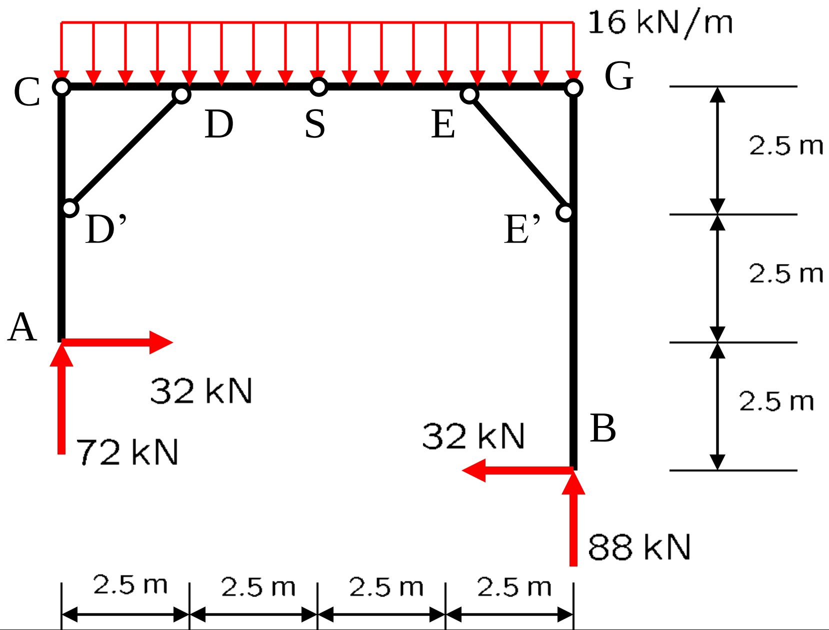
Today:

Virtual Work

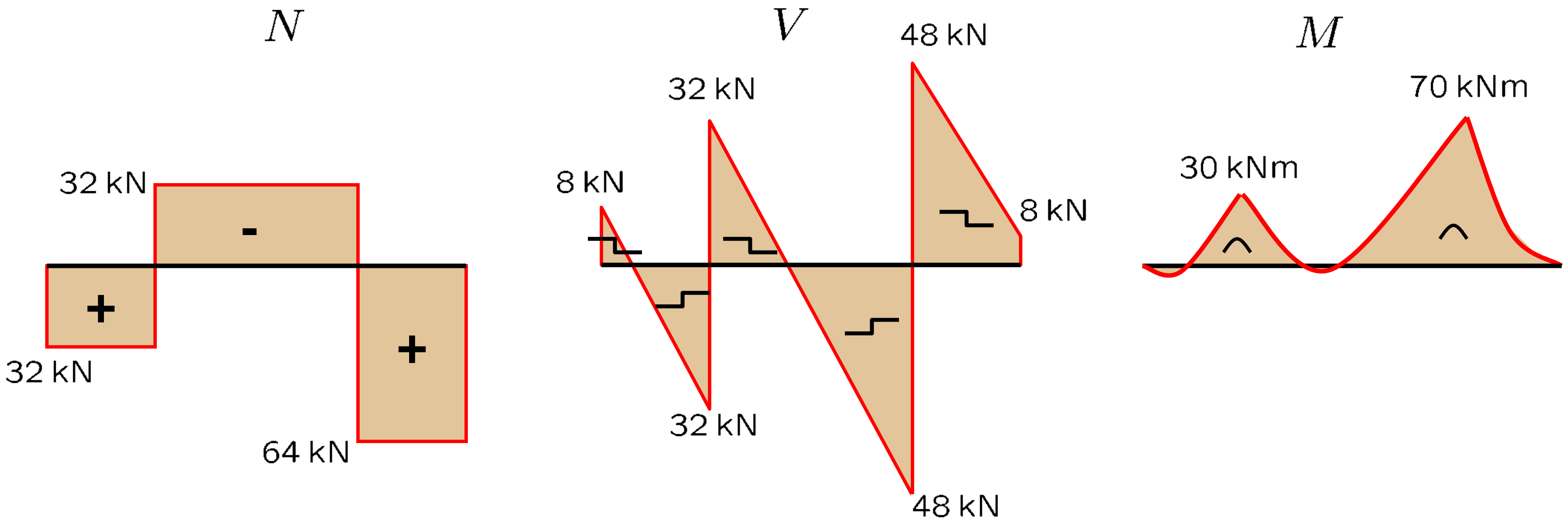
Book: Chapter 11.1-11.3 + hand-out



a) Calculate the reactions in A and B

b) Calculate the normal forces in DD’ and EE’ with the correct signs for tension and compression.

