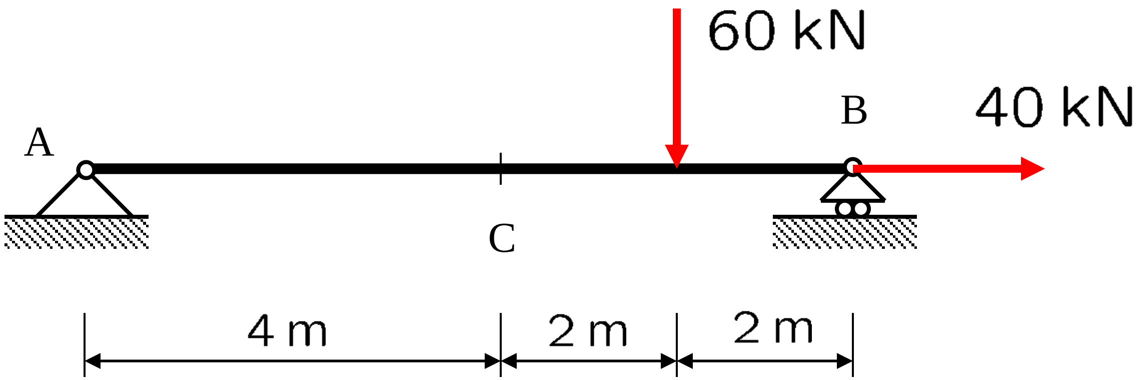
Today:

Internal effects in beams

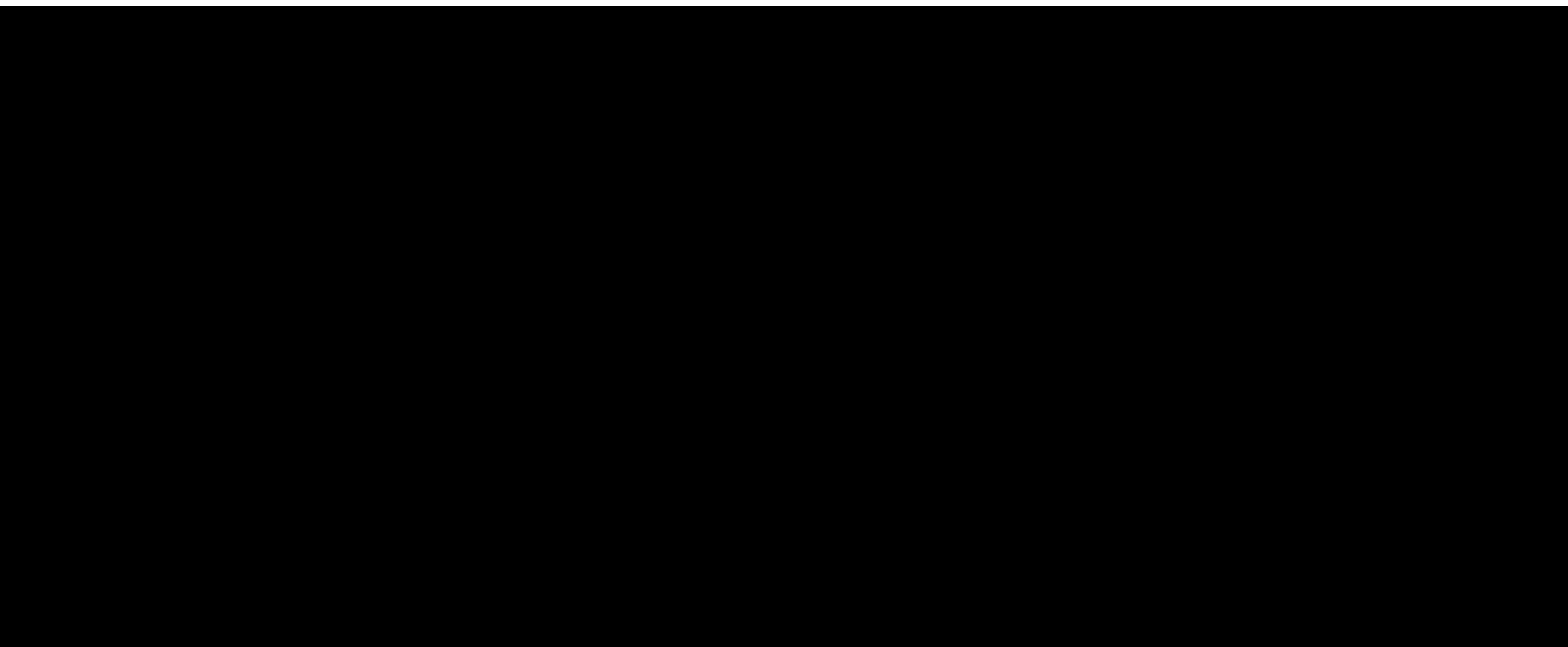
Book: Chapter 7.1-7.3 + hand outs

Calculate the internal forces in C.



Name convention

A cutting plane is ***positive*** when the perpendicular coordinate axis (e.g. the x-axis) is pointing outward.

