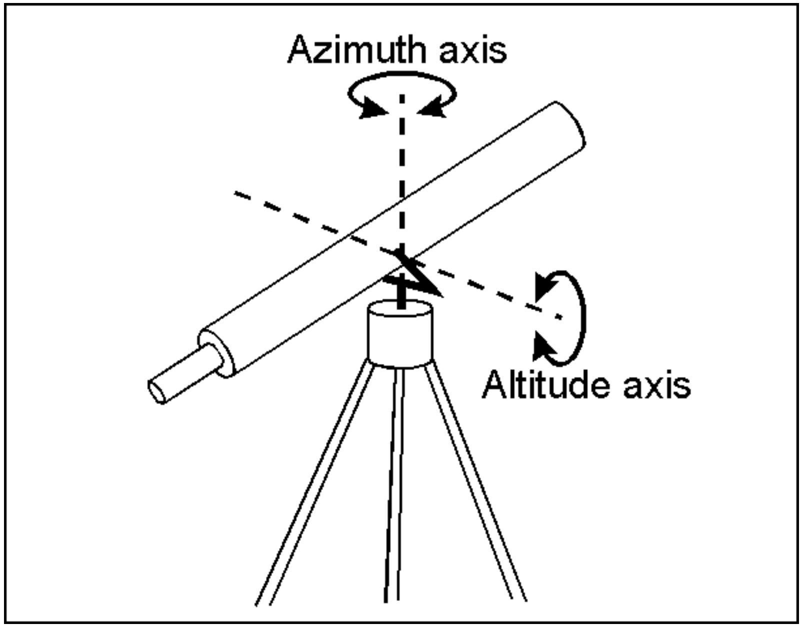
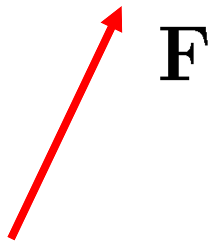
Today:

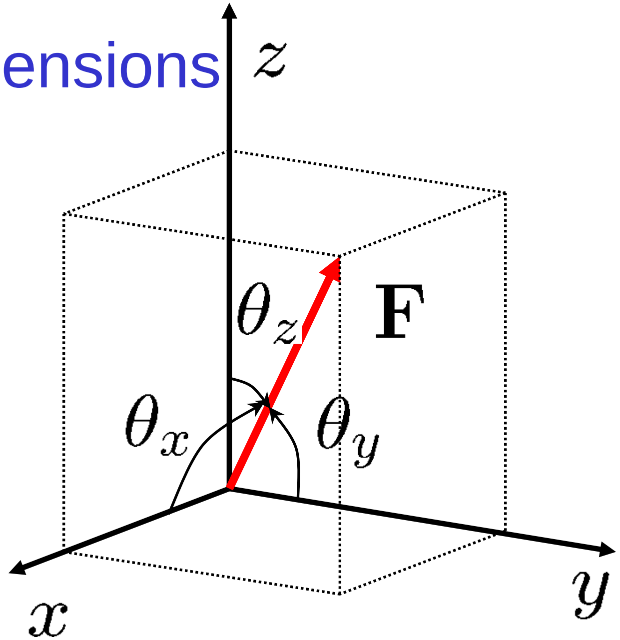
- Forces & moments in 3D

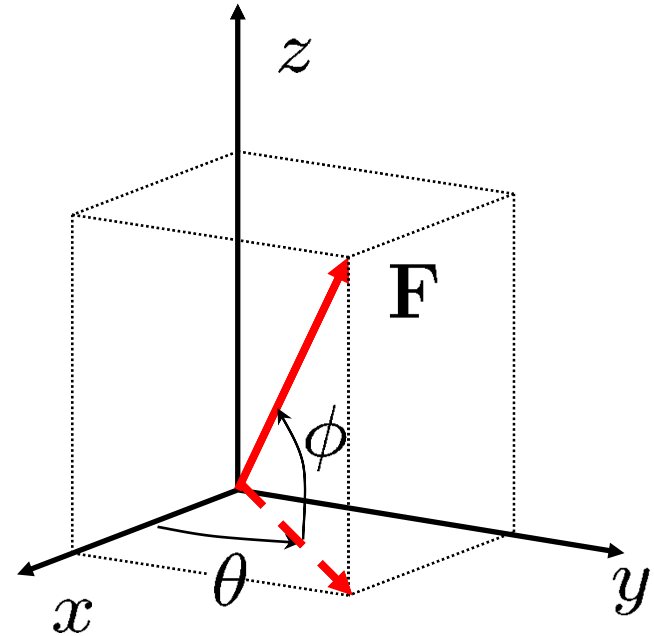
Book: Chapter 2.8-2.9, 3.4, 4.4-4.9 (3D)