c) Draw the *N*, *V* and *M* lines for CDSEG


N
32 kN
-
+
32 kN
+
64 kN
V
48 kN
32 kN
8 kN
8 kN
32 kN
48 kN
M
70 kNm
30 kNm

A few tips…

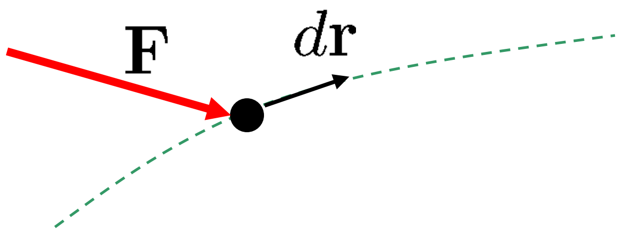
- When drawing N,V and M lines, clearly denote the jumps and kinks in the lines. Give the values of ‘special’ points on the curves (maxima, transition points).

A few tips…

- Use enough paper, work neatly. It will reduce the number of unnecessary mistakes.
- Before filling in the answering sheet, draw the N,V and M diagrams on a piece of scrap paper.

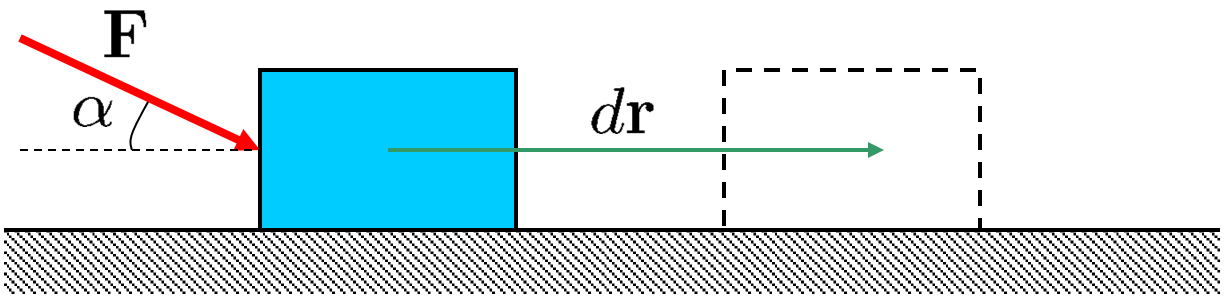
Work of a force (infinitesimal movement)

The work done by a force ***F*** acting on a particle with a differential displacement *d**r*** is defined as:



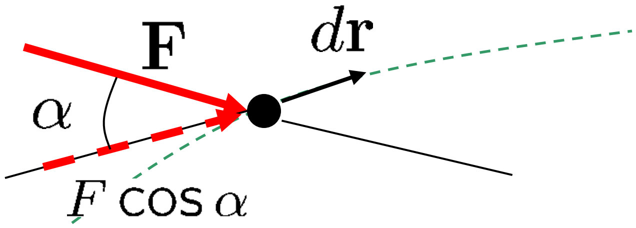
$$dW = \mathbf{F} \cdot d\mathbf{r}$$

W = [Nm] = kg m^2/s^2] = [J]

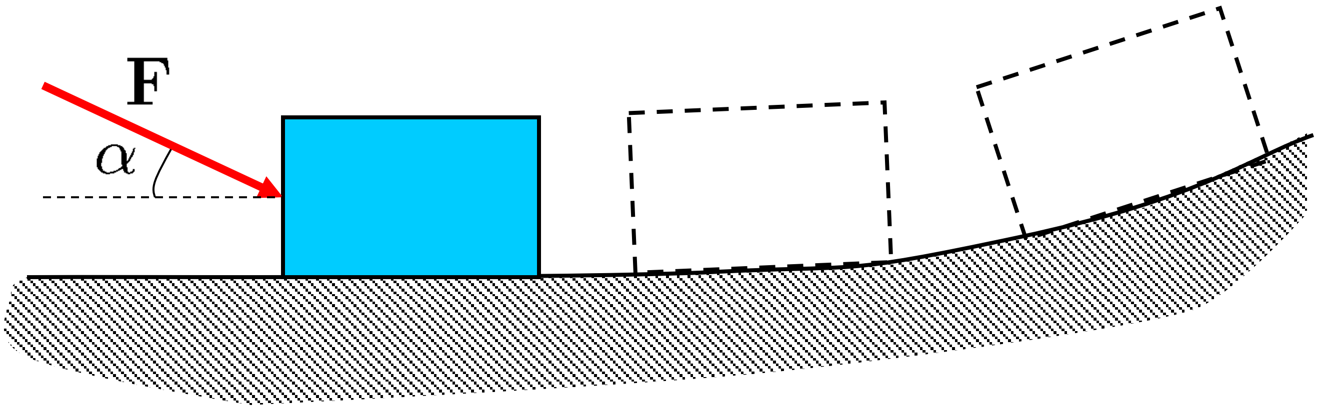

F
α
dr

Work of a force (infinitesimal movement)