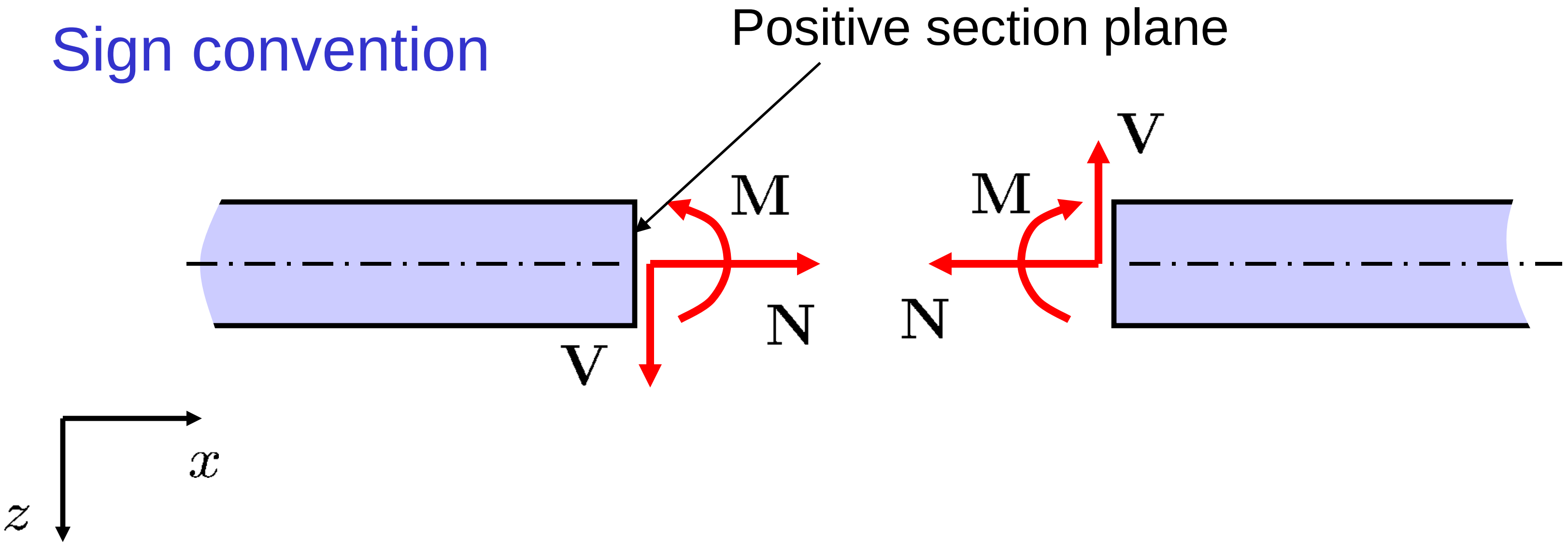


Sign convention

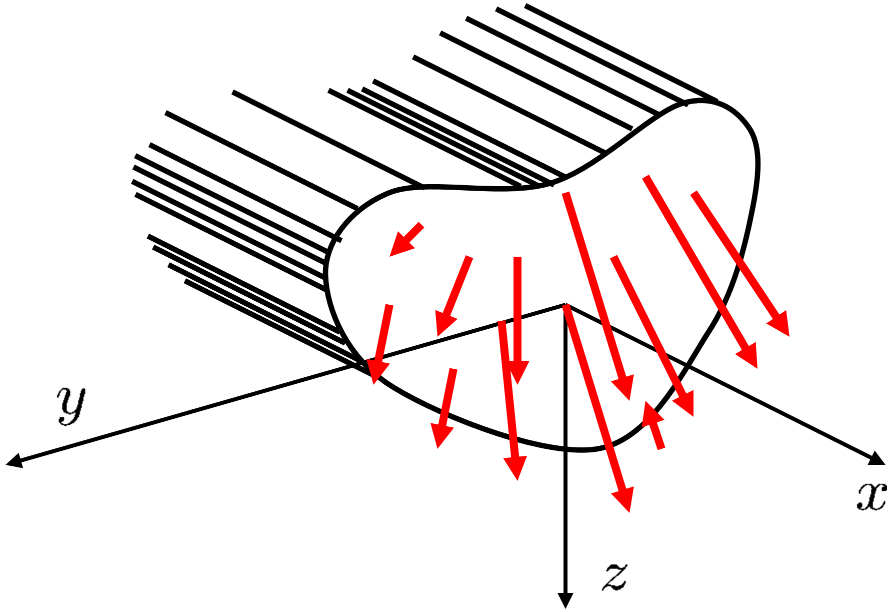
Normal and shear forces are ***positive*** when they are acting on a ***positive*** cutting-plane in the ***positive*** *x*- or *y*-direction, or when it is acting on a ***negative*** cutting-plane in the ***negative*** direction.

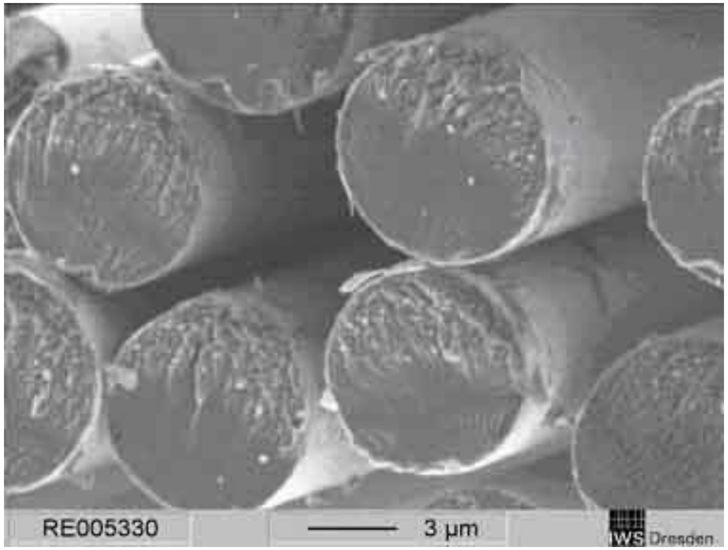
A bending moment is ***positive*** when it gives rise to a **tensile** stress in the side of the beam on the ***positive*** *z*-direction.