Azimuth axis
Altitude axis

# Force in 3 dimensions



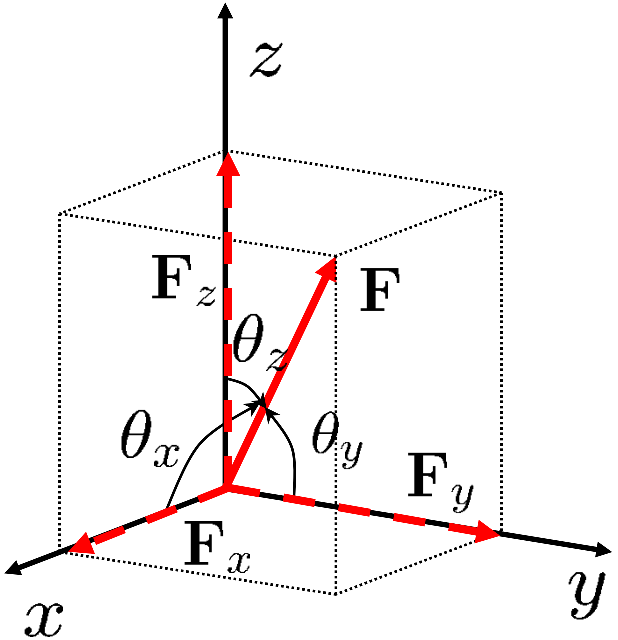
# Force in 3 dimensions

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

$$F_x = F\cos\theta_x$$

$$F_y = F\cos\theta_y$$

$$F_z = F\cos\theta_z$$



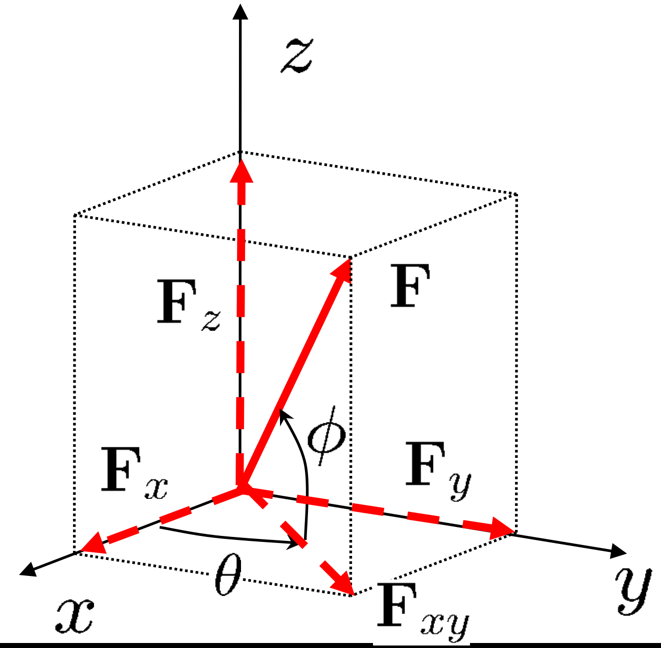
# Force in 3 dimensions

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

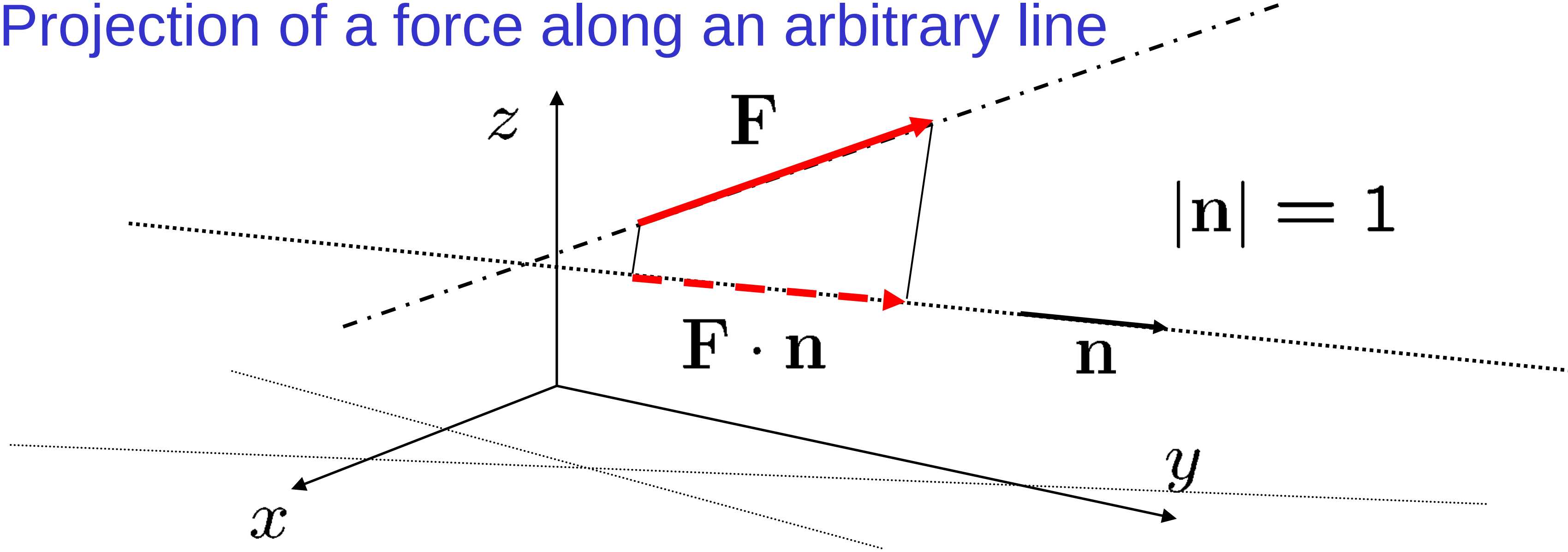
$$F_x = F\cos\phi\cos\theta$$

