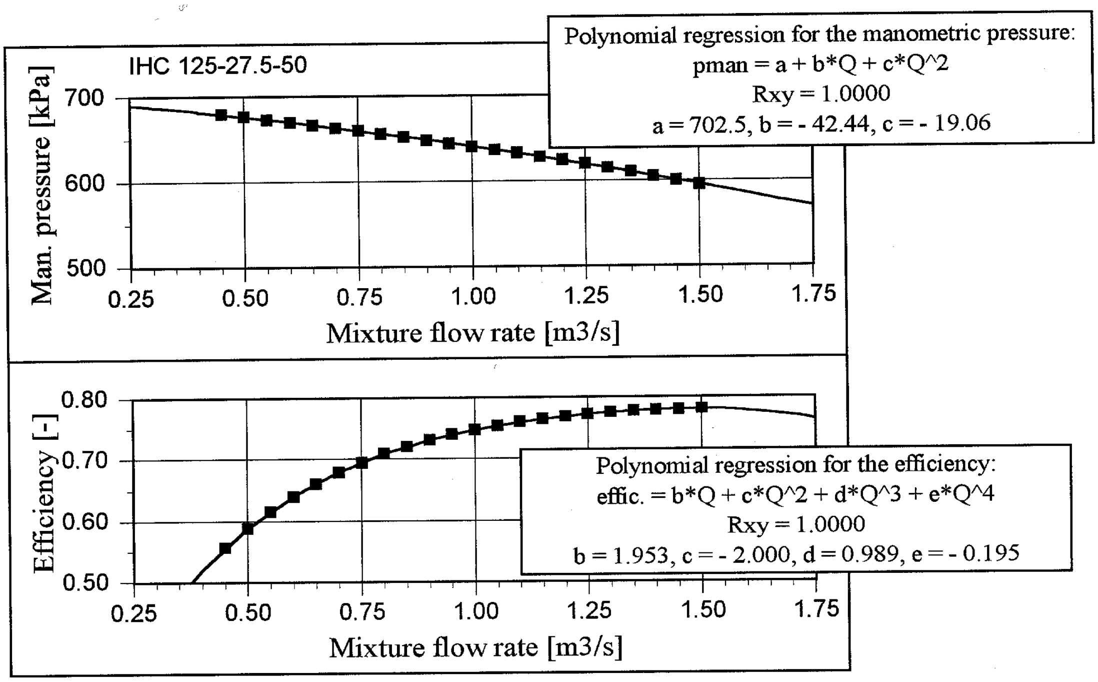

Figure C7.1. Characteristics of the dredging pump IHC 125-27.5-50 at the maximum speed 475 rpm.

The affinity laws

$$\frac{Q_{m,n\,max}}{Q_m} = \frac{n_{max}}{n}, \qquad \frac{P_{man,f,n\,max}}{P_{man,f}} = \left(\frac{n_{max}}{n}\right)^2, \qquad \frac{\eta_{f,n\,max}}{\eta_f} = 1$$

enable to produce the constant-speed curves ($P_{man,f}$–Q_m) and efficiency curves (η_f–Q_m) for different constant speeds, n, of the pump (see Fig. C7.2 for speeds 475, 450, 425, 400 and 375 rpm).