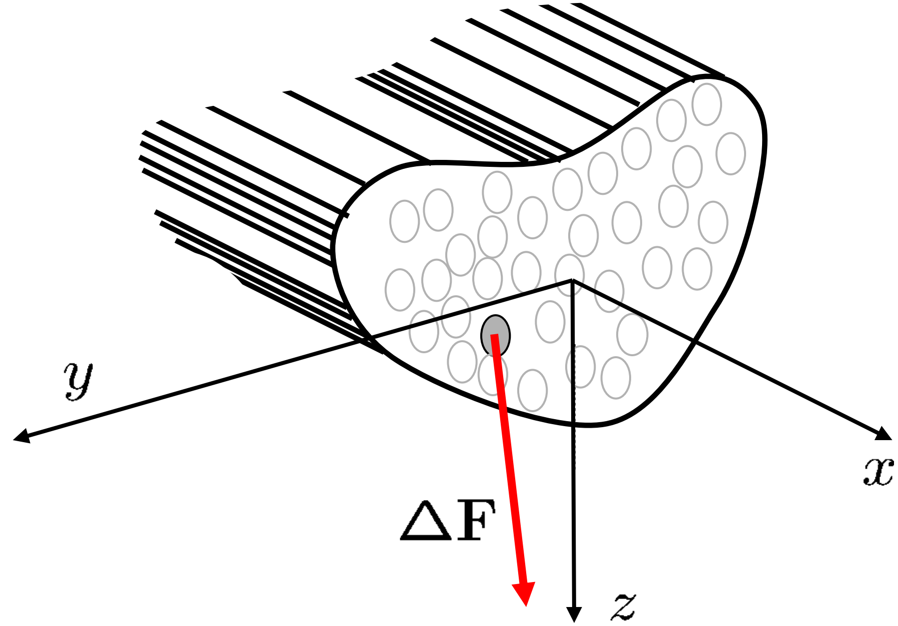
Sign convention

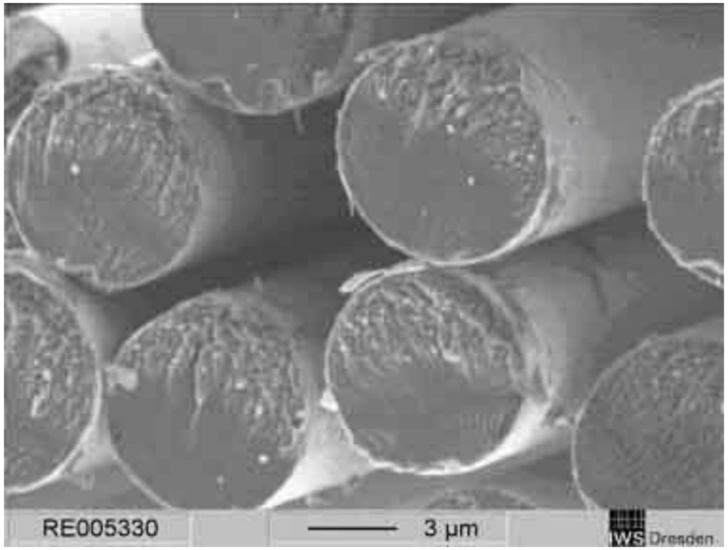


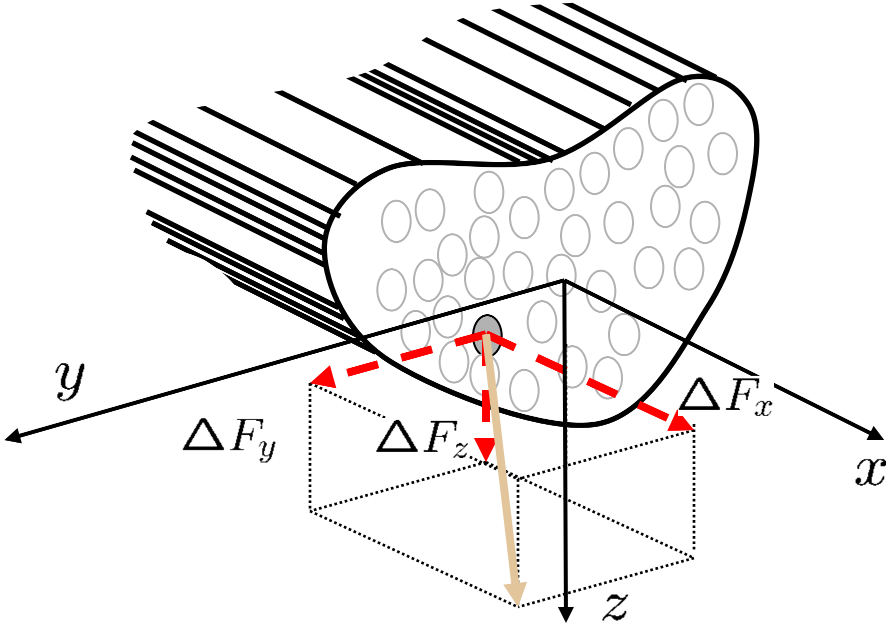
Interaction

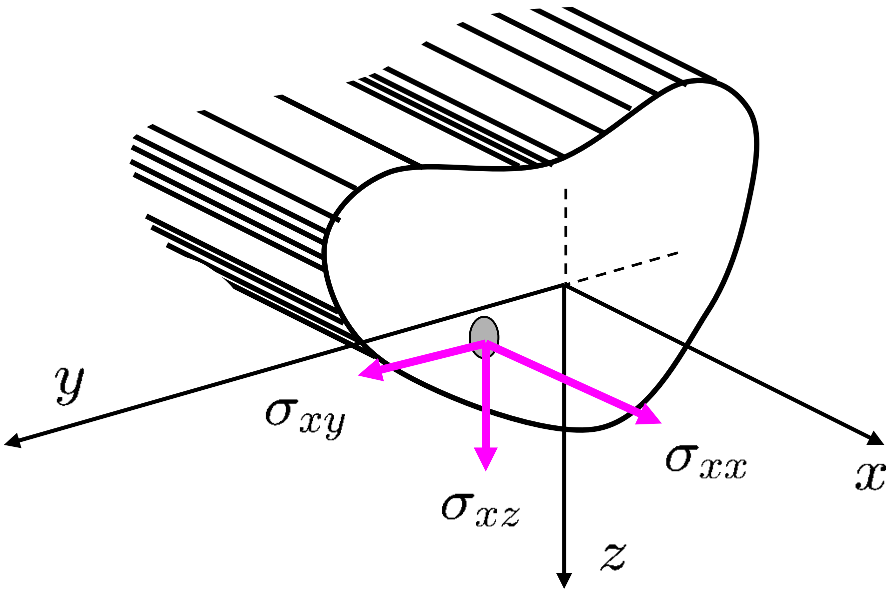



RE005330
3 µm
IWS Dresden


y
x
$\Delta \mathbf{F}$
z


RE005330
3 µm