$$F_y = F\cos\phi\sin\theta$$

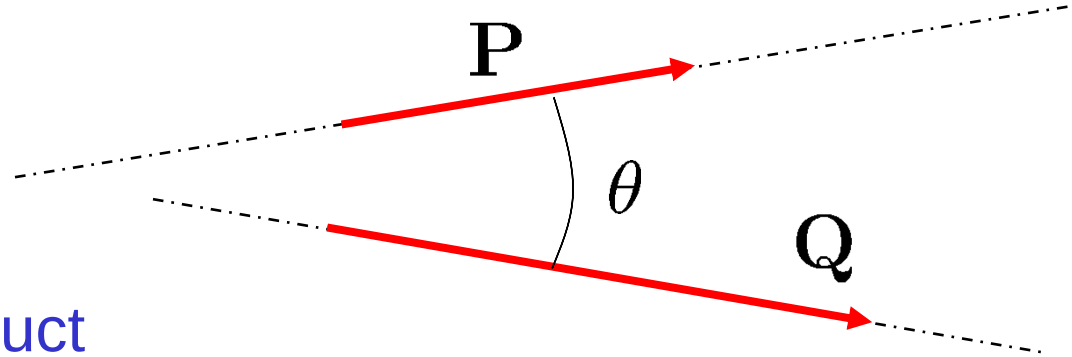
$$F_z = F\sin\phi$$



# Projection of a force along an arbitrary line

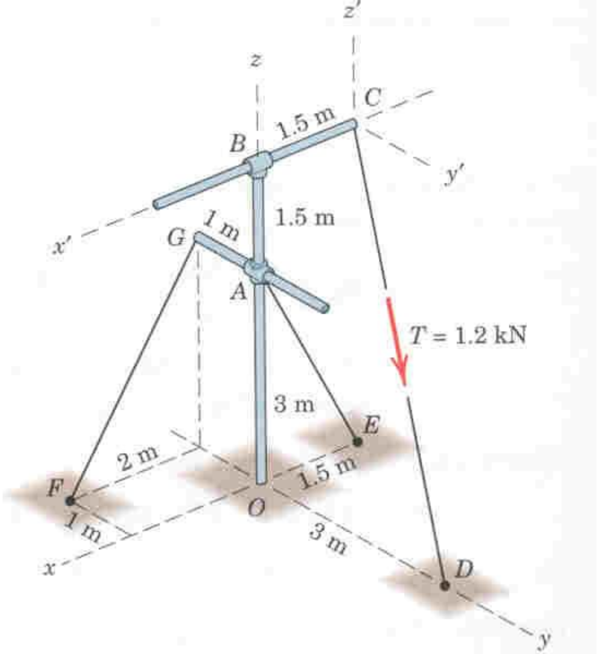


## Angle between two vectors




## Scalar product

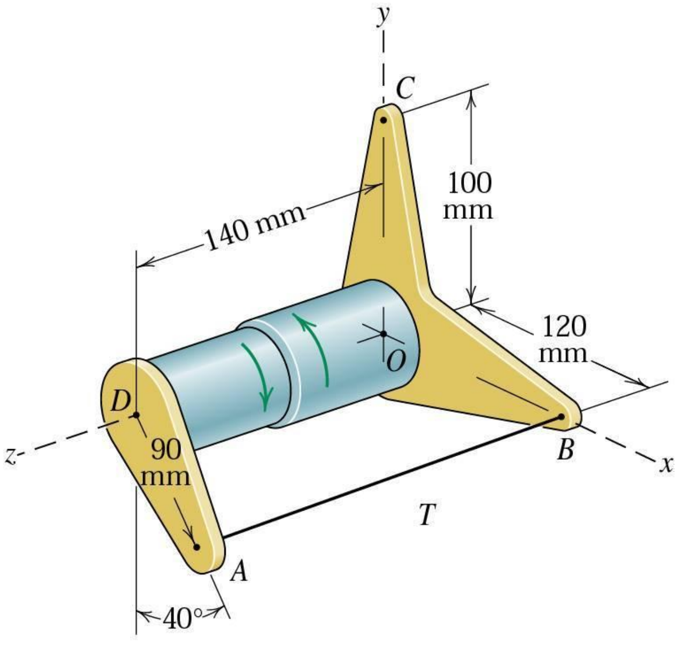
$$\mathbf{P} \cdot \mathbf{Q} = P_x Q_x + P_y Q_y + P_z Q_z = PQ \cos\theta$$




The tension T in the cable CD is equal to 1.2 kN.

