Today:

Distributed loads

Area moments of Inertia

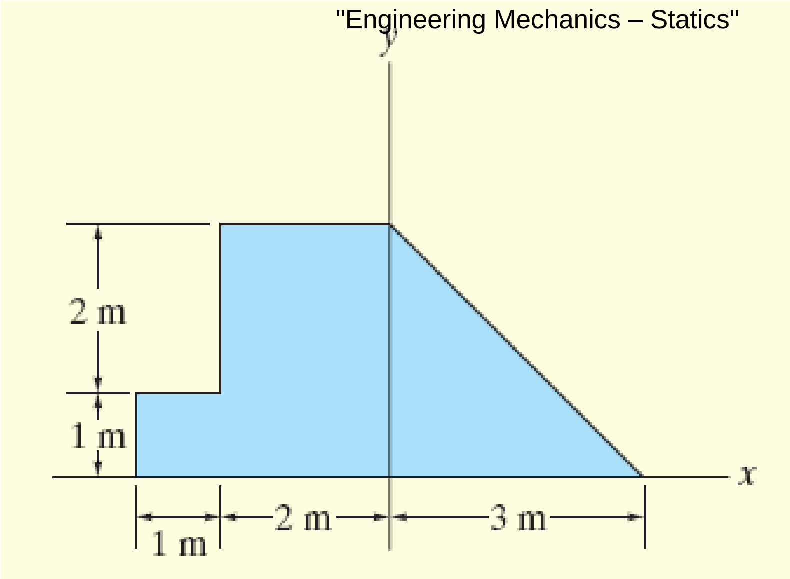
Steiner theorem

Book: Chapter 4.9, 10.1,10.2 & 10.4

Source: R.C. Hibbeler, "Engineering Mechanics – Statics"

For an area A composed of n parts:

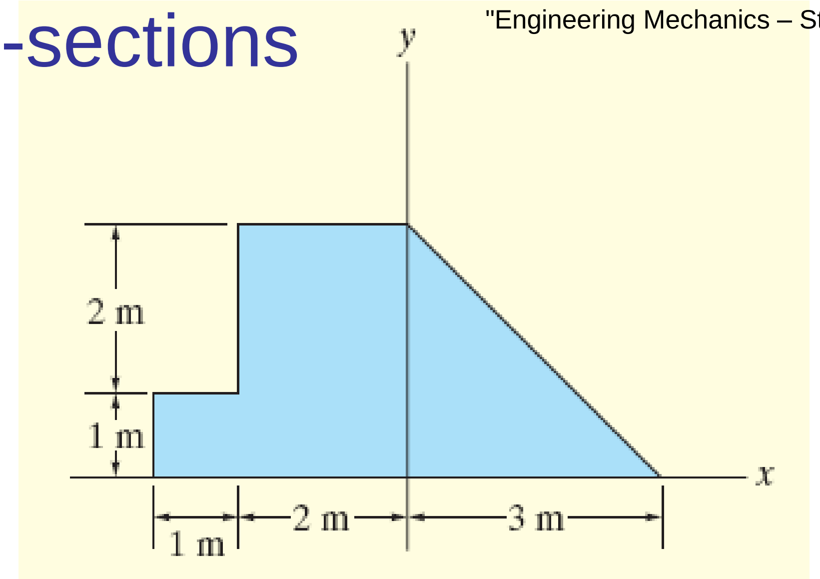
$$\bar{x} = \frac{\sum_{i=1}^{n} \tilde{x}_i A_i}{\sum_{i=1}^{n} A_i}; \qquad \bar{y} = \frac{\sum_{i=1}^{n} \tilde{y}_i A_i}{\sum_{i=1}^{n} A_i}$$



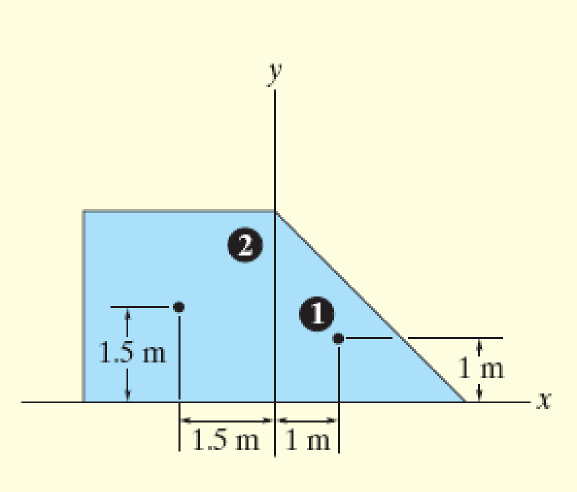
Source: R.C. Hibbeler, "Engineering Mechanics – Statics"