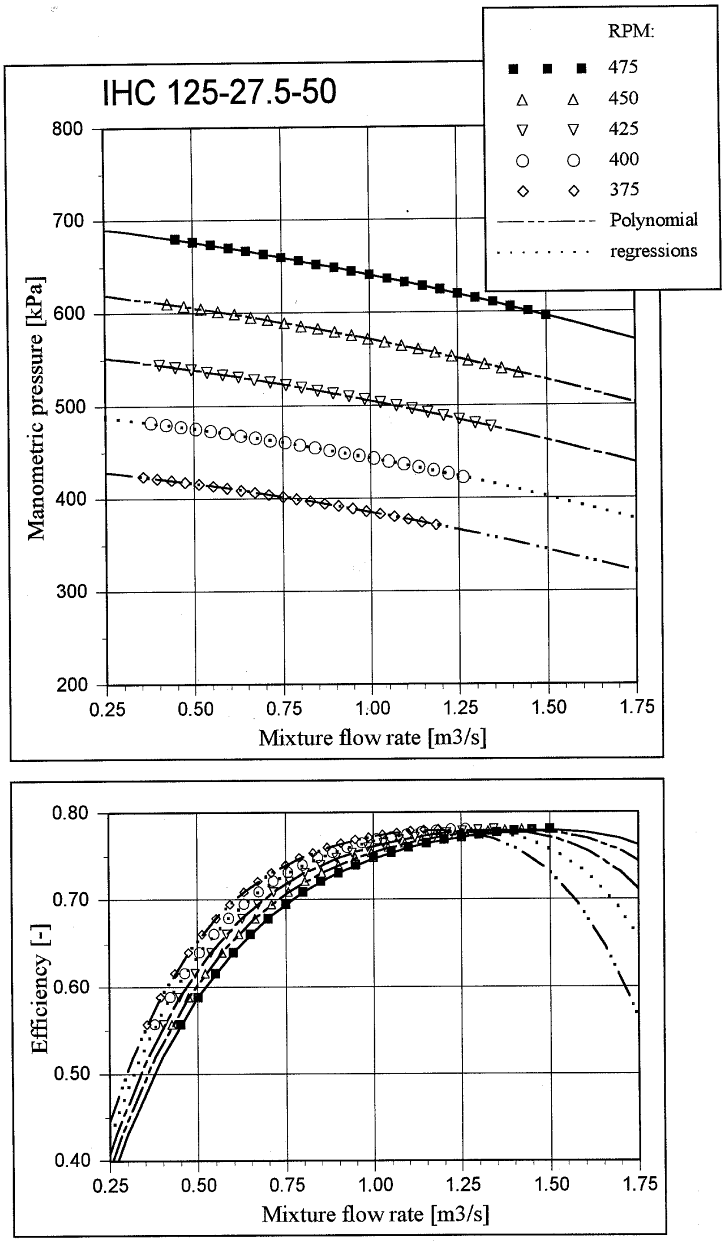

Figure C7.2. Characteristics of the dredging pump pumping water at different constant values of pump speed. (An application of affinity laws).

The power delivered by a drive to the pump is limited by a value $W_{in,max}$ = 1000 kW at the maximum speed n_{max} = 475 rpm. This maximum-power value is reached at the flow rate Q_m = 1.244 m^3/s (the Q_m value is calculated using $\eta_f = P_{man}Q_m/W_{in,max}$ combined with Eqs. C7.1 & C7.2). For higher flow rates the diesel engine, that drives the pump, can not maintain the constant speed and the engine operates at the constant torque. Thus the revolutions of a shaft drop if the flow rate grows above Q_m = 1.244 m^3/s. The constant torque has a value equal to $60W_{in,max}/(2\pi n_{max})$ = 20 104 Nm.

The *constant-torque curve* of a pump-drive set is obtained from Eqs. C7.1 & C7.2 for the condition

$$P_{man,f}\frac{Q_m}{\eta_f}\frac{60}{2\pi n} = W_{in,max}\frac{60}{2\pi n_{max}} \quad \text{where } P_{man,f} = \frac{n^2}{n_{max}^2}P_{man,f,n\,max},$$

$$\frac{Q_{m,n\,max}}{Q_m} = \frac{n_{max}}{n} \quad \text{and} \quad \frac{\eta_{f,n\,max}}{\eta_f} = 1$$

The condition is fulfilled for a set of [$P_{man,f}$, Q_m] points (see Tab. C7.2 and Fig. C7.3) that can be approximated (correlation coefficient Rxy = 0.9976) by the equation

$$P_{man,f} = 20950 - 43120Q_m + 30690Q_m^2 - 7365Q_m^3 \qquad \text{[kPa]} \quad \text{(C7.3).}$$