**Express T as a vector.**

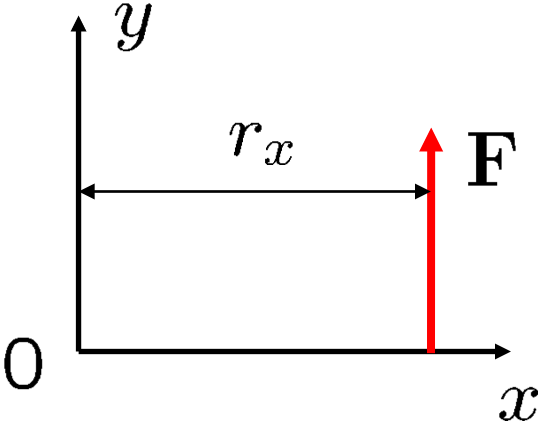
Source: R.C. Hibbeler,
"Engineering Mechanics – Statics"

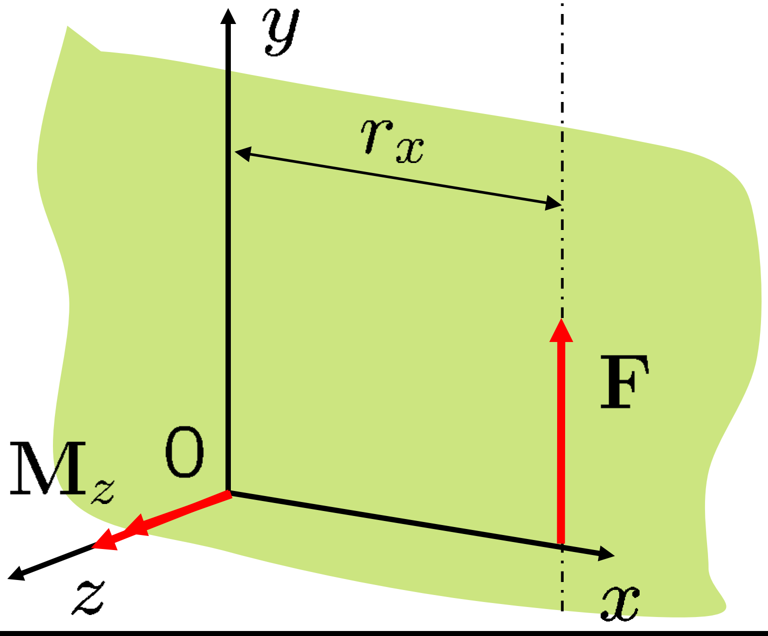



The shafts and attached brackets are twisted in opposite directions to maintain a tension **T** of 500 N in the wire joining *A* and *B*. Express the tension, considered as a force acting on *A*, as a vector and determine the projection of **T** onto the line *DC*.