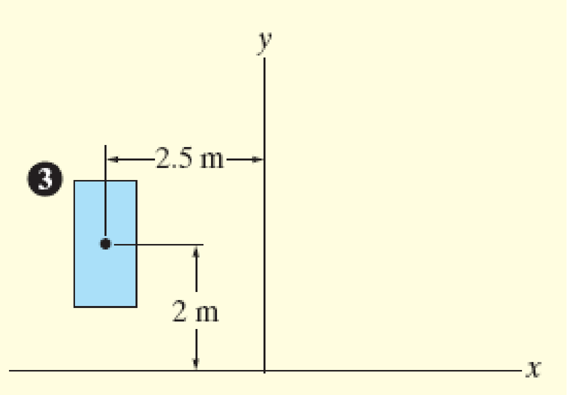
Example: composite cross-sections

Locate the *x*- and *y* – coordinates of the centroid



Example continued:



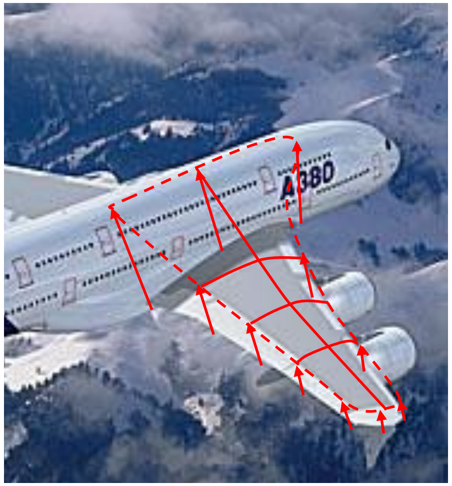


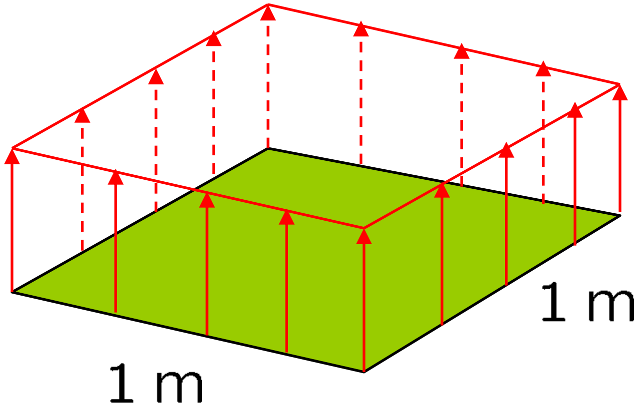

A380
A380
F


A380
A380

