

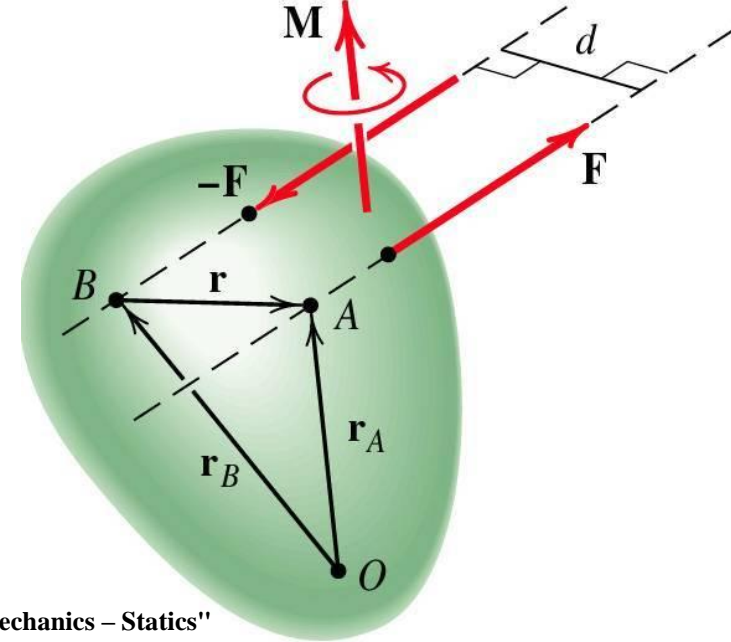
Today:

3D continued

Book: Chapter 5.5-5.6

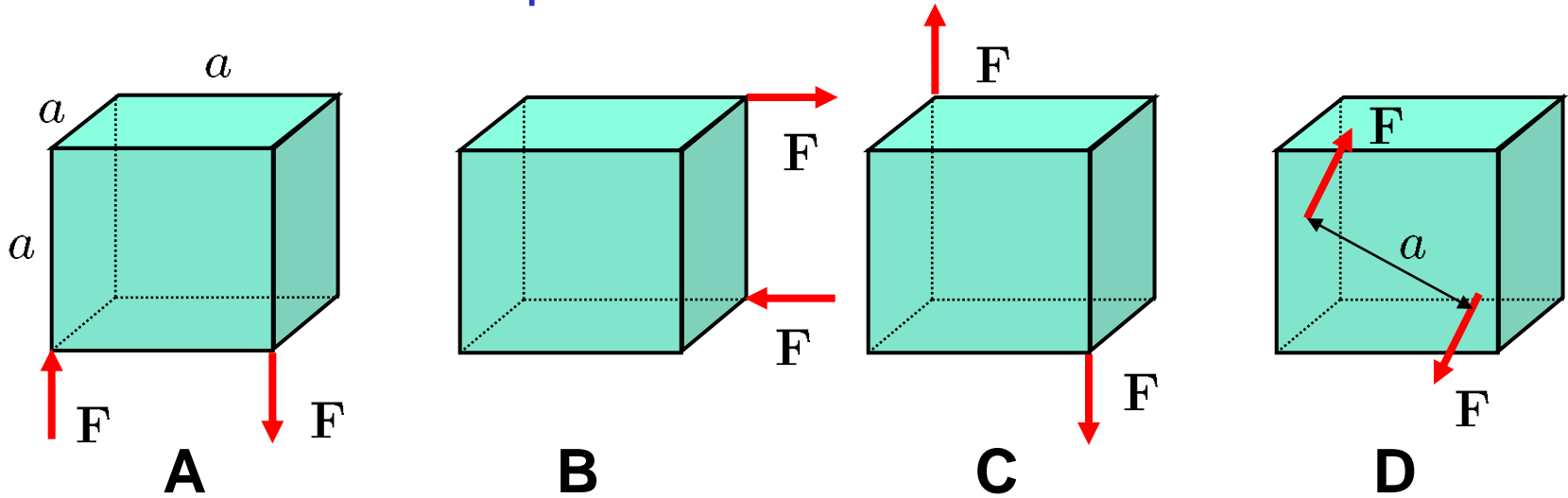
Couple in 3D

$$\begin{aligned}\mathbf{M} &= \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = \\ &(\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F} = \mathbf{r} \times \mathbf{F}\end{aligned}$$