Source: R.C. Hibbeler, "Engineering Mechanics – Statics"

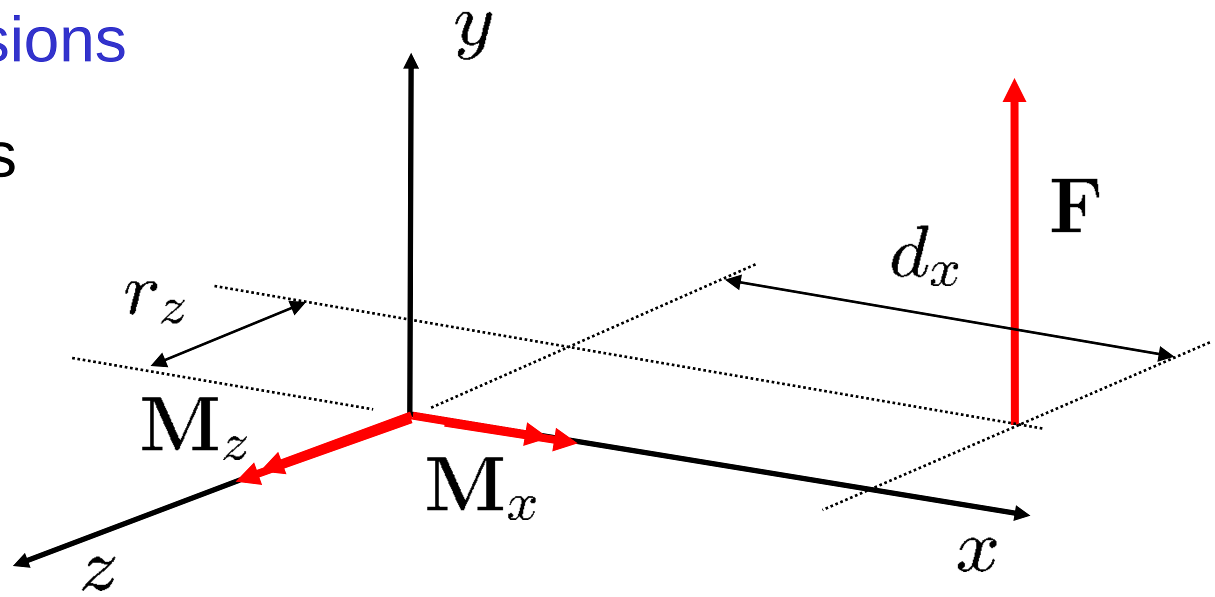
# Moment due to a force