Distributed force: Area distribution

Aerodynamic pressure

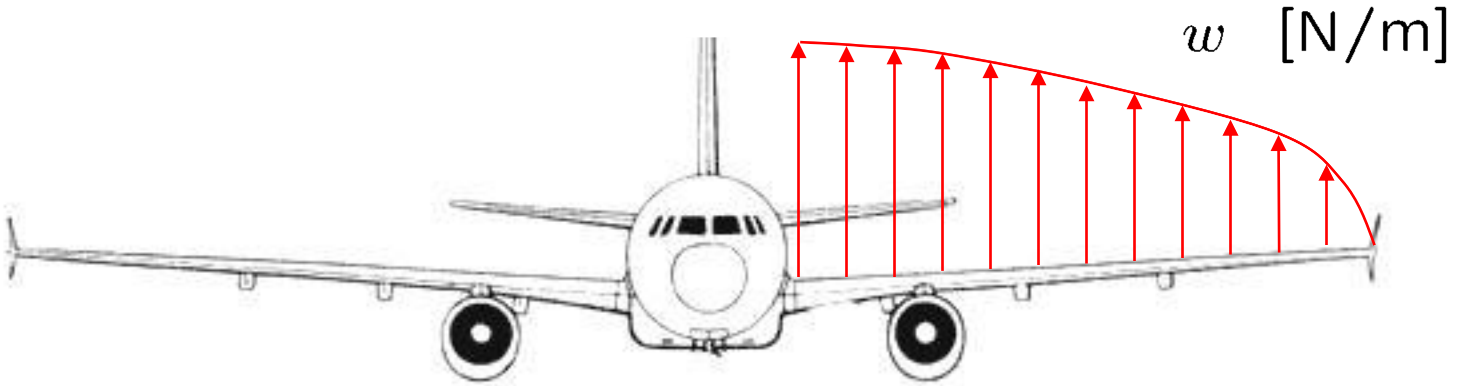




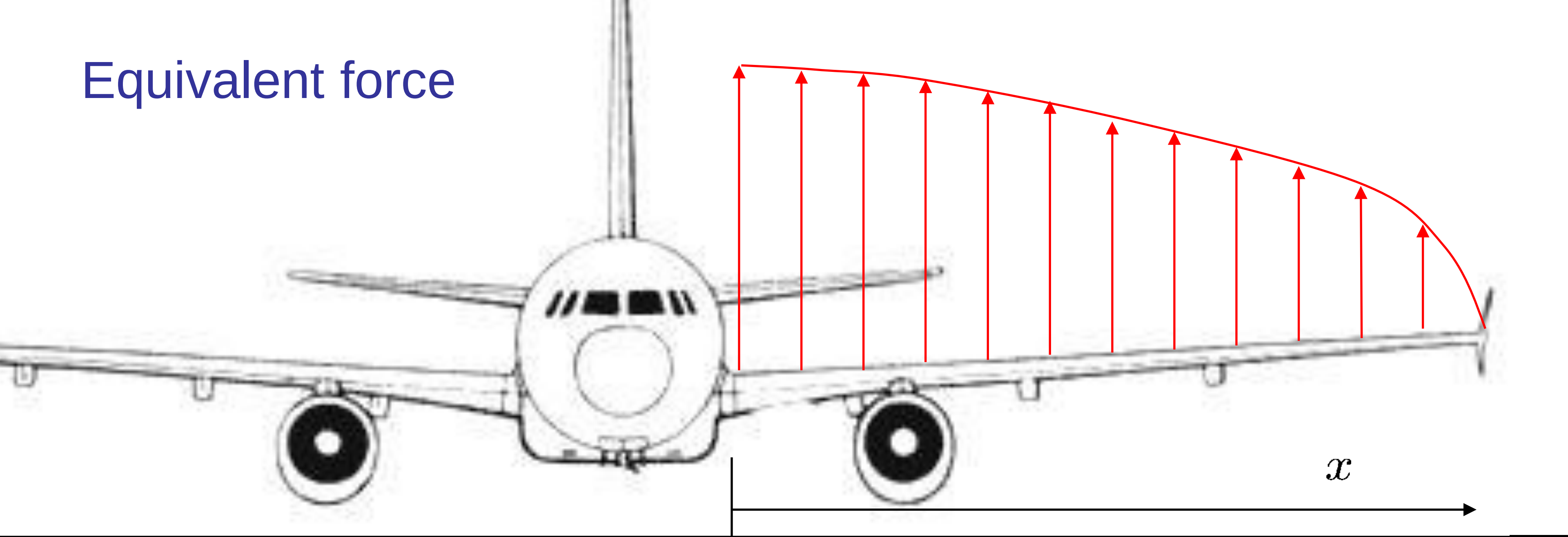
$$p = 1\,\mathrm{N/m^2}$$

$$F = p * A$$

Distributed force: Line distribution

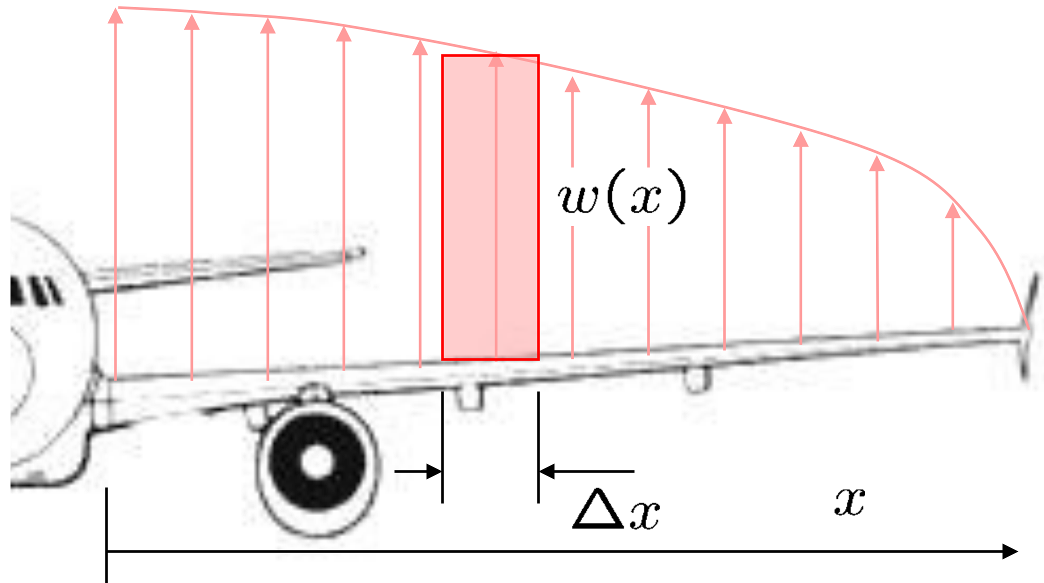