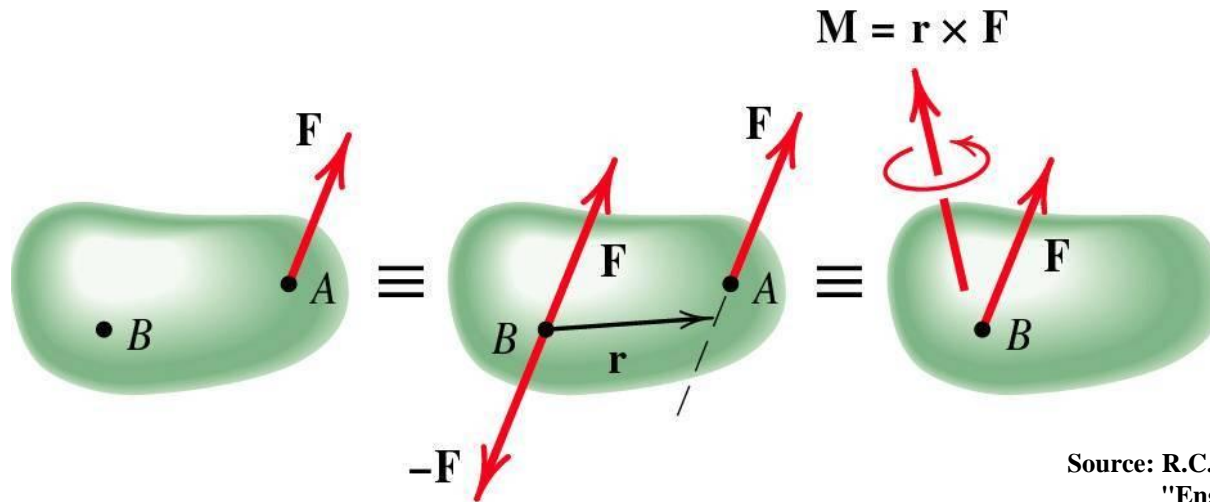


Source: R.C. Hibbeler,
"Engineering Mechanics – Statics"

Which of these couples is different?



Force-Couple systems



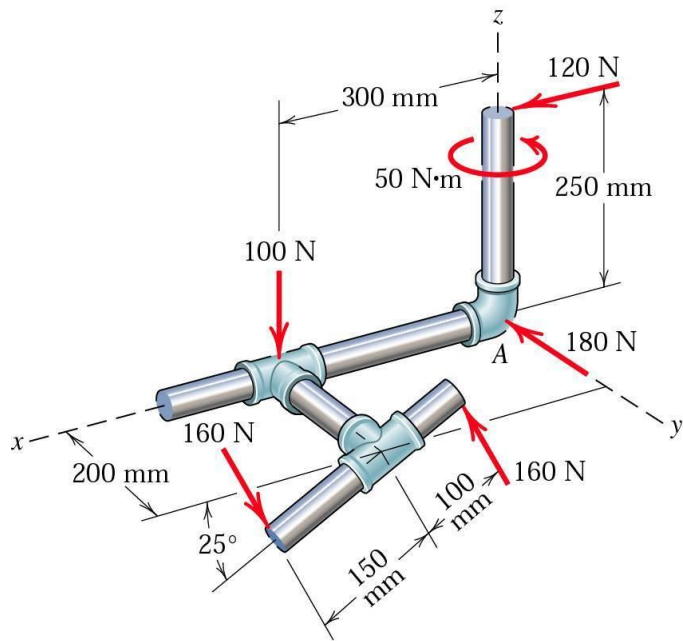
Source: R.C. Hibbeler,
"Engineering Mechanics – Statics"

Equivalent Force-Couple system

A single resultant force and a moment that replaces all forces and couples in a body

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \dots = \sum \mathbf{F}$$

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \dots = \sum (\mathbf{r} \times \mathbf{F})$$



Represent the resultant of the force system acting on the pipe assembly by a single force **R** at A and a couple **M**.

Source: R.C. Hibbeler,
"Engineering Mechanics – Statics"

Static equilibrium of a body (Newton's first law)

The equivalent force and moment in any point is equal to zero.

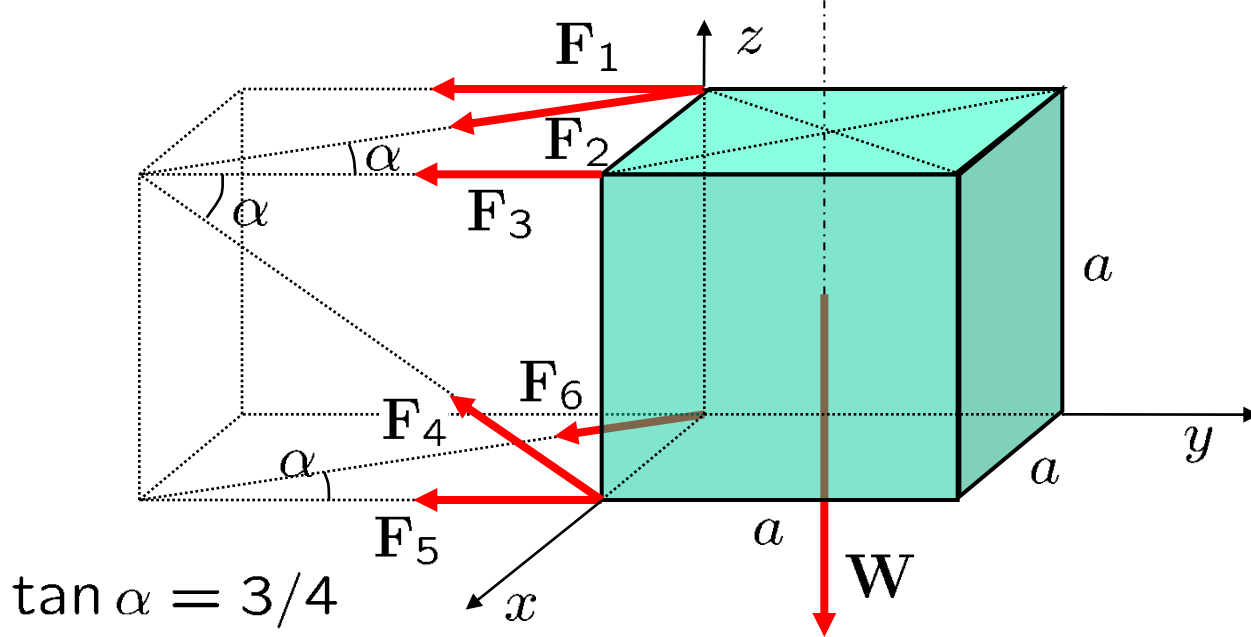
$$\mathbf{R} = \sum \mathbf{F} = \mathbf{0}$$