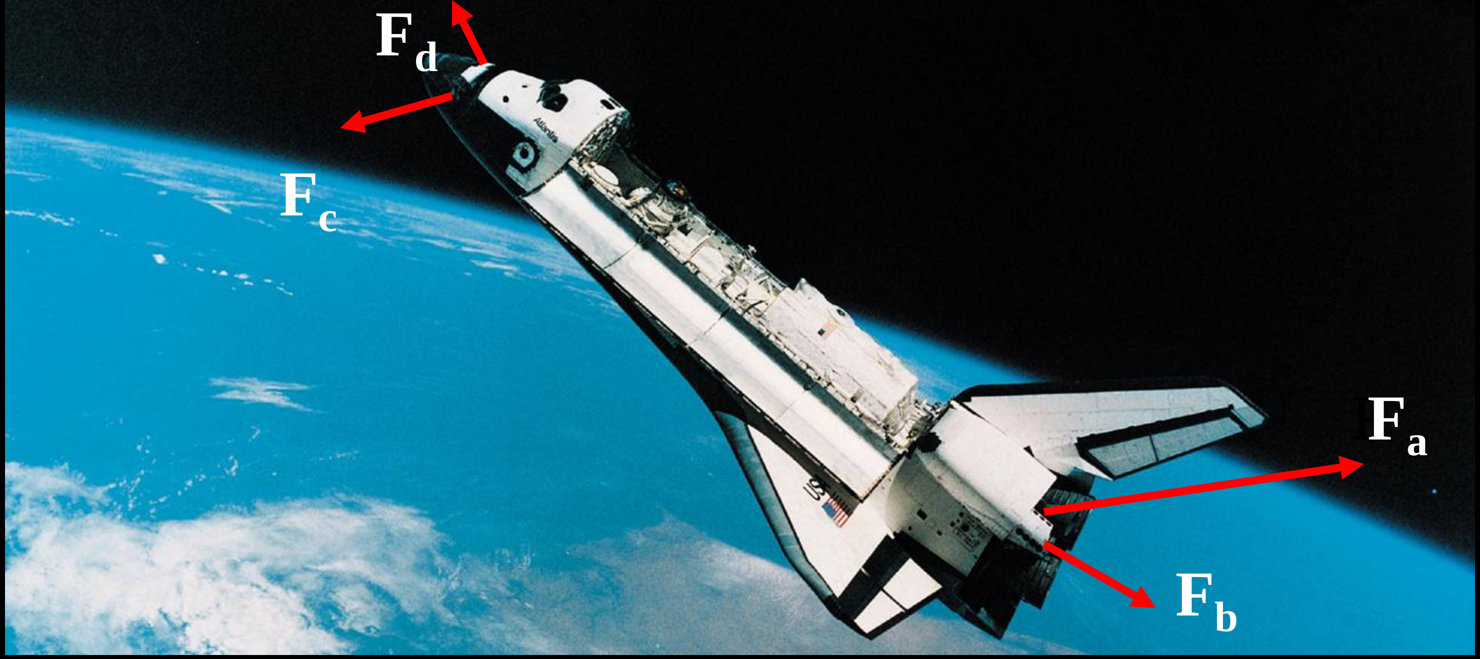


# Moment in 3 dimensions

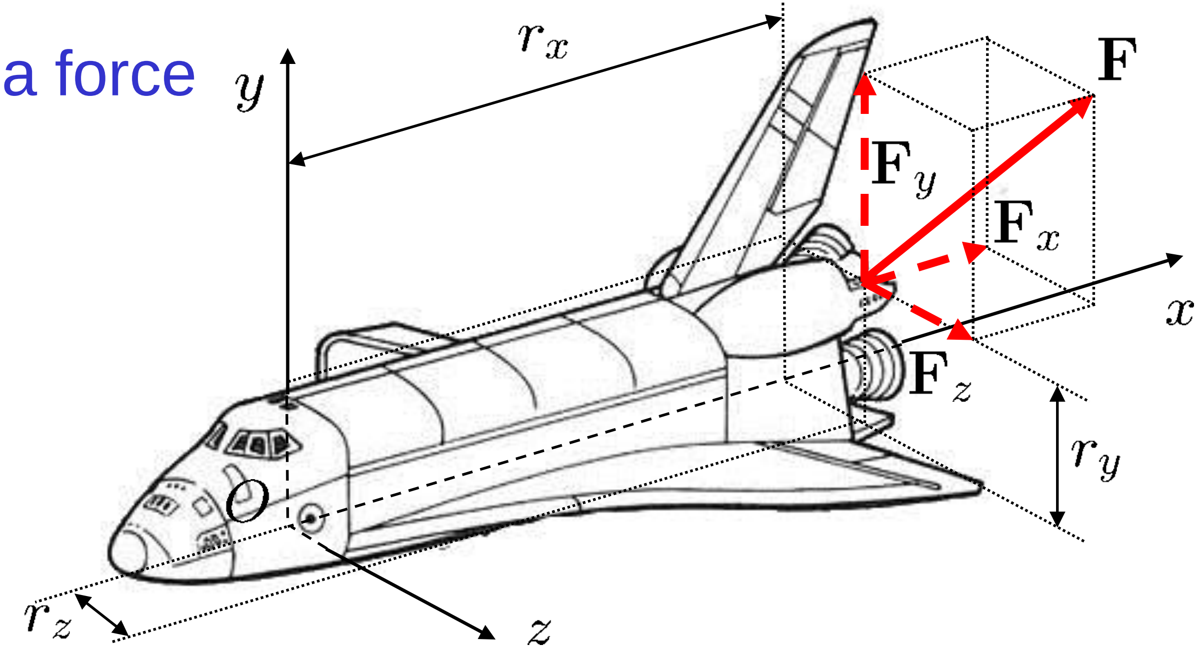
F is parallel to *y*-axis
but not in *xy* plane