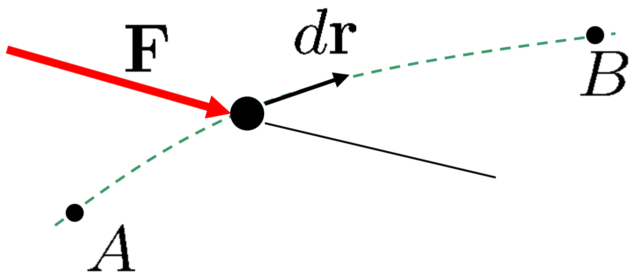
The work done by a force $\boldsymbol{F}$ acting on a particle with a differential displacement $d\boldsymbol{r}$ is defined as:



$$dW = |\mathbf{F}||d\mathbf{r}|\cos\alpha$$


F
α

Work of a force (finite movement)



$$W = \int \mathbf{F} d\mathbf{r} = \int \left(F_x dr_x + F_y dr_y + F_z dr_z \right)$$

Static equilibrium of a particle

$$\sum \mathbf{F}_x = 0$$

$$\sum \mathbf{F}_y = 0$$

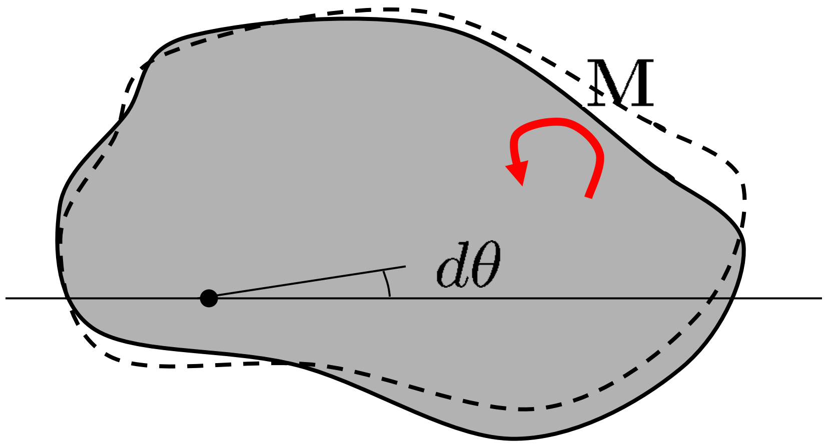
$$\sum \mathbf{F}_z = 0$$

The principle of virtual work (particle)

A particle is in equilibrium when for any variational displacement, the virtual work is equal to zero.

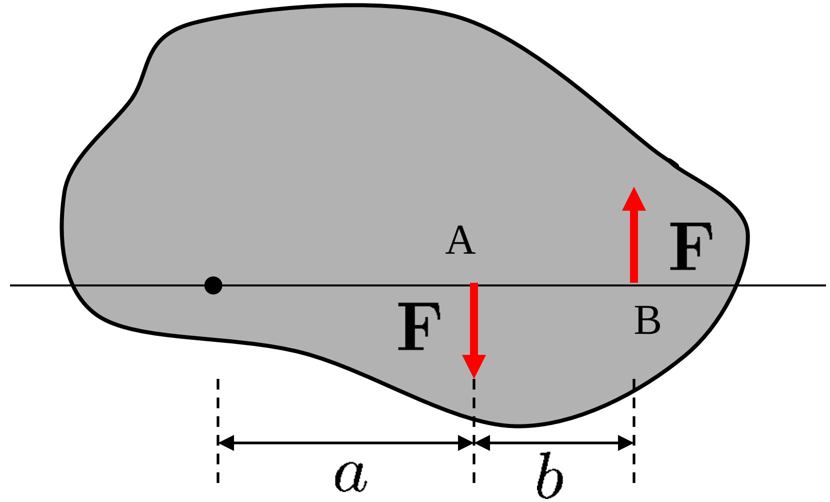
$$\delta W = \delta u_x \sum F_x + \delta u_y \sum F_y + \delta u_z \sum F_z = 0$$