Equivalent force



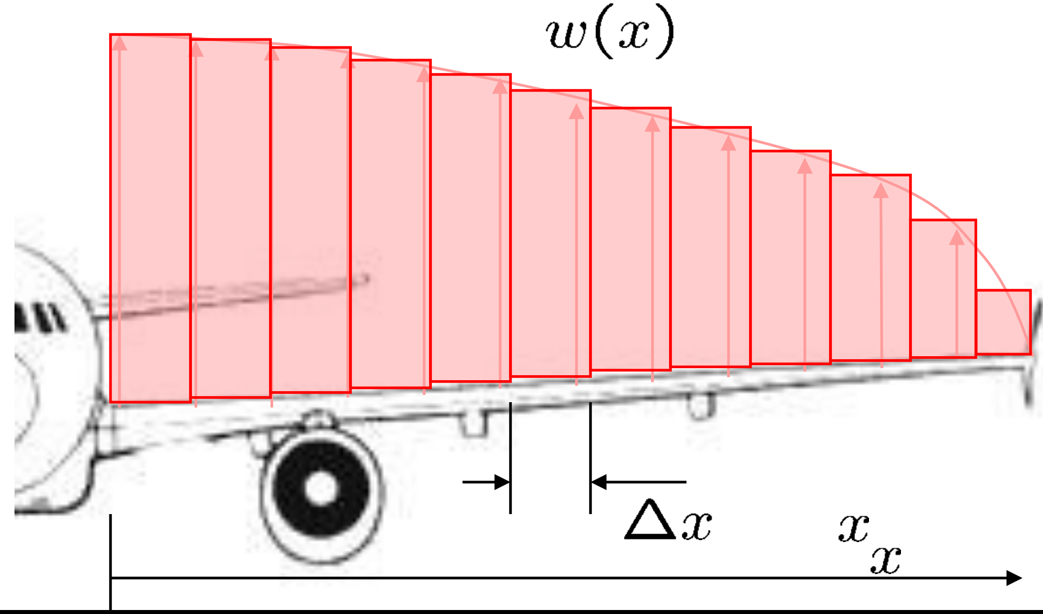
Equivalent force

$$\Delta R = w(x)\Delta x$$



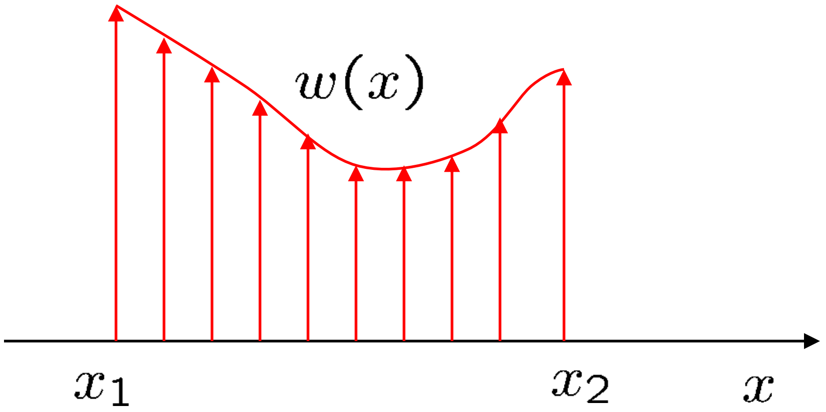
Equivalent force

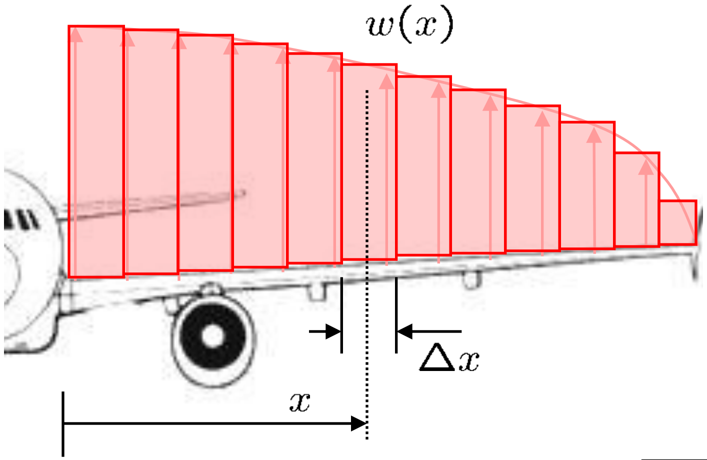
$$R = \sum \Delta R = \sum \left(w(x) \Delta x \right)$$