$F_d$
$F_c$
$F_a$
$F_b$

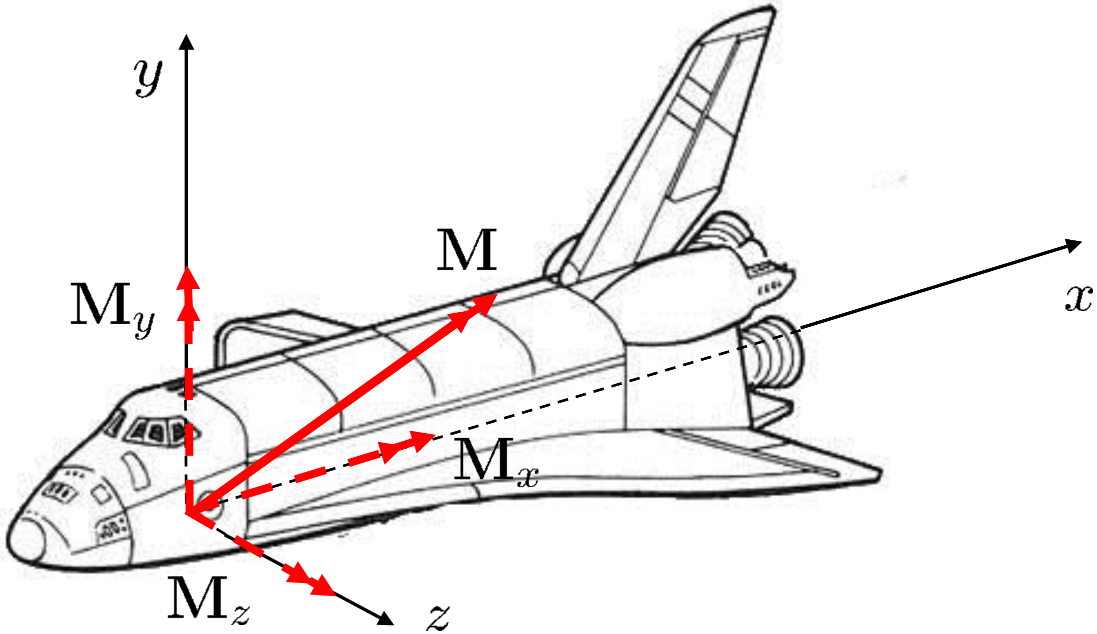
# Moment due to a force



# Moment

M =

$$(r_y F_z - r_z F_y)\mathbf{i} + (r_z F_x - r_x F_z)\mathbf{j} + (r_x F_y - r_y F_x)\mathbf{k}$$