Work of a concentrated moment



Work of a concentrated moment

$$M = bF$$



Small-Angle approximations

$$\theta << 1 \text{ radians}$$

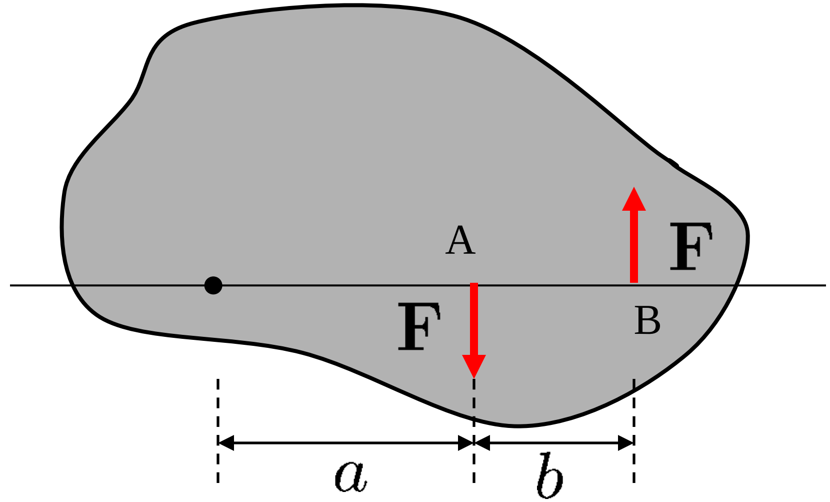
$$\sin\theta \approx \theta$$

$$\tan\theta \approx \theta$$

$$\cos\theta \approx 1$$

Work of a concentrated moment

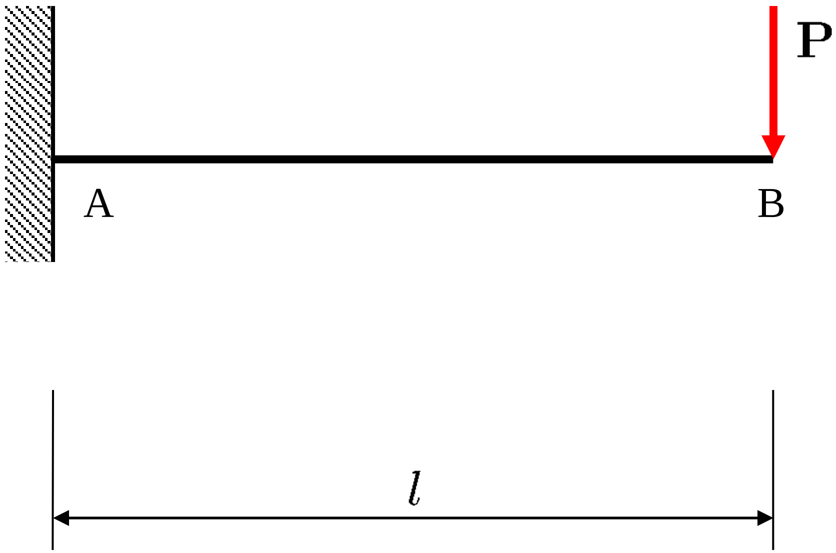
$$M = bF$$



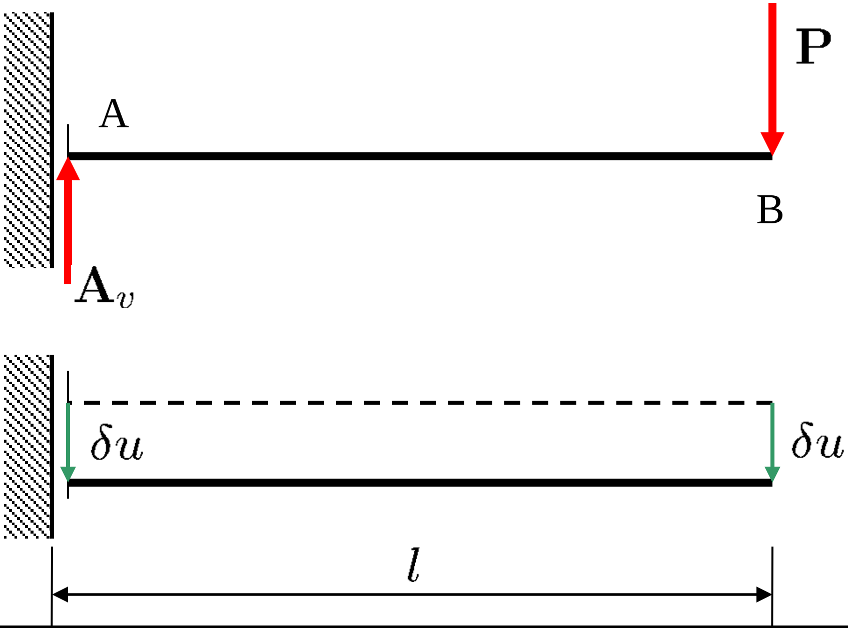
The principle of virtual work (body in two dimensions)