$$\mathbf{M} = \sum (\mathbf{r} \times \mathbf{F}) = \mathbf{0}$$

Static equilibrium

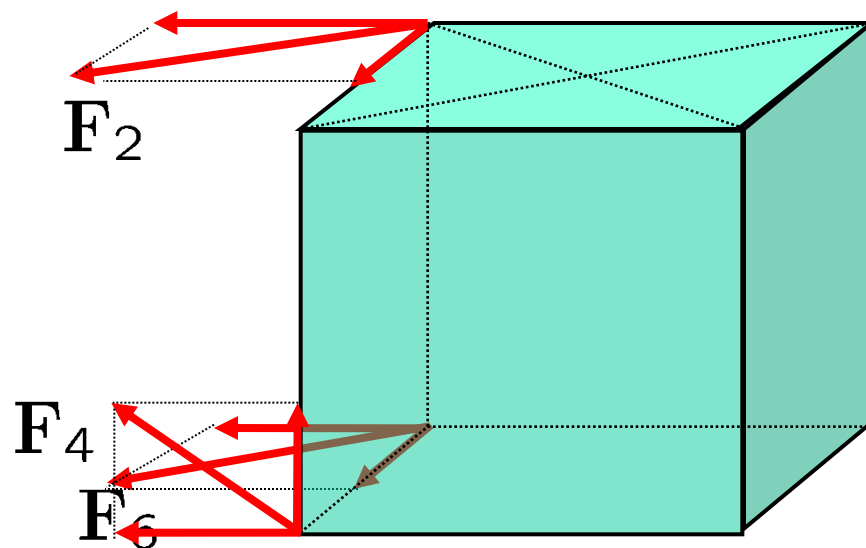
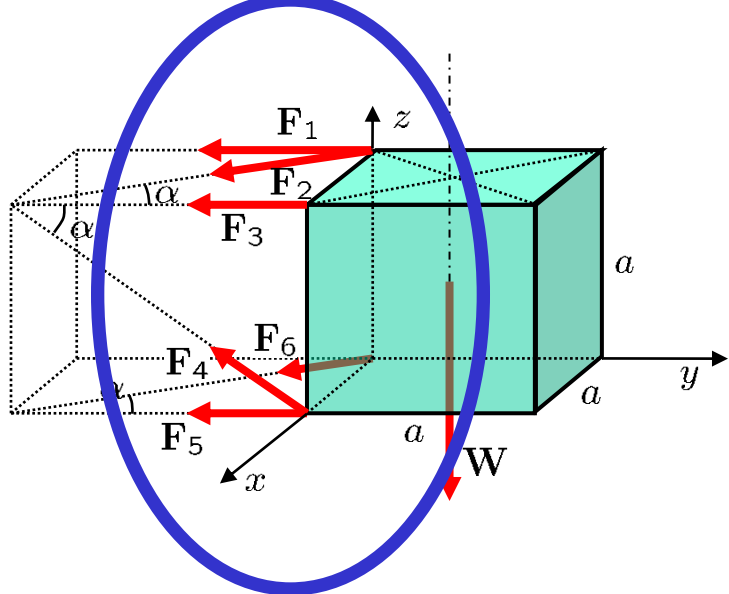
$$\sum \mathbf{F} = \mathbf{0} \quad \text{or} \quad \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases}$$

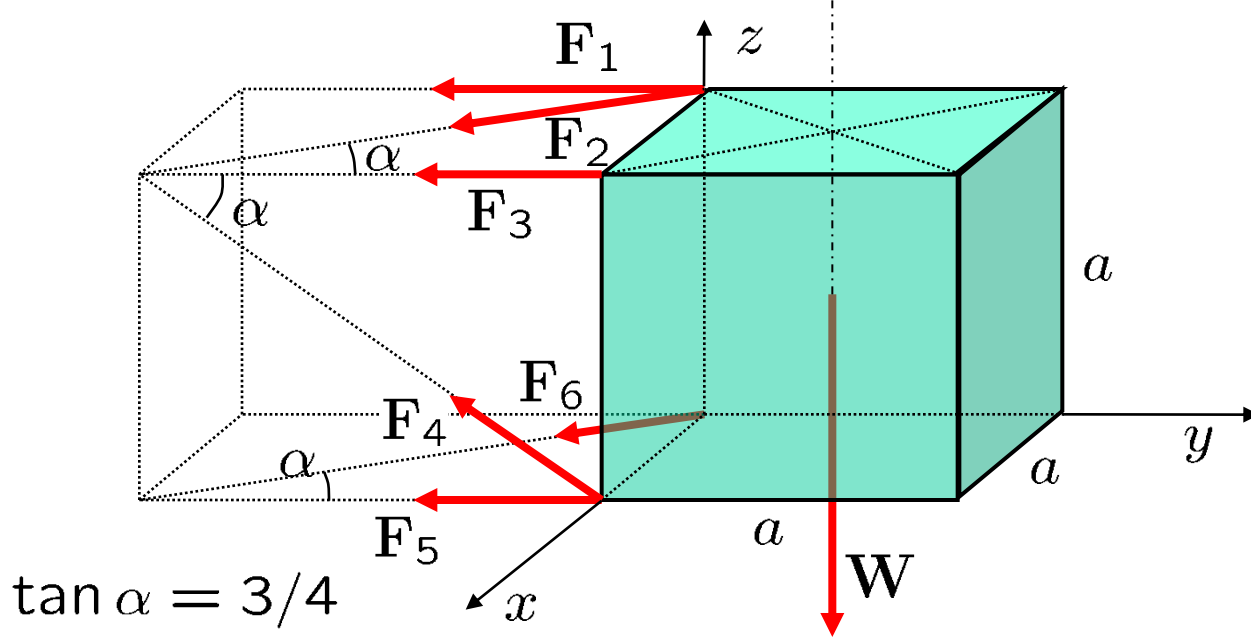
$$\sum \mathbf{M} = \mathbf{0} \quad \text{or} \quad \begin{cases} \sum M_x = 0 \\ \sum M_y = 0 \\ \sum M_z = 0 \end{cases}$$