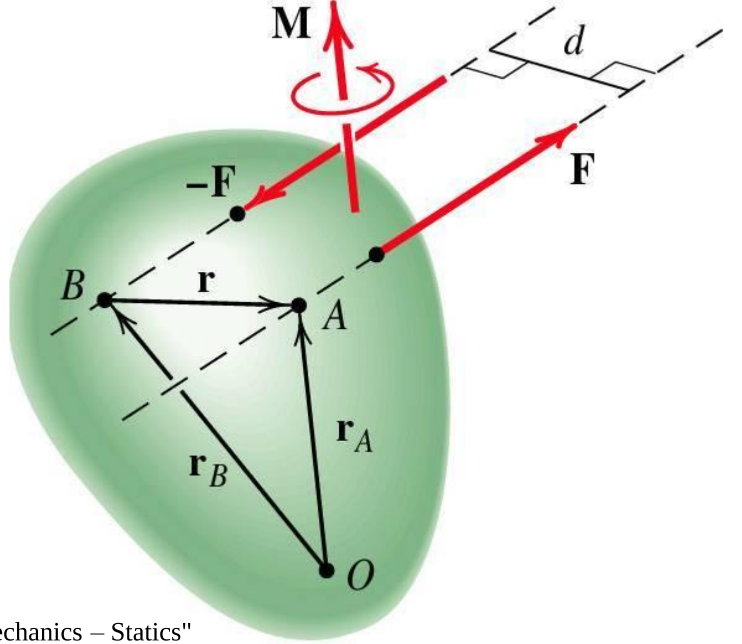


y
M
$M_y$
x
$M_x$
$M_z$
z

# Couple in 3D

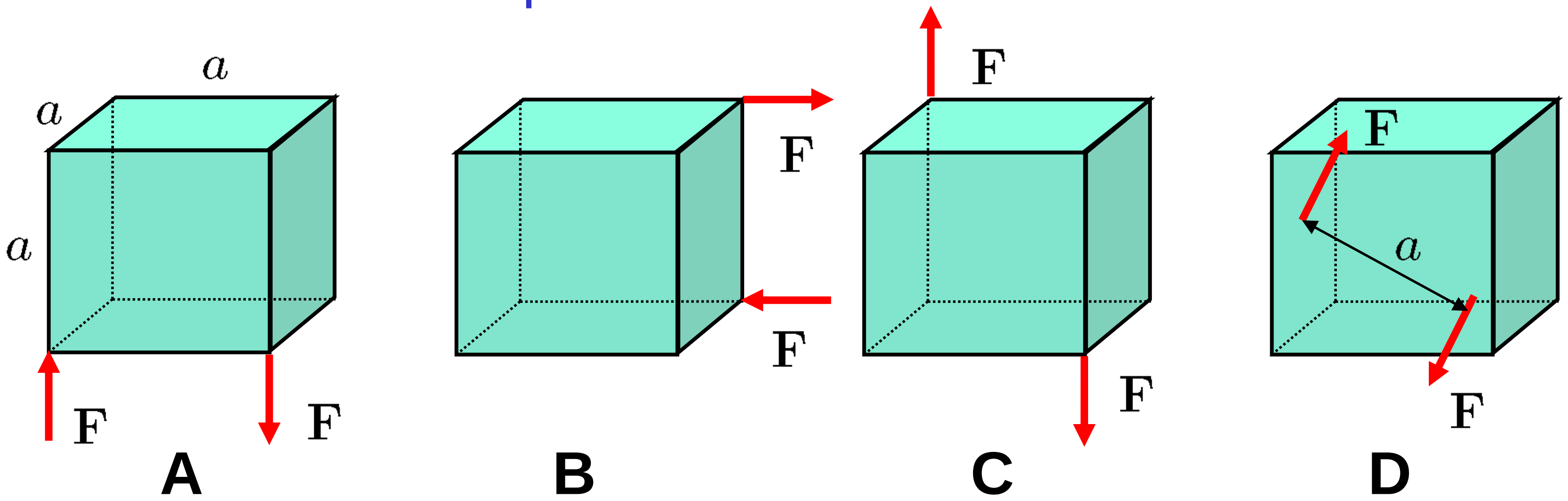
$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) =$$

$$(\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F} = \mathbf{r} \times \mathbf{F}$$

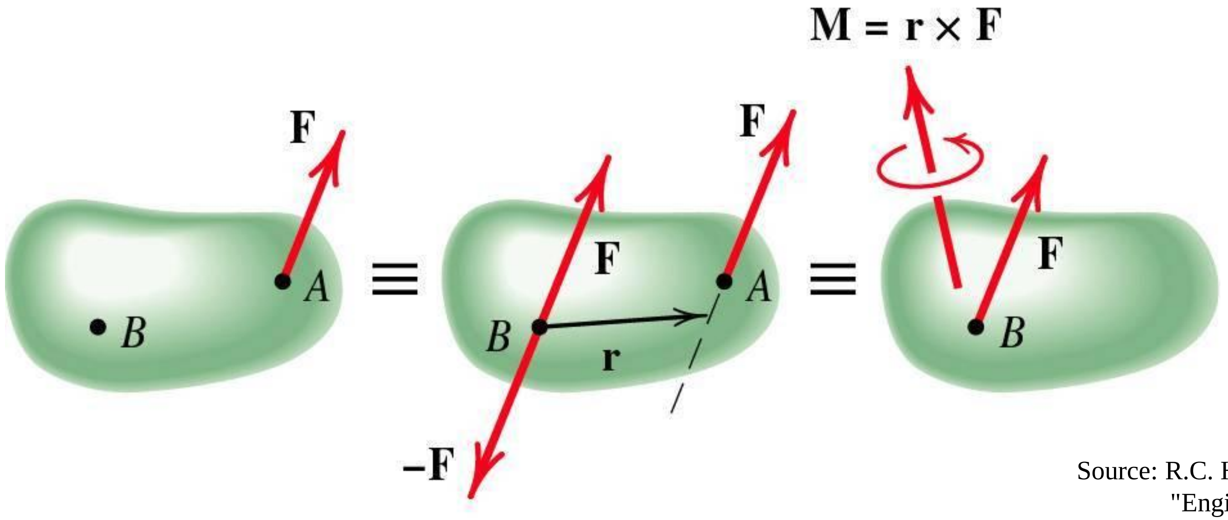



Source: R.C. Hibbeler, "Engineering Mechanics – Statics"

Which of these couples is different?



# Force-Couple systems




Source: R.C. Hibbeler,
"Engineering Mechanics – Statics"

# Equivalent Force-Couple system

A single resultant force and a moment that replaces all forces and couples in a body

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \ldots = \sum \mathbf{F}$$

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \ldots = \sum (\mathbf{r} \times \mathbf{F})$$