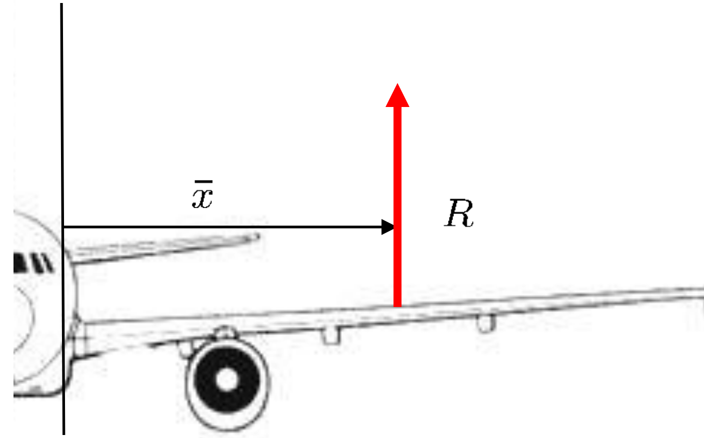


Magnitude of the equivalent force of a line distributed load

$$R = \int_{x_1}^{x_2} w(x)\,dx$$



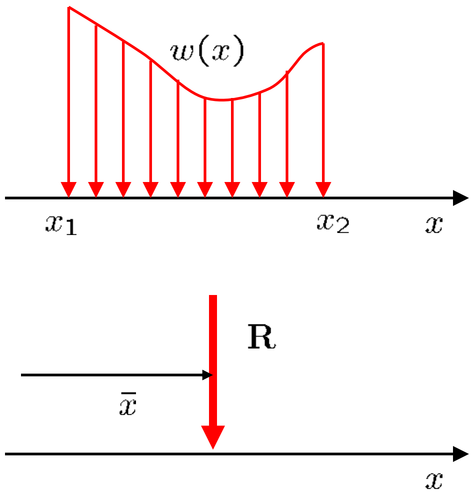




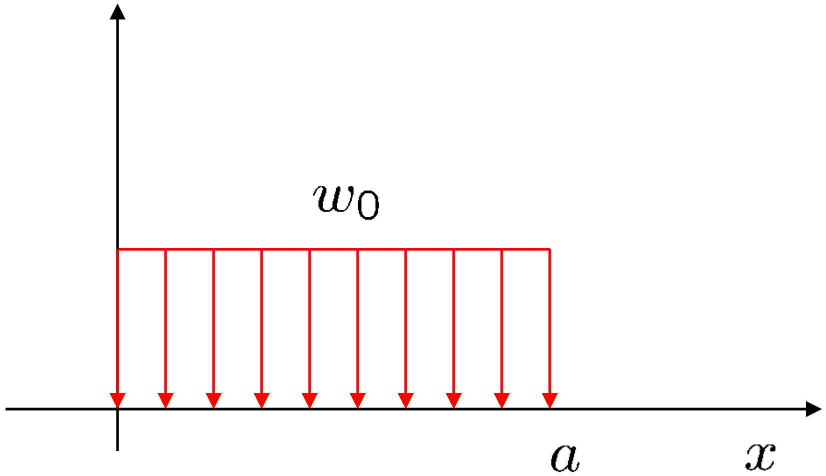
$$\bar{x}R = \sum x \Delta R = \sum x w(x) \Delta x$$

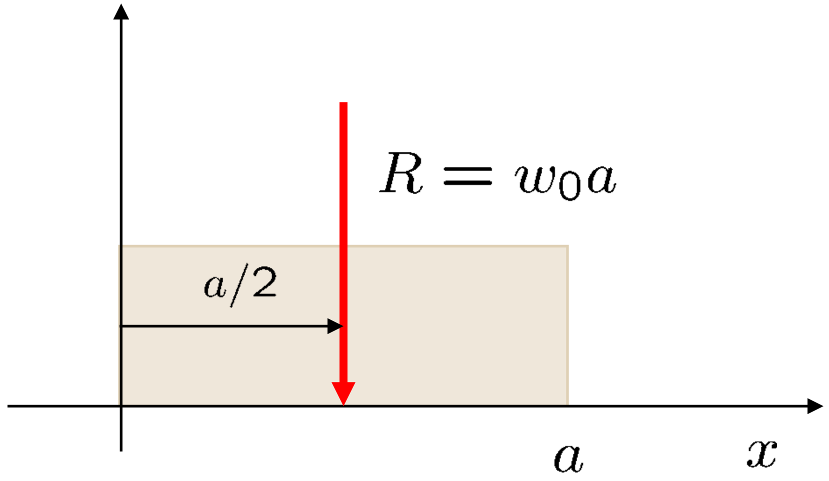
Position of the line of action of the equivalent force

