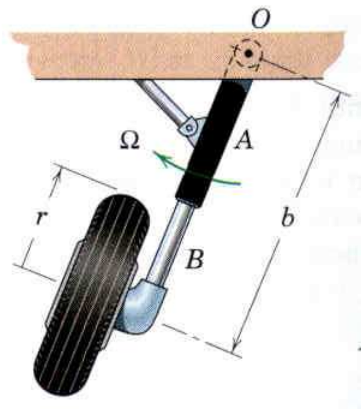
Dynamics and Stability AE3-914

Sample problem—Week 5

Landing gear

Statement

An aeroplane has just taken off with speed v. The spinning wheel has mass m and radius of gyration k about its axle. As seen from the back of the aeroplane, the landing gear is folded into the wing about its pivot O at a constant rate Ω. Calculate the torsional moment in the member AB during the operation, with the proper sign.



Model

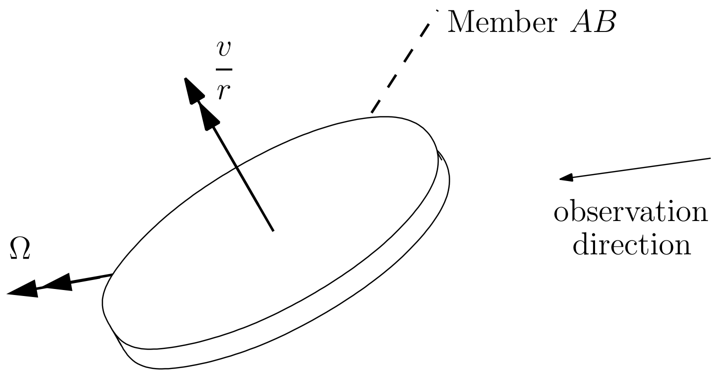
The problem is essentially about the gyroscopic effects due to the spinning wheel. The axle direction, about which the wheel is spinning, is taken as z-axis, which coincides with the third principal direction. One then has

$$I_3 = mk^2. \tag{1}$$

Since no information is provided on the actual shape of the wheel it *cannot* be assumed to be a thin disk. The only information available is that there is axial symmetry about the z-axis and, consequently,

$$I_1 = I_2 = I \quad \text{(unavailable)} \tag{2}$$

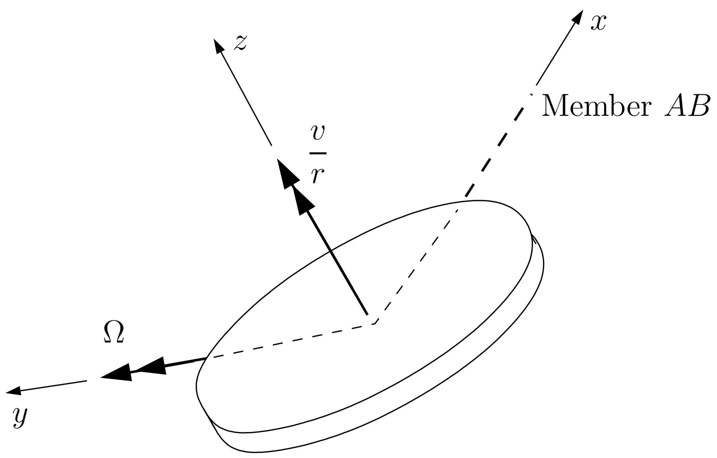
The mass centre of the wheel is translating with speed Ωb relative to the aeroplane. This information is irrelevant to this problem, because the effects asked for are purely related to the rotational contribution of the motion. The rotations of the wheel are schematised as



where it is understood that the vector with magnitude Ω is in the plane of the wheel and perpendicular to member AB, and that the spin vector is orthogonal to the wheel.

Coordinate axes

The z-axis is already chosen perpendicular to the wheel. A natural choice for the x- and y-axes would be to make them coincide with the member AB and the vector with magnitude Ω.



Referring to the handout on kinematics of a spinning body, one can now identify the vectors

$$\mathbf{p} = \frac{v}{r}\mathbf{k}; \qquad \boldsymbol{\Omega} = \Omega\mathbf{j} \tag{3}$$

or, specifying the components,

$$p = \frac{v}{r}; \qquad \Omega_1 = 0; \qquad \Omega_2 = \Omega; \qquad \Omega_3 = 0. \tag{4}$$

This corresponds to a total angular velocity vector $\boldsymbol{\omega}$ with components

$$\omega_1 = 0; \qquad \omega_2 = \Omega; \qquad \omega_3 = \frac{v}{r}. \tag{5}$$

The axes are *attached* to the wheel and the situation sketched above is a *snapshot* at the considered instant.