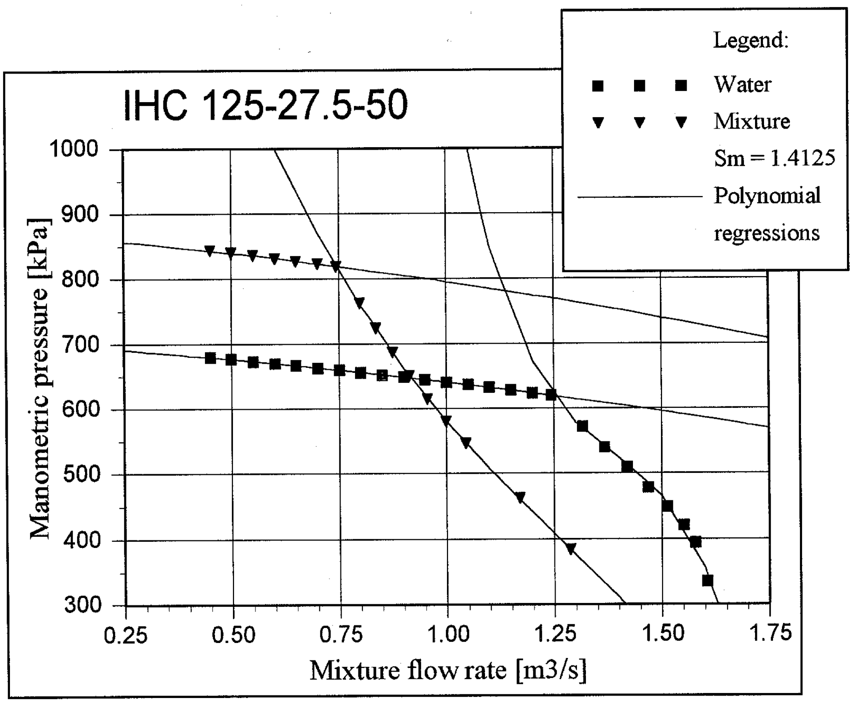

Figure C7.3. Characteristics of the dredging pump IHC 125-27.5-50 when pumping water, mixture of S_m = 1.4125 respectively, in the range of maximum speed of the pump and in the range of constant torque of a drive.

Table C7.2:
Constant torque points for the performance of the pump-drive set pumping water at different pump speeds:

n [rpm]	Q_m [m^3/s]	$P_{man,f}$ [kPa]	η_f [-]
475	1.244	620.21	0.771
460	1.317	571.64	0.777
450	1.368	539.83	0.780
440	1.420	508.53	0.779
430	1.469	478.13	0.776
420	1.514	448.73	0.768
410	1.551	420.72	0.756
400	1.578	394.31	0.739
375	1.606	334.88	0.681

Pump characteristics if mixture is pumped instead of water:
The *constant-speed curve* is obtained (see Eqs. 7.4 & 7.5) from

$$P_{man,m} = P_{man,f} S_m \left[1 - \frac{S_m - 1}{S_s - 1}\left(0.8 + 0.6 \log d_{50}\right) \right] \qquad \text{[kPa]},$$

thus for $d_{50} = 0.30$ mm and $S_s = 2.65$

$$P_{man,m} = P_{man,f} S_m \left[1 - 0.2947\left(S_m - 1\right)\right] \qquad \text{[kPa]} \quad \text{(C7.4)}$$

and for $S_m = 1.4125$

$$P_{man,m} = 1.2408 P_{man,f} \qquad \text{[kPa]}.$$

The *constant-torque curve* fulfils the condition

$$P_{man,f} S_m \frac{Q_m}{\eta_f} \frac{60}{2\pi n} = W_{in,max} \frac{60}{2\pi n_{max}} \quad \text{where } P_{man,f} = \frac{n^2}{n_{max}^2} P_{man,f,n\,max},$$

$$\frac{Q_{m,n\,max}}{Q_m} = \frac{n_{max}}{n} \quad \text{and} \quad \frac{\eta_{f,n\,max}}{\eta_f} = 1.$$

The condition is fulfilled for a set of [$P_{man,m}$, Q_m] points that can be approximated (correlation coefficient Rxy = 1.00) for $S_m = 1.4125$ (see Tab. C7.3 and Fig. C7.3) by the equation

$$P_{man,m} = 2408 - 3457 Q_m + 2205 Q_m^2 - 575.4 Q_m^3 \quad \text{[kPa]} \quad \text{(C7.5)}$$

valid for flow rate values $Q_m > 0.743$ m^3/s.

Table C7.3:
Constant torque points for the performance of the pump-drive set pumping mixture of S_m = 1.4125 at different pump speeds:

n [rpm]	Q_m [m^3/s]	$P_{man,m}$ [kPa]	η_m [-]
475	0.743	819.47	0.609
460	0.798	761.72	0.628
450	0.836	724.08	0.639
440	0.874	687.24	0.649
430	0.914	651.00	0.657
420	0.955	615.45	0.665
410	0.999	580.41	0.672
400	1.045	545.96	0.677
375	1.170	462.26	0.685
350	1.287	384.14	0.671

Remark:

The variable speed in the constant-torque regime of the engine can be approximated (correlation coefficient Rxy = 0.9998) for S_m = 1.4125 (see Tab. C7.3) by the equation

$$n = 723 - 396.1Q_m + 83.08Q_m^2 \qquad \text{[rpm]}$$

valid for flow rate values $Q_m > 0.743$ m^3/s.