$$\bar{x} = \frac{\int_{x_1}^{x_2} x w(x) dx}{\int_{x_1}^{x_2} w(x) dx}$$

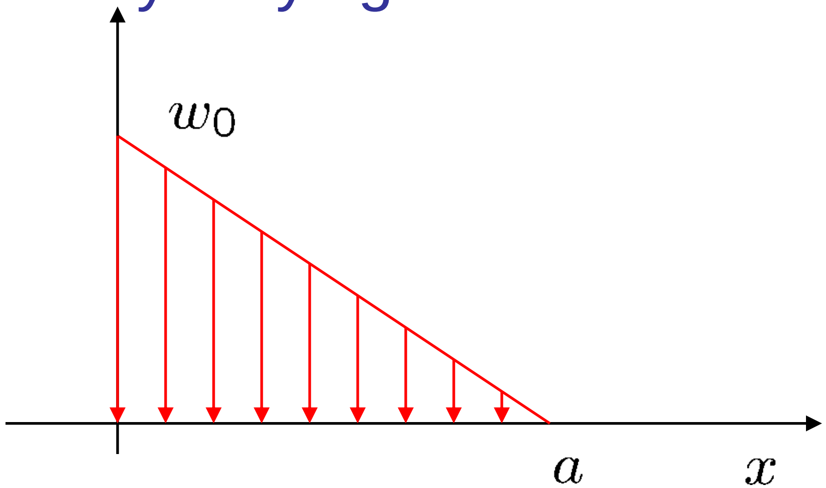


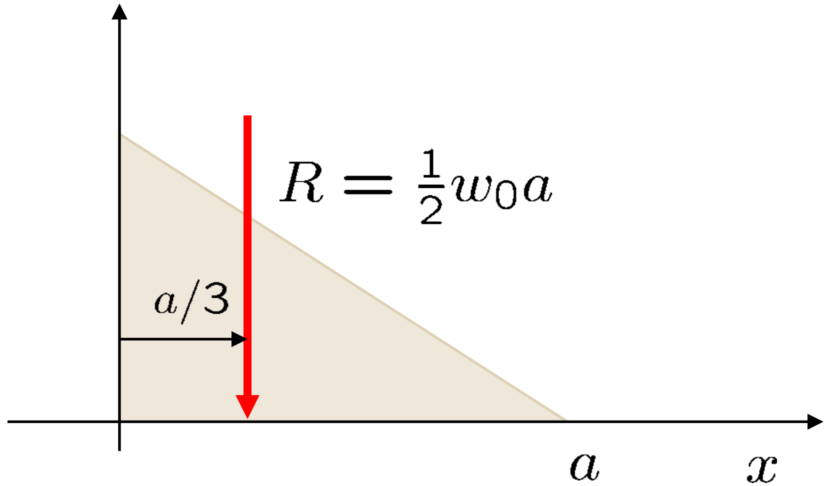
Uniform load distribution



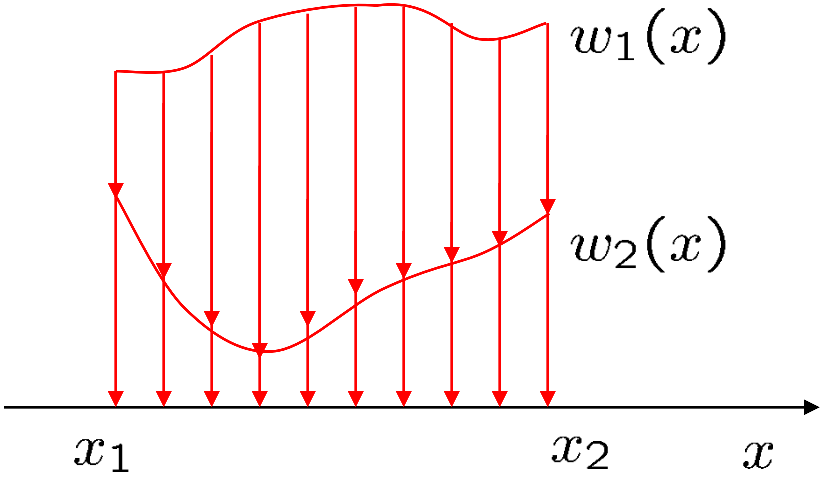


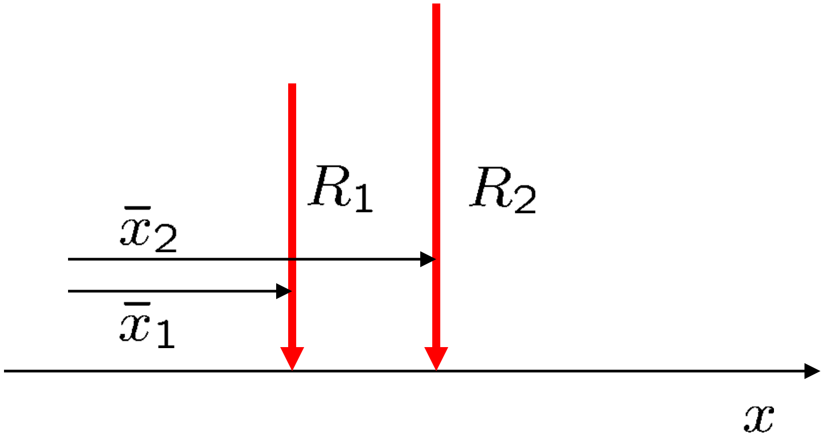
Linearly varying load distribution

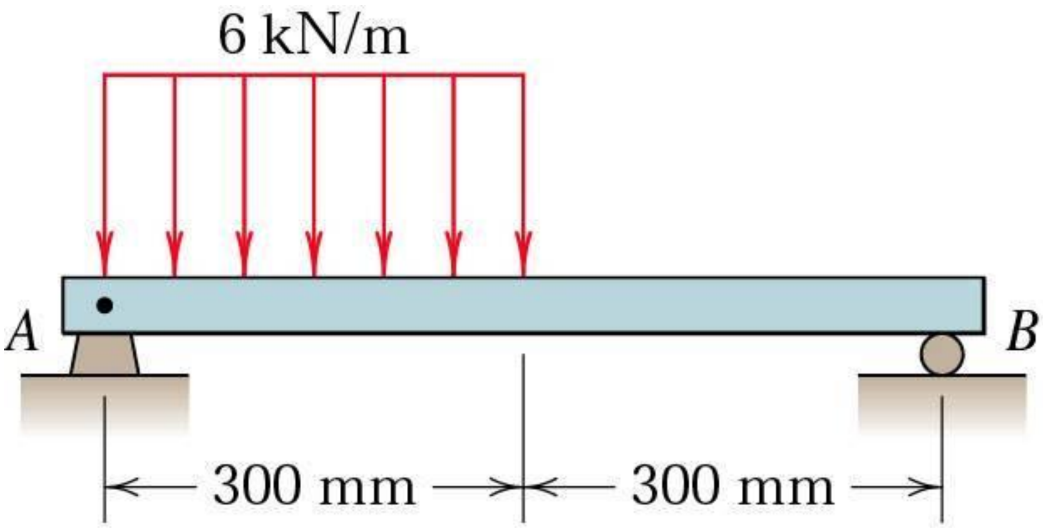




Superposition principle

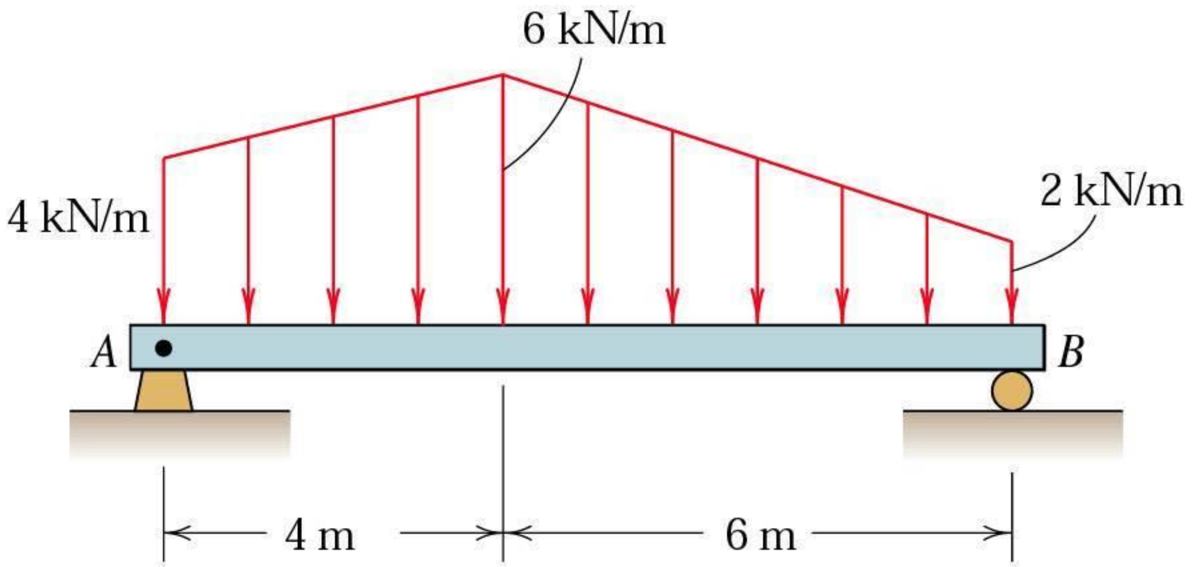