The cube with weight $W=24\text{ N}$ and dimensions $a=1\text{ m}$ is in equilibrium.

Calculate F_1 to F_6 .



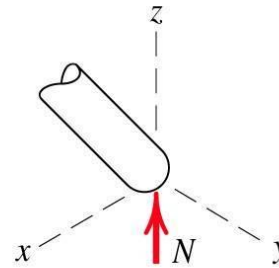
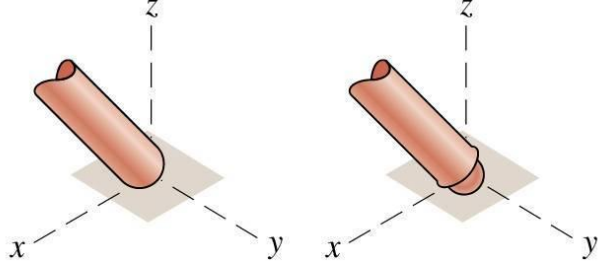


The cube with weight $W=24\text{ N}$ and dimensions $a=1\text{ m}$ is in equilibrium.

Calculate F_1 to F_6 .

Constraints in 3 dimensions

1. Member in contact with smooth surface, or ball-supported member

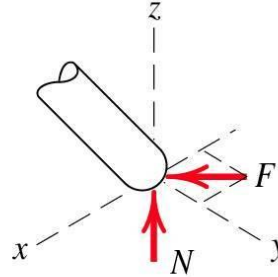
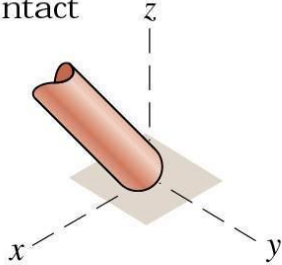


Force must be normal to the surface and directed toward the member.

Source: R.C. Hibbeler,
"Engineering Mechanics – Statics"

Constraints in 3 dimensions

2. Member in contact with rough surface

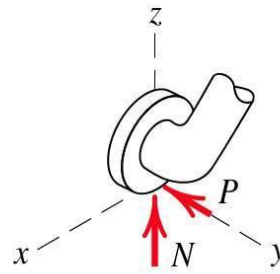
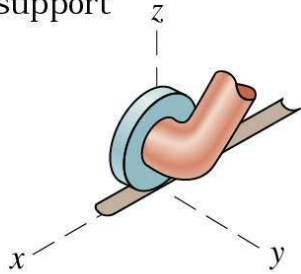


The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N .

Source: R.C. Hibbeler,
"Engineering Mechanics – Statics"

Constraints in 3 dimensions

3. Roller or wheel support with lateral constraint

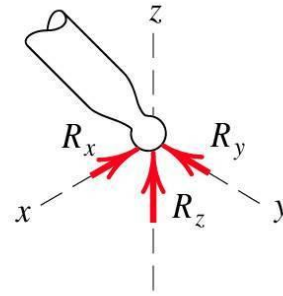
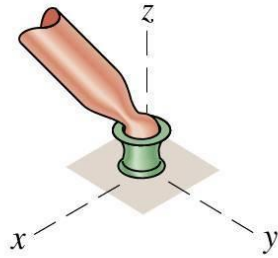


A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N .

Source: R.C. Hibbeler,
"Engineering Mechanics – Statics"

Constraints in 3 dimensions

4. Ball-and-socket joint

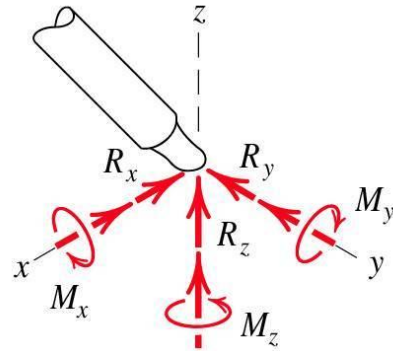
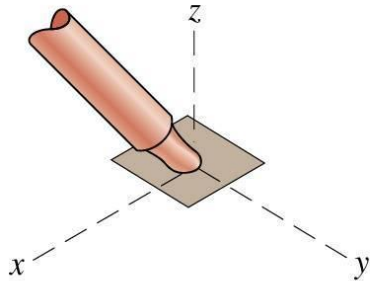


A ball-and-socket joint free to pivot about the center of the ball can support a force \mathbf{R} with all three components.