IWS Dresden


y
x
z
ΔF_y
ΔF_z
ΔF_x



Normal stress

$$\sigma_{xx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A \cdot n_x}$$

Shear stresses

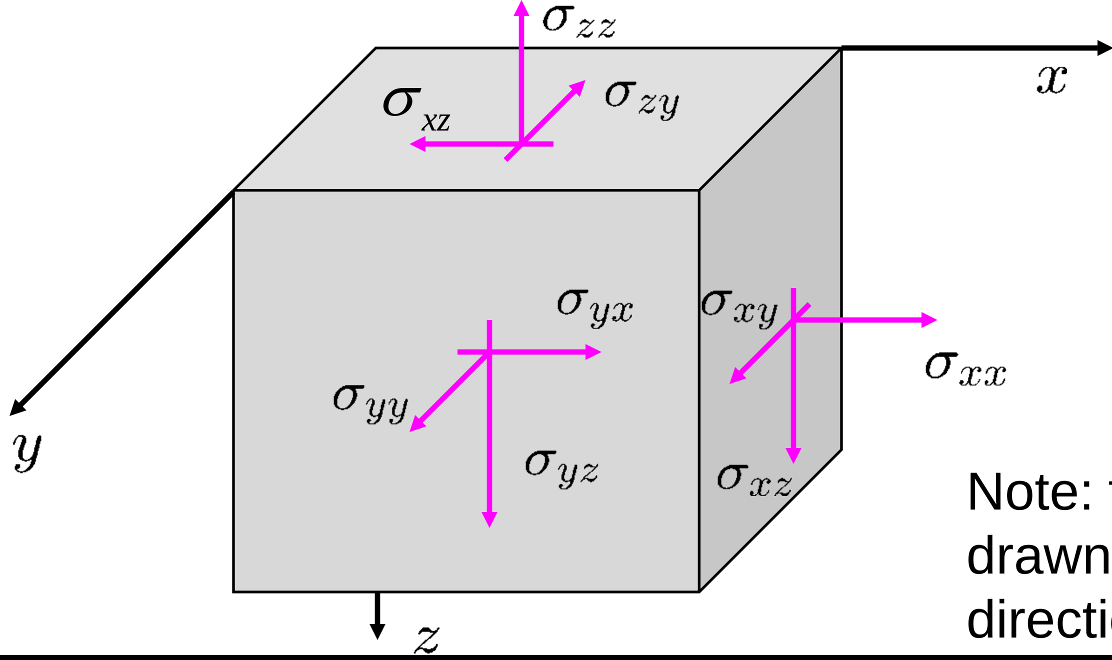
$$\sigma_{xy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A \cdot n_x}$$