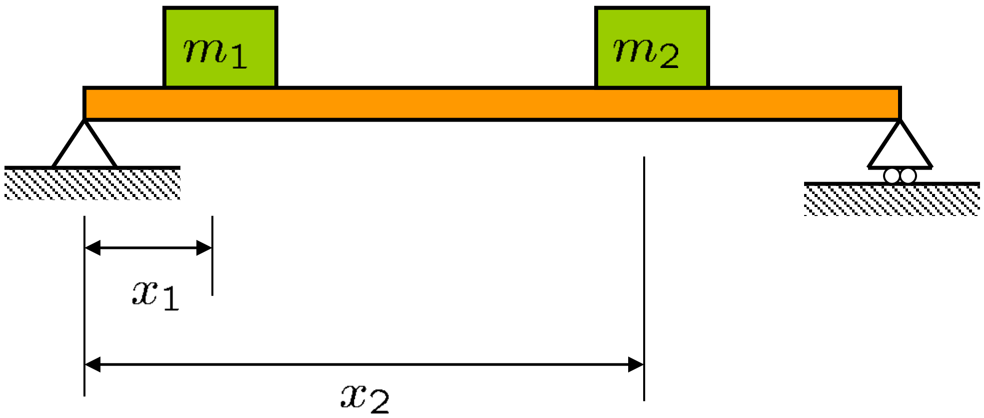
Determine the reactions at A and B for the beam subjected to the uniform load distribution

Source: R.C. Hibbeler, "Engineering Mechanics – Statics"



Determine the reactions at A and B for the beam subjected to the two linearly varying load distributions

Source: R.C. Hibbeler, "Engineering Mechanics – Statics"



Determine the equivalent force and its line of action that replaces the forces exerted by the masses.