Source: R.C. Hibbeler,
"Engineering Mechanics – Statics"

Constraints in 3 dimensions

5. Fixed connection (embedded or welded)

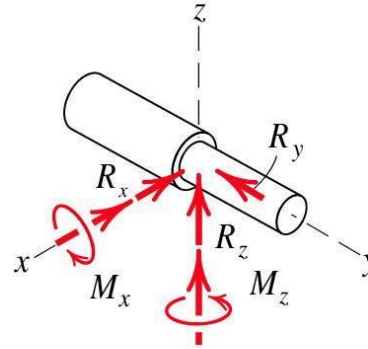
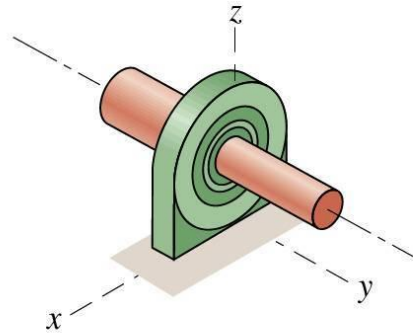


In addition to three components of force, a fixed connection can support a couple \mathbf{M} represented by its three components.

Source: R.C. Hibbeler,
"Engineering Mechanics – Statics"

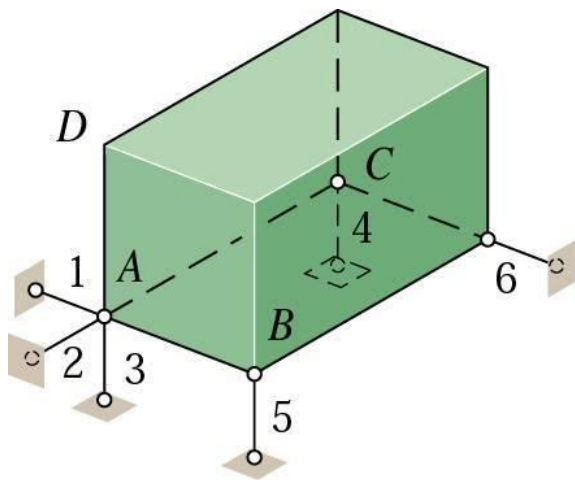
Constraints in 3 dimensions

6. Thrust-bearing support

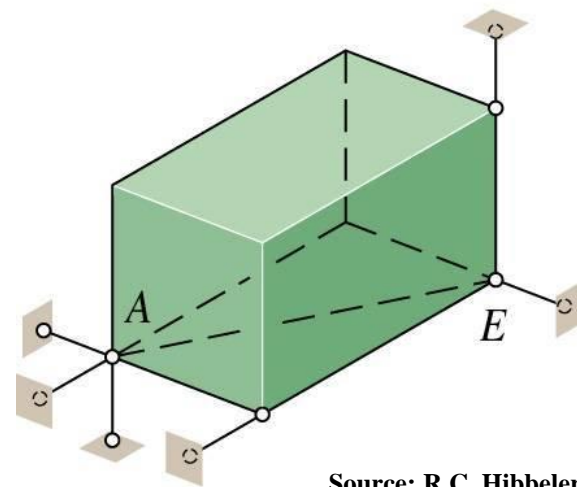


Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z . Couples M_x and M_z must, in some cases, be assumed zero in order to provide statical determinacy.

Source: R.C. Hibbeler,
"Engineering Mechanics – Statics"

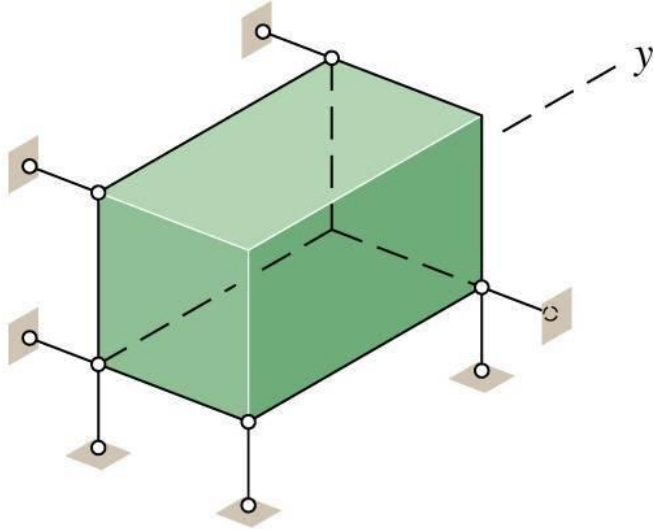


(a) Complete fixity
Adequate constraints

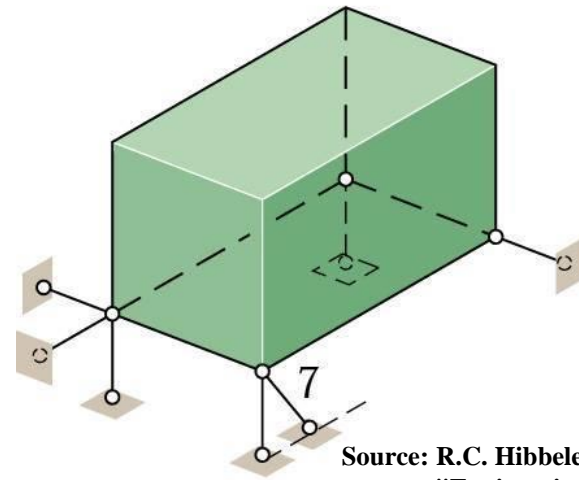


Source: R.C. Hibbeler,
"Engineering Mechanics – Statics"