$$\sigma_{xz} = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A \cdot n_x}$$

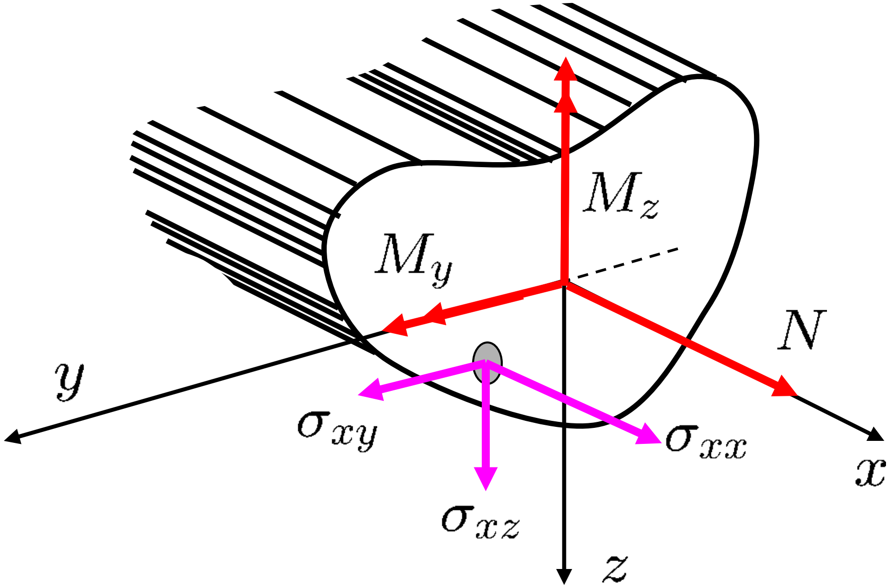
Sign convention

A stress component is ***positive*** when it is acting on a ***positive*** cutting-plane in the ***positive*** direction, or when it is acting on a ***negative*** cutting-plane in the ***negative*** direction.

A stress component is ***negative*** otherwise.



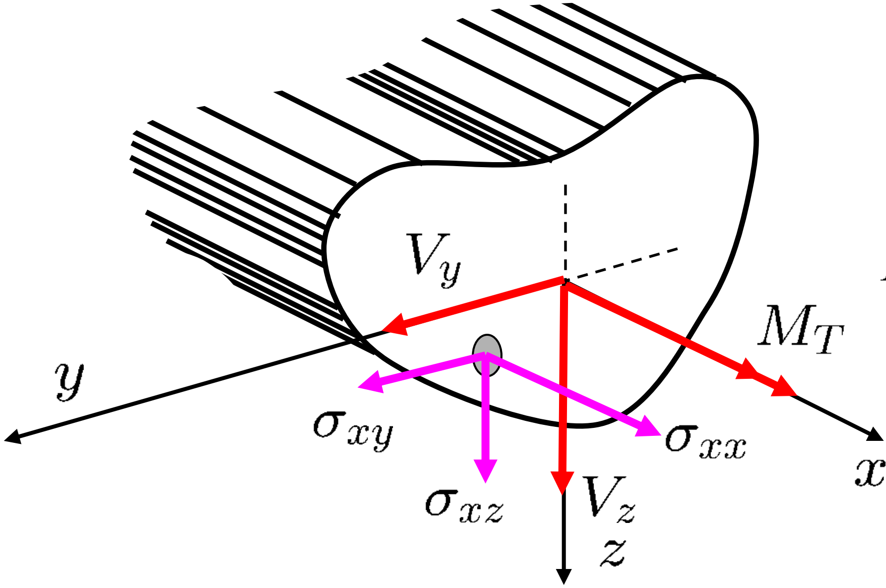
Note: the stresses are drawn in their positive directions.