Angular accelerations

The magnitude of the considered vectors is constant, but the direction is changing and so does the orientation of the axes as a consequence of the spin $\mathbf{p}$. The only vector which will not change during the process is $\boldsymbol{\Omega}$. Consequently, one can write (see handout)

$$\frac{d\boldsymbol{\Omega}}{dt} = \mathbf{0} \tag{6}$$

which is elaborated as (see handout)

$$\frac{d\mathbf{\Omega}}{dt} = \dot{\Omega}_1\mathbf{i} + \dot{\Omega}_2\mathbf{j} + \dot{\Omega}_3\mathbf{j} + \boldsymbol{\omega} \times \mathbf{\Omega} = \mathbf{0} \tag{7}$$

resulting into

$$\dot{\omega}_1 = p\Omega_2; \qquad \dot{\omega}_2 = -p\Omega_1; \qquad \dot{\omega}_3 = 0, \tag{8}$$

which after substituting the values specified in equation (4) yields

$$\dot{\omega}_1 = \frac{v}{r}\Omega; \qquad \dot{\omega}_2 = 0; \qquad \dot{\omega}_3 = 0. \tag{9}$$

Euler equations

The angular velocities and accelerations found in equations (4) and (9) are now substituted into Euler equations

$$\begin{aligned} I_1\dot{\omega}_1 - (I_2 - I_3)\omega_2\omega_3 &= M_1 \\ I_2\dot{\omega}_2 - (I_3 - I_1)\omega_3\omega_1 &= M_2 \\ I_3\dot{\omega}_3 - (I_1 - I_2)\omega_1\omega_2 &= M_3 \end{aligned} \tag{10}$$

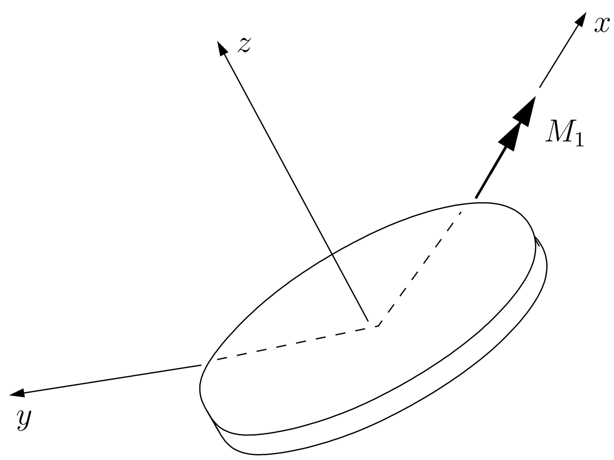
to obtain

$$\begin{aligned} M_1 &= I\frac{v}{r}\Omega - (I - mk^2)\frac{v}{r}\Omega \\ M_2 &= I \cdot 0 - (mk^2 - I)\frac{v}{r} \cdot 0 \\ M_3 &= mk^2 \cdot 0 - (I - I) \cdot 0 \cdot \Omega \end{aligned} \tag{11}$$

which ends up in

$$M_1 = mk^2\frac{v}{r}\Omega; \qquad M_2 = 0; \qquad M_3 = 0. \tag{12}$$

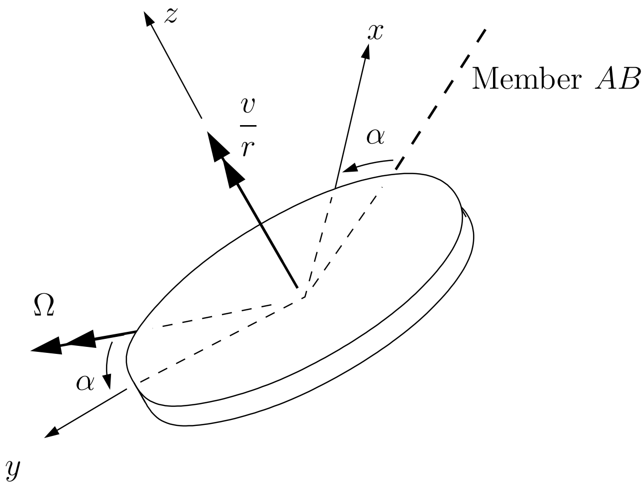
which leads to the following free-body diagram for the wheel



in which a positive torsional moment with magnitude $mk^2\frac{v}{r}\Omega$ is identified in the direction of member AB.

Remark

The reader is invited to carry out the whole calculation again by selecting the x- and y-axes along any other, arbitrary direction in the plane of the wheel forming an angle α with member AB when projecting the angular velocities.



Observe that the same results are obtained for the torsional moment.