(b) Incomplete fixity
Partial constraints

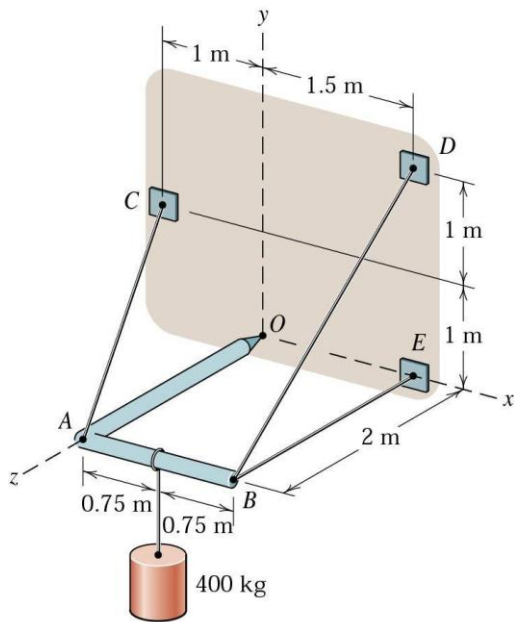


(c) Incomplete fixity
Partial constraints



(d) Excessive fixity
Redundant constraints

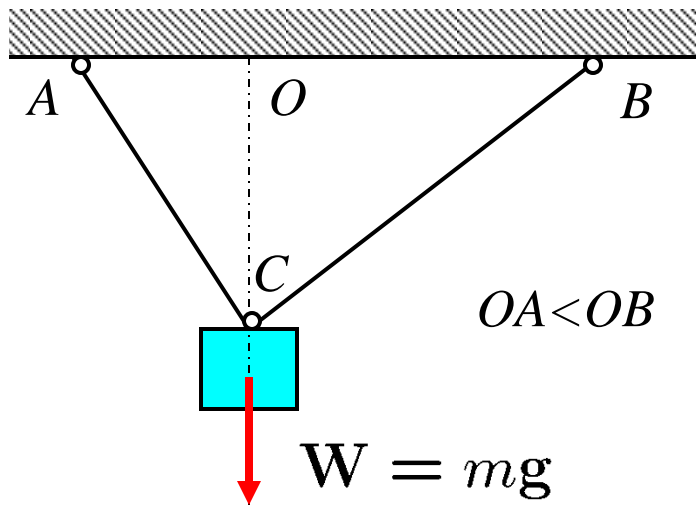
Source: R.C. Hibbeler,
"Engineering Mechanics – Statics"



The light right-angle boom which supports the 400-kg cylinder is supported by three cables and a ball- and socket joint at O attached to the vertical x - y surface.

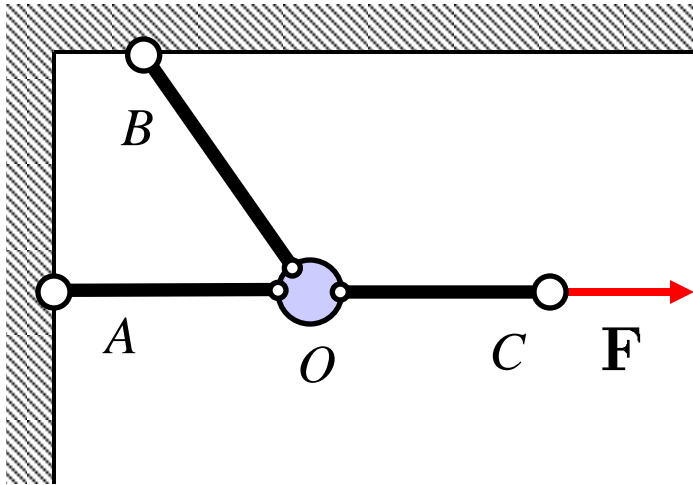
Determine the reactions at O and the cable tensions.

Source: R.C. Hibbeler,
"Engineering Mechanics – Statics"



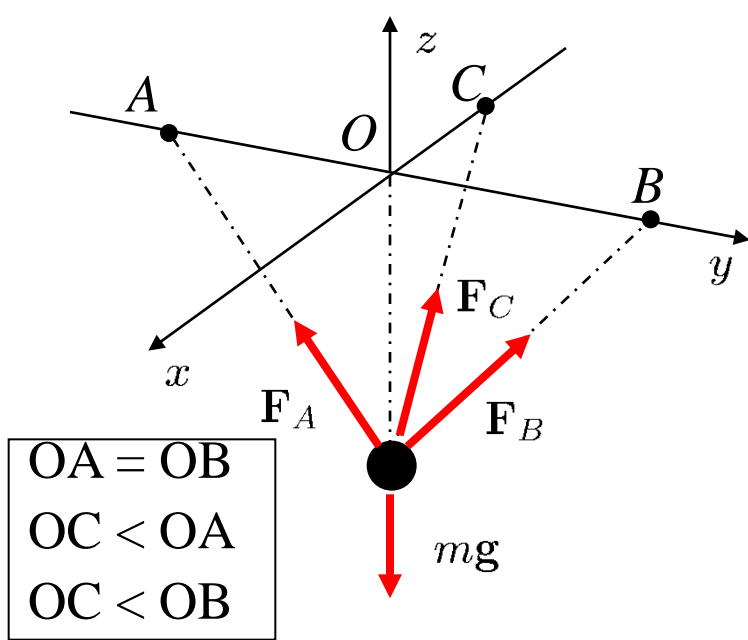
A particle is subjected to its own weight and is kept in equilibrium by two cables AC and BC . The tension in the cables are denoted T_{AC} and T_{BC} respectively. Which of the following statements is true?