$$N = \sum \Delta N = \sum \sigma_{xx} \Delta A = \int_A \sigma_{xx} dA$$

$$M_y = \sum z \Delta N = \sum z \sigma_{xx} \Delta A = \int_A z \sigma_{xx} dA$$

$$M_z = \sum y \Delta N = \sum y \sigma_{xx} \Delta A = \int_A y \sigma_{xx} dA$$



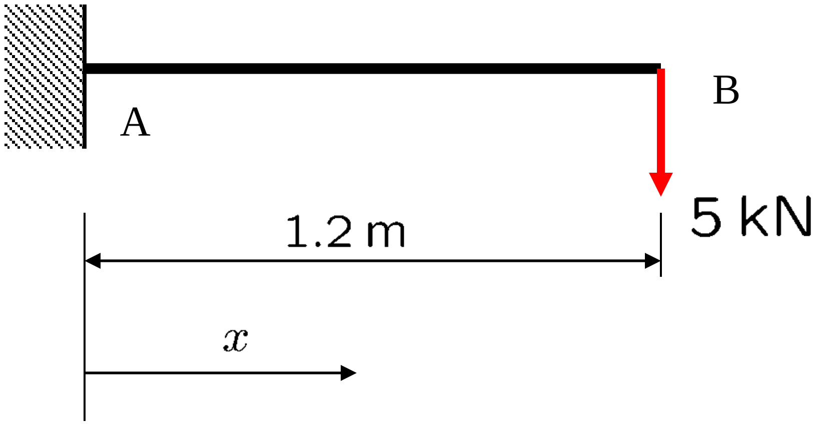
$$V_y = \sum \Delta V_y = \sum \sigma_{xy} \Delta A = \int_A \sigma_{xy} dA$$

$$V_z = \sum \Delta V_z = \sum \sigma_{xz} \Delta A = \int_A \sigma_{xz} dA$$

$$M_T = \sum y \Delta V_z - z \Delta V_y = \int_A (y\sigma_{xz} - z\sigma_{xy}) dA$$

Interaction force in two-dimensional calculations

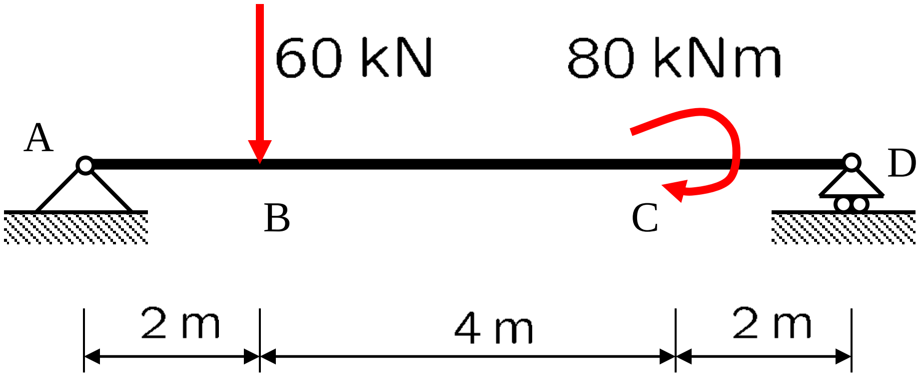
Normal force	N
Shear force	$V_z = V$
Bending moment	$M_y = M$