A particle is in equilibrium when for any variational displacement and/or rotation, the virtual work is equal to zero.

$$\delta W = \sum \delta u_{x_i} F_{x_i} + \sum \delta u_{y_i} F_{y_i} + \sum \delta \theta_i M_i = 0$$

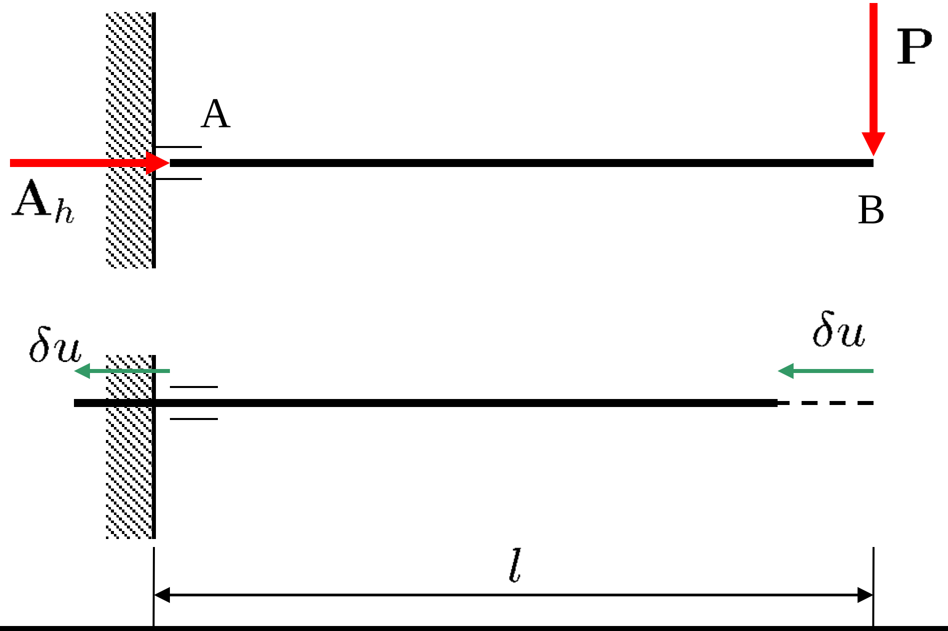


Calculate the reaction forces and moment in *A* using the principle of virtual work.



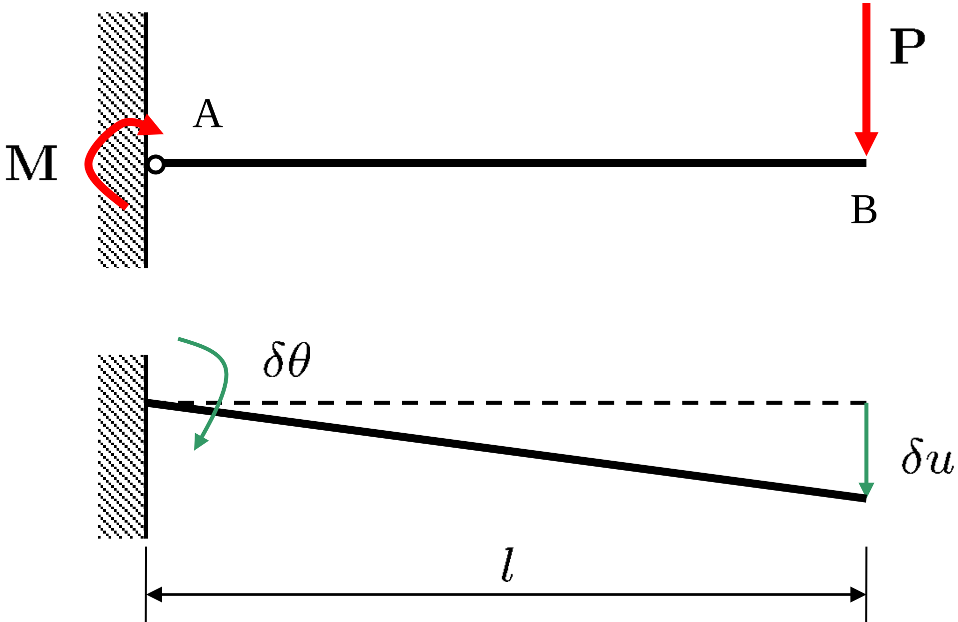
Calculate the reaction forces and moment in *A*.