- A) $T_{AC} < T_{BC}$
- B) $T_{AC} > T_{BC}$
- C) $T_{AC} = T_{BC}$



Consider the following experimental set-up in the International Space Station (no gravity!!!). Particle O is connected to 3 bars (links). Bar OC is subjected to a force F . Which of the following statements is true?

- A) OA is loaded in compression
- B) $T_{OA} = T_{OB}$
- C) $T_{OB} = 0$
- D) $T_{OC} > T_{OA}$



A particle is subjected to its own weight and is kept in equilibrium by three forces with magnitude F_A , F_B and F_C . Points A and B are on the y -axis and C is on the x -axis. Which of the following is true?

- A) $F_C > mg$
- B) $F_C > F_A$ and $F_C > F_B$
- C) $F_A + F_B = 0$
- D) $F_C = 0$

Design exercise

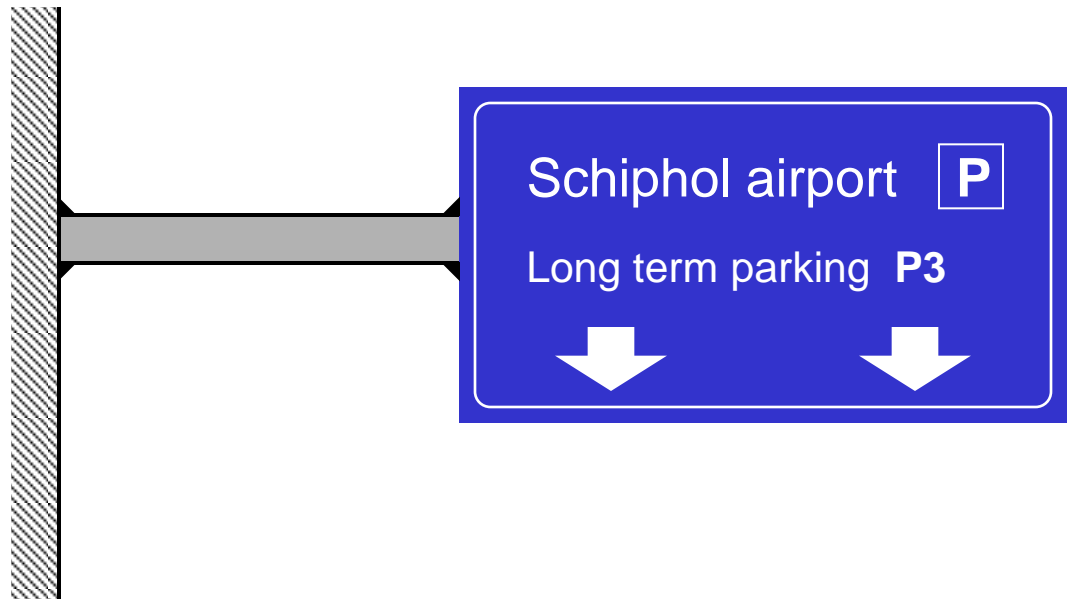
Design an efficient (=low weight) structure to attach the road sign to the wall. Take into account that the sign has a considerable mass and is subject to wind loads.

?



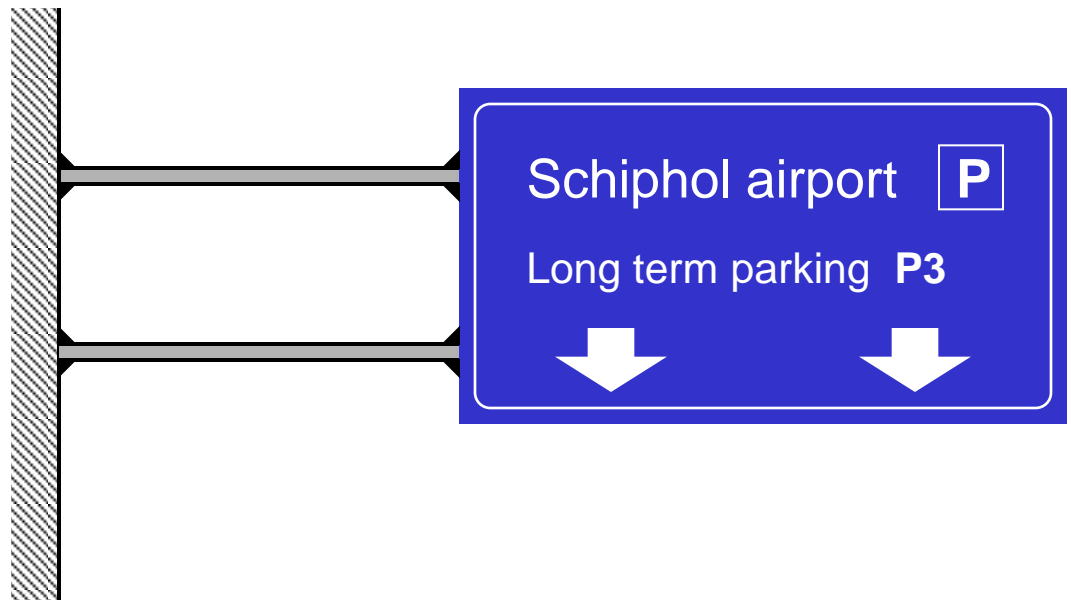
Design exercise

Design a structure to attach the road sign to the wall.