b. Pipeline characteristics

b.1 Major losses in a pipeline

Water: Darcy-Weisbach equation (Eq. 1.20):

$$I_f = \frac{\lambda_f}{D}\frac{V_m^2}{2g} = \frac{0.011}{0.5}\frac{V_m^2}{19.62} = 0.00112V_m^2 \qquad \text{[-]},$$

For $V_m = \dfrac{Q_m}{A} = \dfrac{4Q_m}{\pi D^2} = \dfrac{4Q_m}{3.1416\text{x}0.5^2} = 5.09296Q_m$,

thus

$$I_f = 0.02905\,Q_m^2 \qquad \text{[-]} \qquad \text{(C7.6)}.$$

Mixture: Wilson model for heterogeneous flow in horizontal pipe (Eqs. 4.16 - 17):
M = 1.7

$$V_{50} \approx 3.93(d_{50})^{0.35}\left(\frac{S_s - 1}{1.65}\right)^{0.45} = 3.93(0.30)^{0.35}1 = 2.58 \text{ m/s}.$$

$$\frac{I_m - I_f}{(S_m - 1)} = 0.22\left(\frac{V_m}{V_{50}}\right)^{-M} \Rightarrow \frac{I_m - I_f}{(S_m - 1)} = 0.22\left(\frac{V_m}{2.58}\right)^{-1.7}$$

$$I_m = I_f + 1.10199(S_m - 1)\, V_m^{-1.7} \qquad [-],$$

$$I_m = I_f + 0.06924(S_m - 1)\, Q_m^{-1.7} \qquad [-] \qquad (C7.7),$$

for $S_m = 1.4125$

$$I_m = I_f + 0.02856\, Q_m^{-1.7} \qquad [-].$$

Mixture: Wilson model for heterogeneous flow in inclined pipe (Eq. 6.6):
$M = 1.7$
$\gamma = 0.5$
$\omega = 45$ deg

$$\frac{I_{m\omega} - I_f}{I_m - I_f} = (\cos\omega)^{(1+M\gamma)} = (\cos 45)^{1.85} = 0.52668$$

$$I_{m\omega} = I_f + 0.52668(I_m - I_f) \qquad [-] \qquad (C7.8).$$

Pressure drop, $\Delta p_{major,m}$ [Pa], lost in a pipeline section of the length $L_{section}$:

$$\Delta p_{major,m} = I_m \rho_f g L_{section}.$$

Horizontal pipeline:

$$\Delta p_{major,hor,m} = (\text{Eq. C7.6}) + (\text{Eq. C7.7})$$

$$\Delta p_{major,hor,m} = 0.02905\, Q_m^2\, g L_{horiz} + 0.06924(S_m - 1)\, Q_m^{-1.7}\, g L_{horiz} \qquad [kPa],$$

$$\Delta p_{major,hor,m} = 0.28498\, Q_m^2\, L_{horiz} + 0.67924(S_m - 1)\, Q_m^{-1.7}\, L_{horiz} \quad [kPa] \qquad (C7.9),$$

for $S_m = 1.4125$

$$\Delta p_{major,hor,m} = 0.28498\, Q_m^2\, L_{horiz} + 0.28019\, Q_m^{-1.7}\, L_{horiz} \qquad [kPa],$$

Inclined pipeline: $L_{incl} = \dfrac{\Delta h_{depth}}{\sin\omega}$

$$\Delta p_{major,incl,m} = (\text{Eq. C7.6}) + (\text{Eq. C7.8})$$

$$\Delta p_{major,incl,m} = 0.02905\, Q_m^2\, g L_{incl} + 0.52668[0.06924(S_m - 1)\, Q_m^{-1.7}]\, g L_{incl} \qquad [kPa],$$

$$\Delta p_{major,incl,m} = 0.28498\, Q_m^2 \frac{\Delta h_{depth}}{\sin\omega} + 0.35774(S_m - 1)\, Q_m^{-1.7} \frac{\Delta h_{depth}}{\sin\omega} \qquad [kPa] \qquad (C7.10),$$

for $S_m = 1.4125$

$$\Delta p_{major,incl,m} = 0.28498\, Q_m^2 \frac{\Delta h_{depth}}{\sin\omega} + 0.14756\, Q_m^{-1.7} \frac{\Delta h_{depth}}{\sin\omega} \text{ [kPa].}$$

b.2 Minor losses in a pipeline

Values of minor-loss coefficients for different pipeline sections:

Suction pipeline:	pipe entrance:	$\xi = 0.4$
	all bends, joints etc.:	$\xi = 0.3$
Floating pipeline:	all bends, joints etc.:	$\xi = 0.8$
Shore pipeline:	*all bends, joints etc.:*	$\xi = 1.5$
Total value:		$\Sigma\xi = 3.0$

Remark:
The coefficient of minor losses for a shore pipeline is further considered constant if the length of a pipeline varies.