Vertical reaction



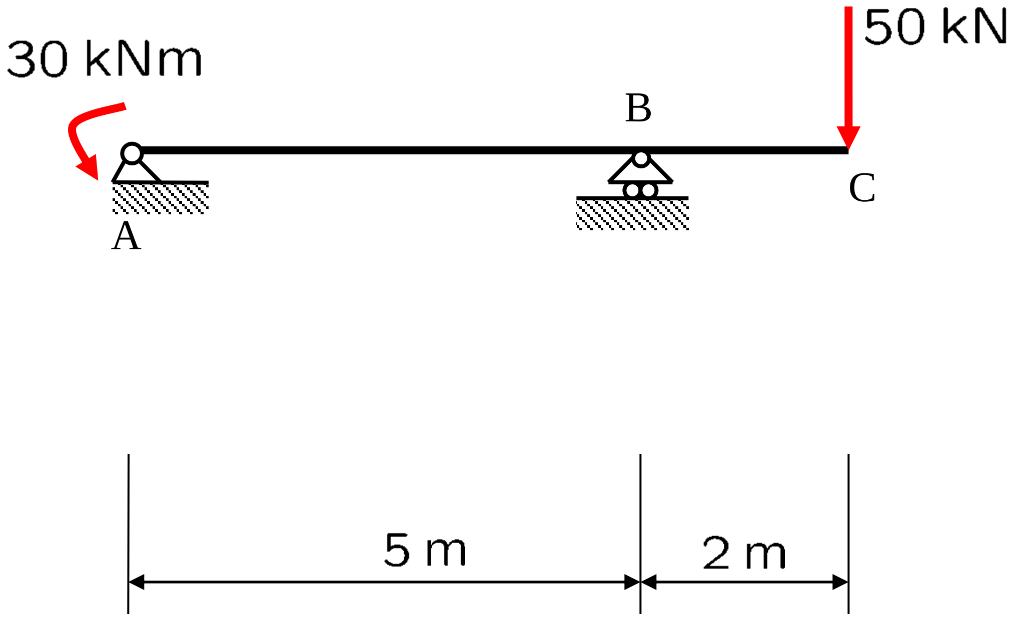
Calculate the reaction forces and moment in *A*.

Horizontal reaction

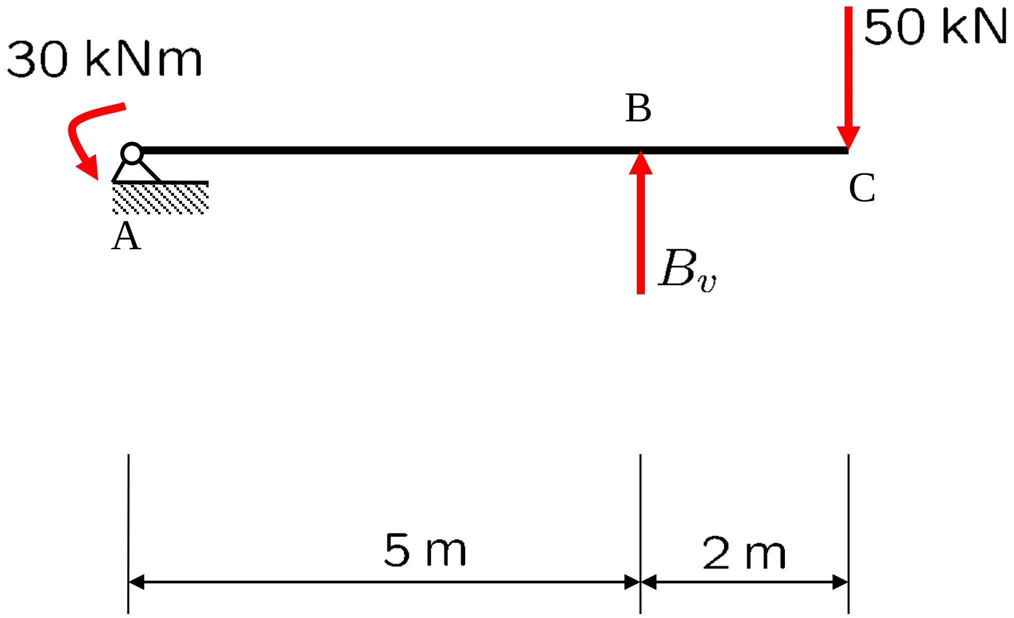


Calculate the reaction forces and moment in *A*.