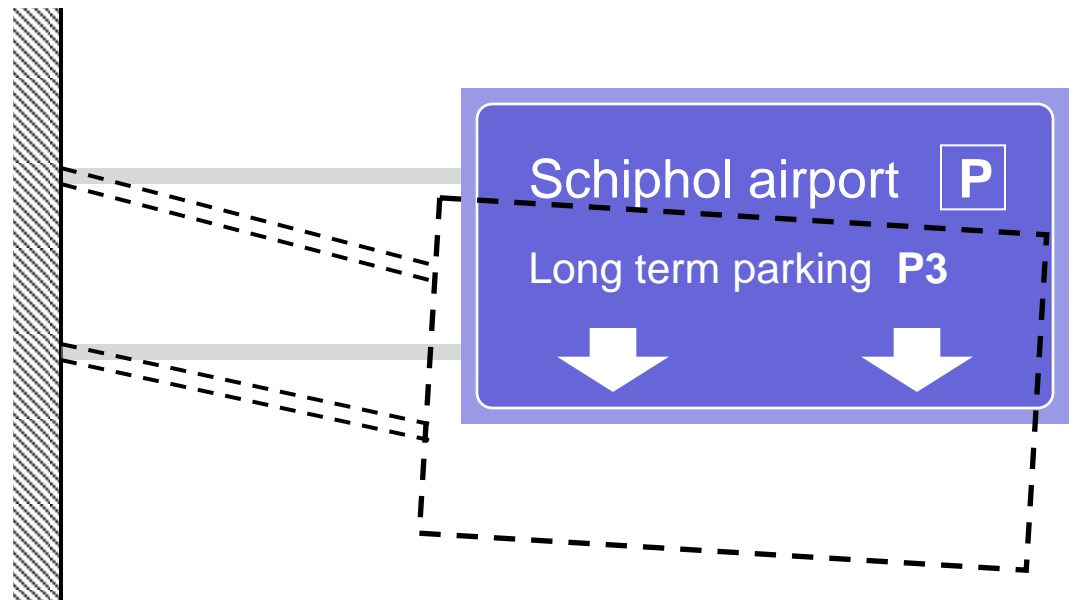
Design exercise

Design a structure to attach the road sign to the wall.



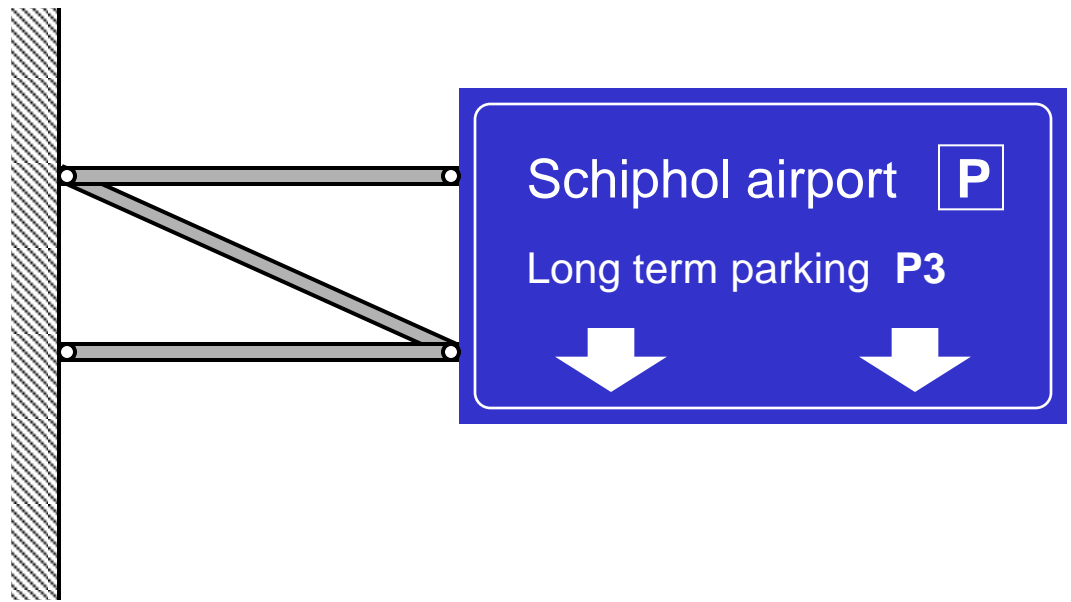
Design exercise

Design a structure to attach the sign to the wall.



Design exercise

Design a structure to attach the sign to the wall.



Truss structure

A framework composed of members joined at their ends to form a rigid structure





Model of a truss structure

- All members are connected by pin joints (even when in reality, the members are connected by welding or riveting).
- All external forces are applied at the pin connections.
- All members are assumed to be straight.