Head loss due to friction in fittings (Eq. 7.9):

$$H_{minor,m} = \Sigma\xi \frac{V_m^2}{2g} \frac{\rho_m}{\rho_f} \text{ [mwc], i.e. } \Delta p_{minor,m} = \Sigma\xi \frac{V_m^2}{2} \rho_m \quad \text{[Pa].}$$

$$\Delta p_{minor,m} = \Sigma\xi \frac{V_m^2}{2} S_m = 12.97\, \Sigma\xi\, S_m\, Q_m^2 \quad \text{[kPa]} \quad \text{(C7.11),}$$

for $S_m = 1.4125$ and $\Sigma\xi = 3.0$

$$\Delta p_{minor,m} = 2.12\, V_m^2 = 54.96\, Q_m^2 \quad \text{[kPa].}$$

b3. Static head in a pipeline

The static head that must be overcome by a pump is

$$H_{static} = S_m\, \Delta h_{depth} - S_f\, \Delta h_{depth} \text{ [mwc], i.e.}$$
$$\Delta p_{static,m} = \rho_m g\, \Delta h_{depth} - \rho_f g\, \Delta h_{depth} \quad \text{[Pa],}$$

$$\Delta p_{static,m} = (S_m - 1) \times 9.81 \times \Delta h_{depth} \quad \text{[kPa]} \quad \text{(C7.12),}$$

for $S_m = 1.4125$

$$\Delta p_{static,m} = 0.4125 \times 9.81 \times \Delta h_{depth} = 4.05\, \Delta h_{depth} \quad \text{[kPa].}$$

b4. Total loss in an entire pipeline

$$\Delta p_{totalpipe,m} = \text{(Eq. C7.9)} + \text{(Eq. C7.10)} + \text{(Eq. C7.11)} + \text{(Eq. C7.12)}$$
$$\Delta p_{totalpipe,m} = \Delta p_{major,hor,m} + \Delta p_{major,incl,m} + \Delta p_{minor,m} + \Delta p_{static,m} \quad \text{[kPa]}$$

$$\Delta p_{totalpipe,m} = 0.28498\, Q_m^2\, (L_{horiz} + L_{incl}) + Q_m^{-1.7}\,(S_m - 1)(0.67924 L_{horiz} + 0.35774 L_{incl}) + 12.97\,\Sigma\xi\, S_m\, Q_m^2 + 9.81\,(S_m - 1)\Delta h_{depth} \quad \text{[kPa]}$$

$$\Delta p_{totalpipe,m} = 0.28498\, Q_m^2\, \left(L_{horiz} + \frac{\Delta h_{depth}}{\sin\omega}\right) + Q_m^{-1.7}\,(S_m - 1)\left(0.67924 L_{horiz} + 0.35774 \frac{\Delta h_{depth}}{\sin\omega}\right) + 12.97\,\Sigma\xi\, S_m\, Q_m^2 + 9.81\,(S_m - 1)\Delta h_{depth} \quad \text{[kPa]}$$

(C7.13).

c. Working point of a pump-pipeline system

Balance:

$P_{man,m} = \Delta p_{totalpipe,m}$
(Eq. C7.4) (Eq. C7.13)
or
(Eq. C7.5)

OUTPUTS:

The maximum attainable length of a pipeline:
For $S_m = 1.4125$: $\mathbf{L_{max} = 1150\ m}$ at $\mathbf{Q_m = 0.756\ m^3/s.}$

The maximum pipeline length in a stable operation regime:
For $S_m = 1.4125$: $\mathbf{L_{max} = 975\ m}$ at $\mathbf{Q_m = 0.897\ m^3/s.}$

Table C7.4:

Flow rates at different lengths of an entire pipeline
(see also Fig. C7.4):