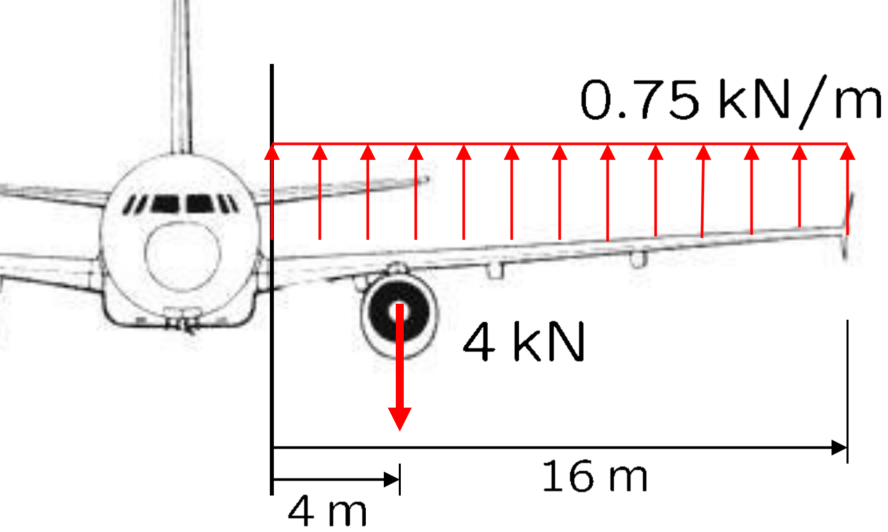


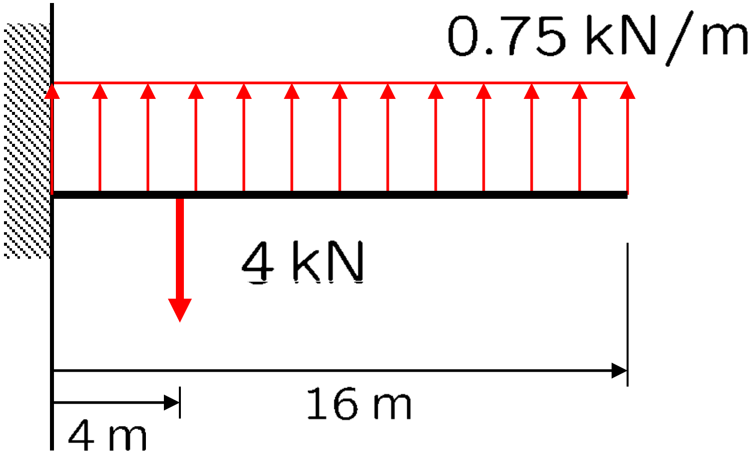
Determine the shear force and moment diagram of the cantilever beam.

(Determine the interaction force *V* and *M* for every position *x* on the beam).

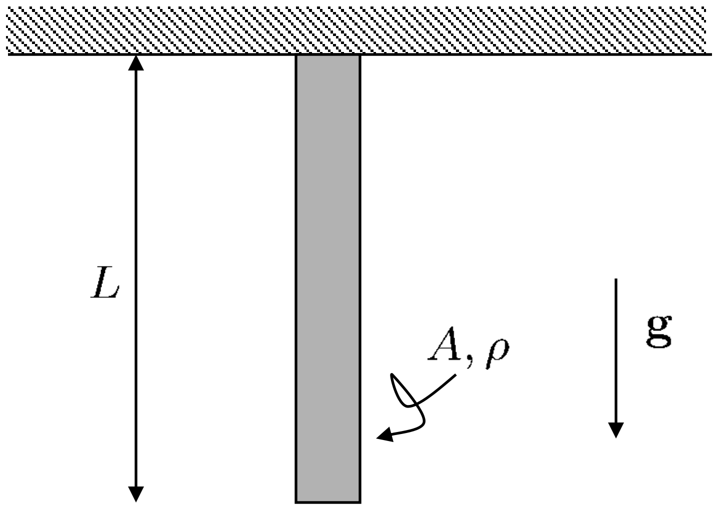
Determine the shear force and moment diagram of this structure.







Determine the shear force and moment diagram of the wing of this airplane.



Determine the normal force diagram in the bar with length *L*, cross-section *A* and density ρ due its own weight.

Bonus: The column is made of steel, ρ=7800 kg/m^3 with an ultimate stress σ_u=400 MPa. For what length *L* will the beam fail due to its own weight?

Handout: Elaboration_pendulum_example_Lecture_5a.pdf