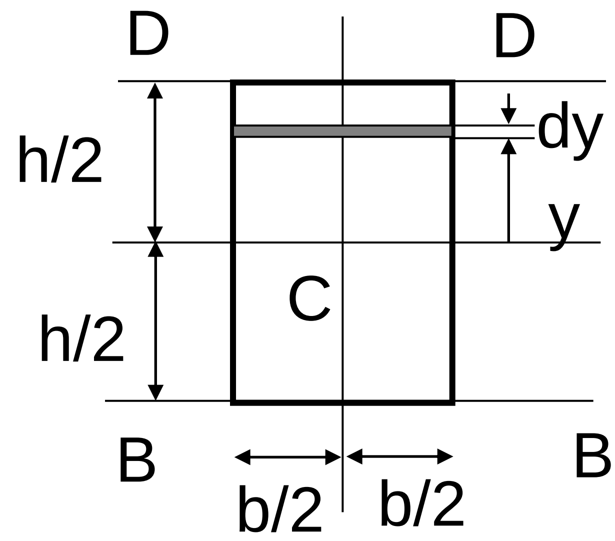
Area Moments of Inertia:

- Cross-sectional property

- Used to indicate resistance of cross-section against bending

Cross-sectional properties

- Dimensions h, b (height, width)
- Area $A = h \ x \ b$ (height x width)
- First moment of Area: $$Q_x = \int_A y\,dA$$ $$Q_y = \int_A x\,dA$$



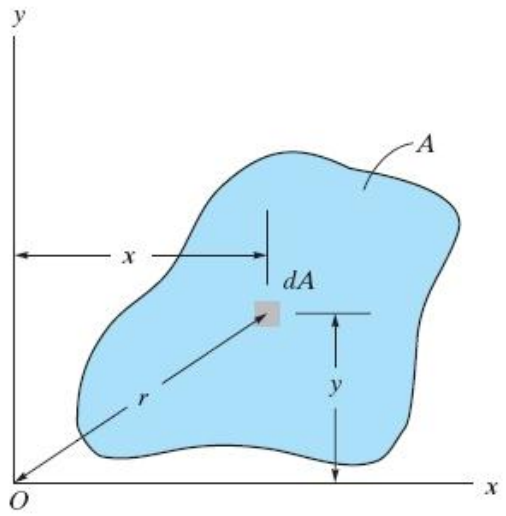
Source: R.C. Hibbeler, "Engineering Mechanics – Statics"

Moments of inertia: Second moment of Area

- Defined as:

$$I_x = \int_A y^2 dA$$

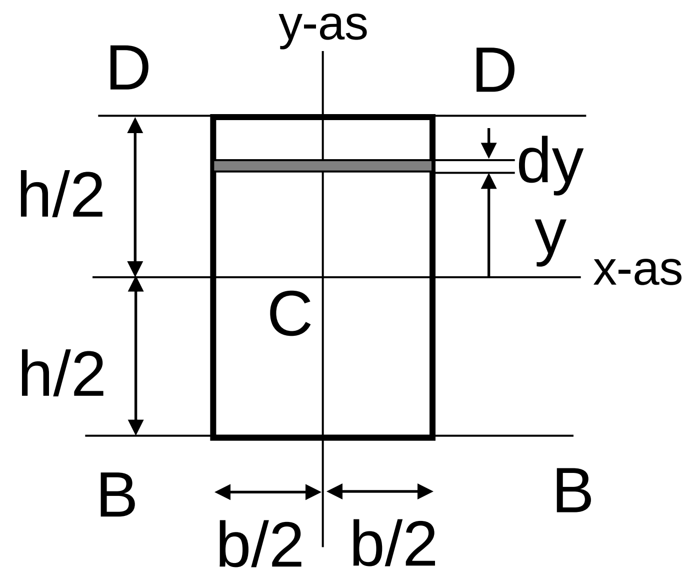
$$I_y = \int_A x^2 dA$$



- Unit: [mm^4]

Rectangle

Calculate I_x and I_y



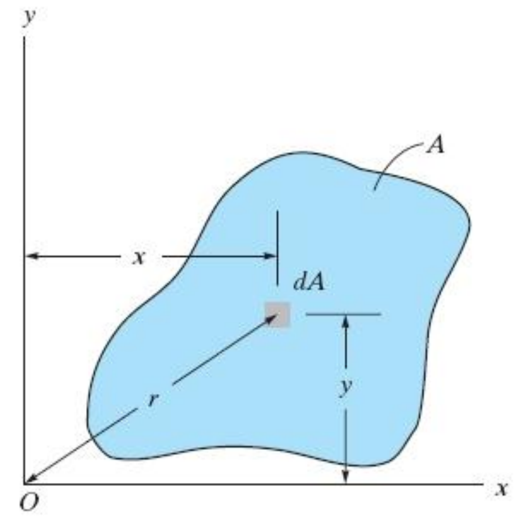
Polar moment of Inertia:

- Area Moment about z- axis

- Signifies resistance against torsion

- Denoted by J_0 or I_p:

$$J_0 = \int_A r^2 dA \quad (= I_x + I_y = \int_A y^2 dA + \int_A x^2 dA)$$



Source: R.C. Hibbeler, "Engineering Mechanics – Statics"

Example: Circle

Calculate I_x, I_y and J_0

Source: R.C. Hibbeler, "Engineering Mechanics – Statics"