Moment

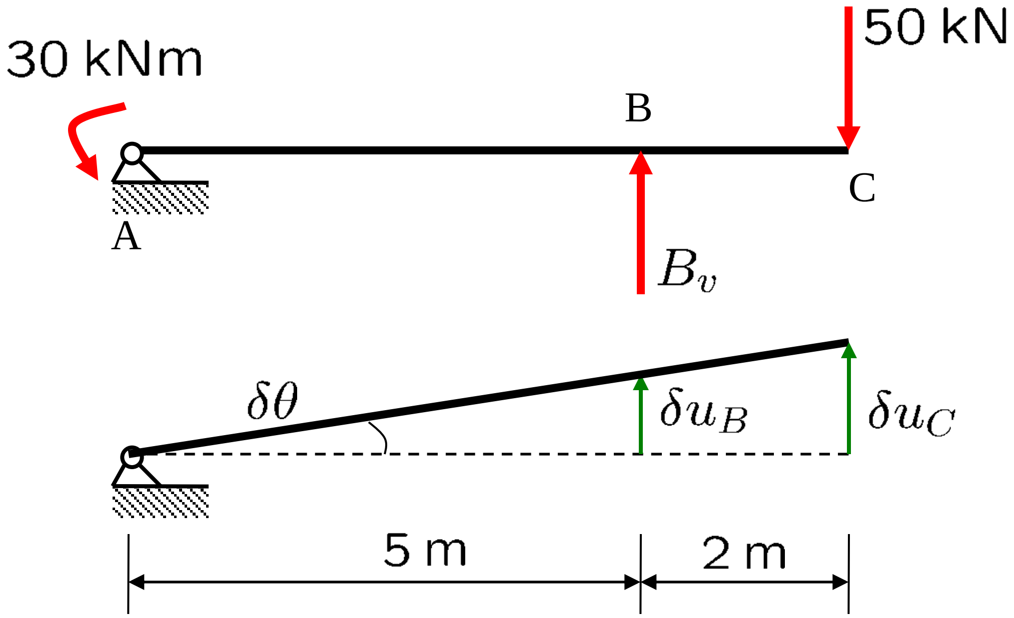


Calculate the reaction in B



Calculate the reaction in B

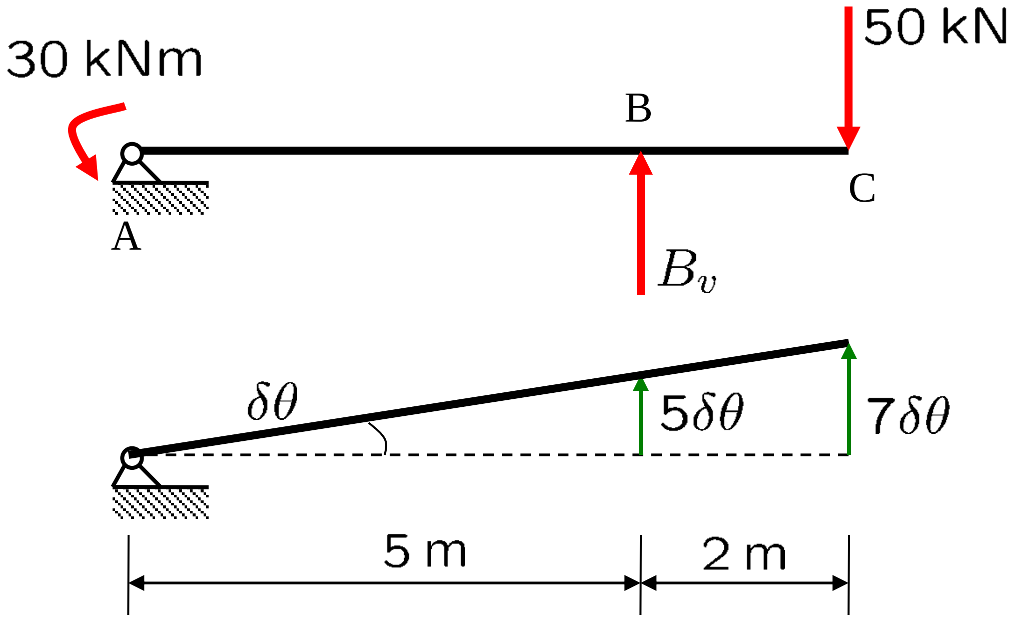
Replace hinge B by a reaction force.



Calculate the reaction in B

Replace hinge B by a reaction force.