S_m [-]	L [m]	Q_m [m^3/s]	n [rpm]
1.4125	400	1.245	359
1.4125	500	1.181	371
1.4125	600	1.121	383
1.4125	700	1.062	396
1.4125	800	1.004	409
1.4125	900	0.944	423
1.4125	950	0.913	431
1.4125	975	0.897	435
1.4125	1000	0.881	439
1.4125	1150	0.756	471

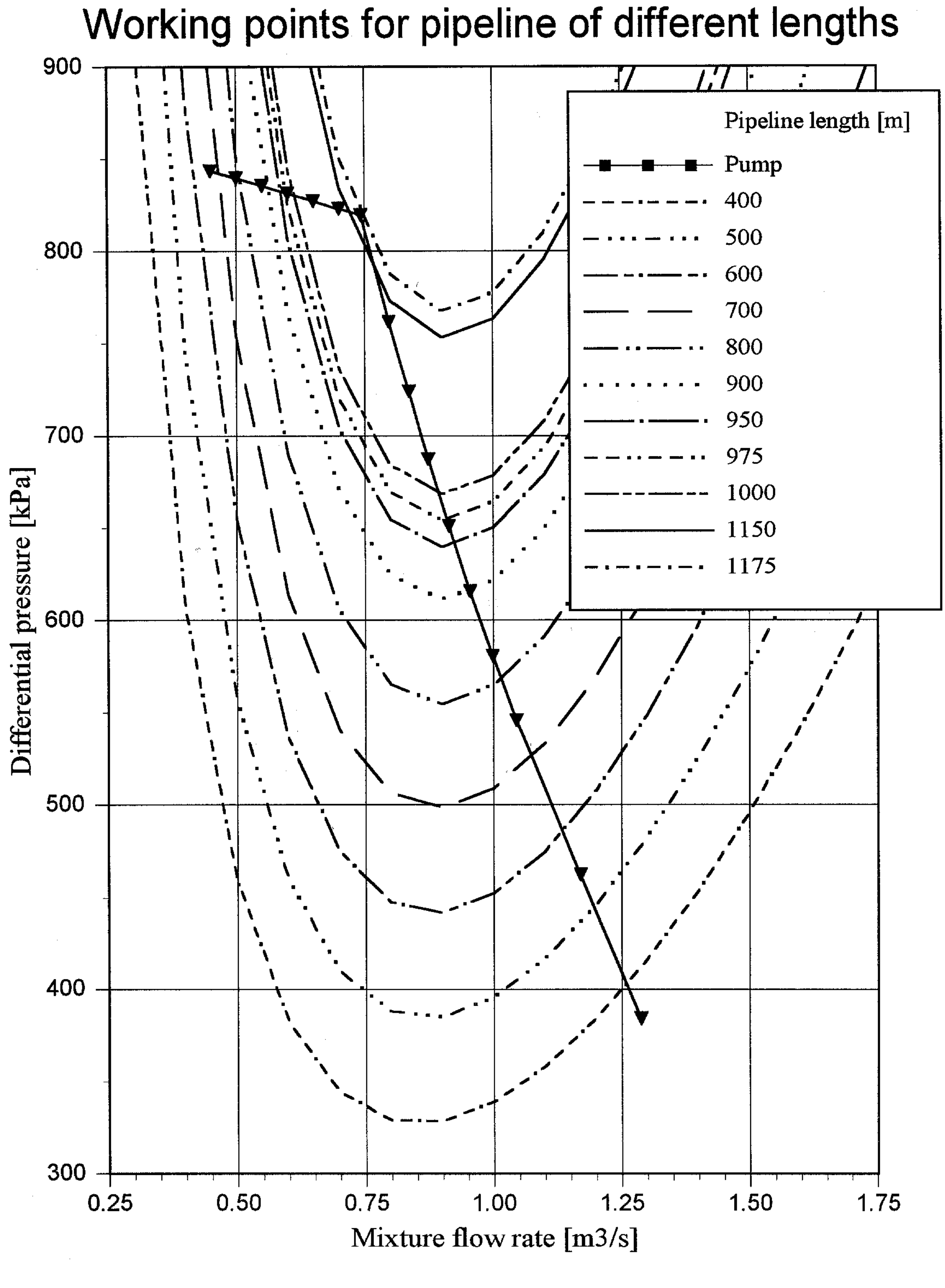

Figure C7.4. Working points of a pump-pipeline system for different lengths of a pipeline. Pump at max. speed (475 rpm) or max. torque.
Pumped mixture of constant density 1412.5 kg/m^3.
Dredging depth: 15 m, pipeline diameter: 500 mm.