Due to the absence of hinge B, the beam can rotate

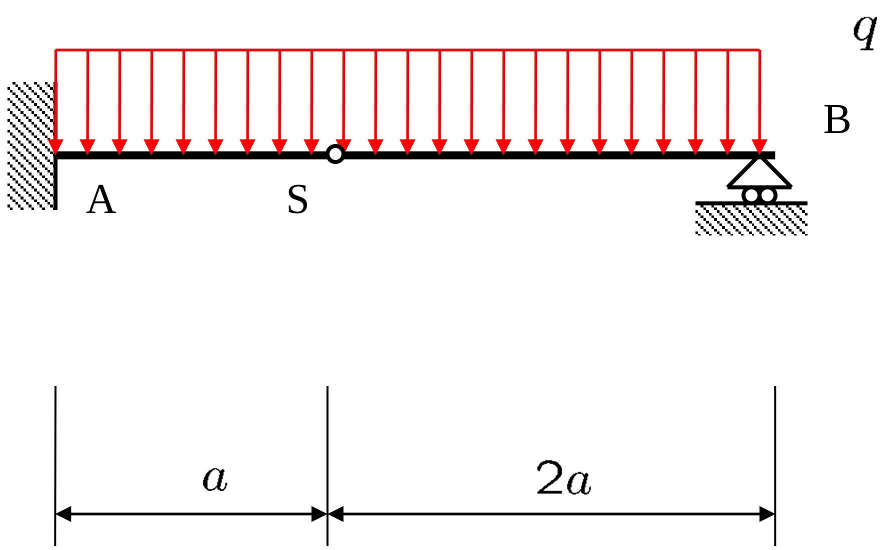


Calculate the reaction in B

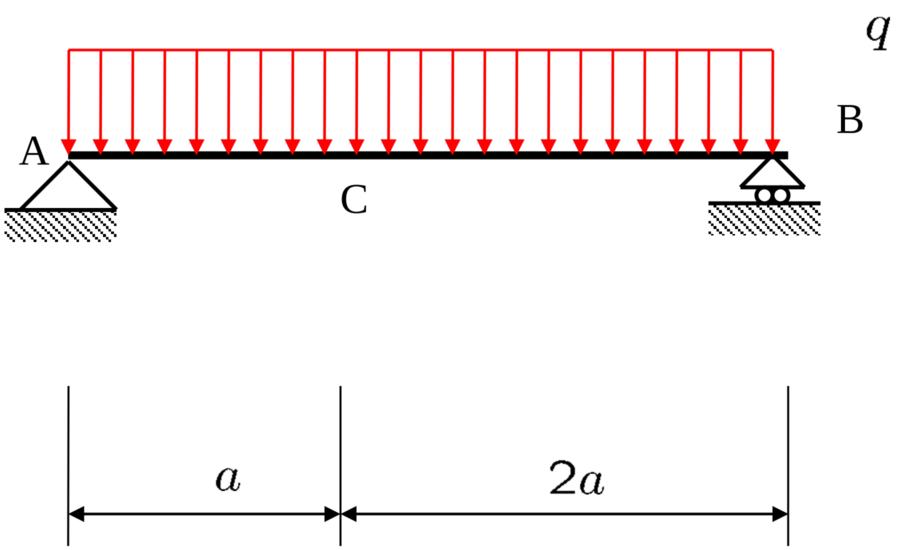
Replace hinge B by a reaction force.

Due to the absence of hinge B, the beam can rotate

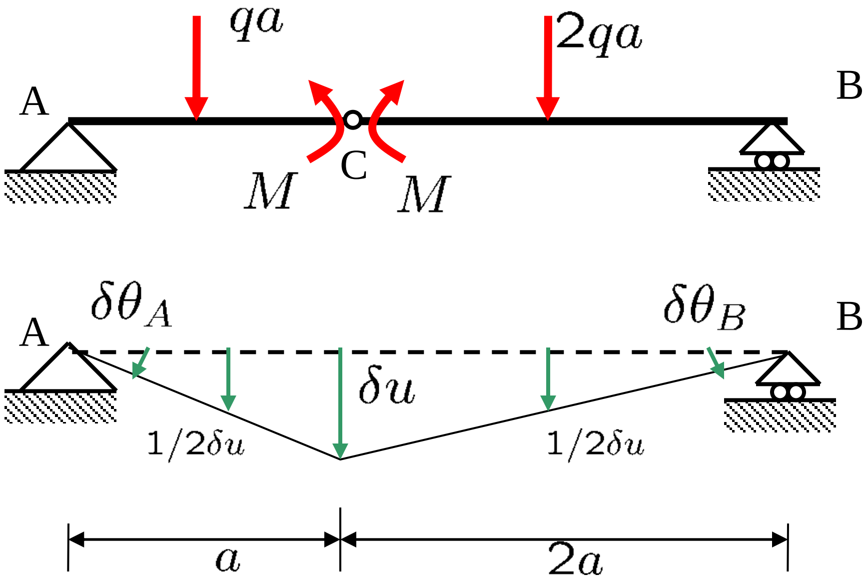
Express all virtual rotations in terms of $\delta\theta$



Calculate the reaction moment in A



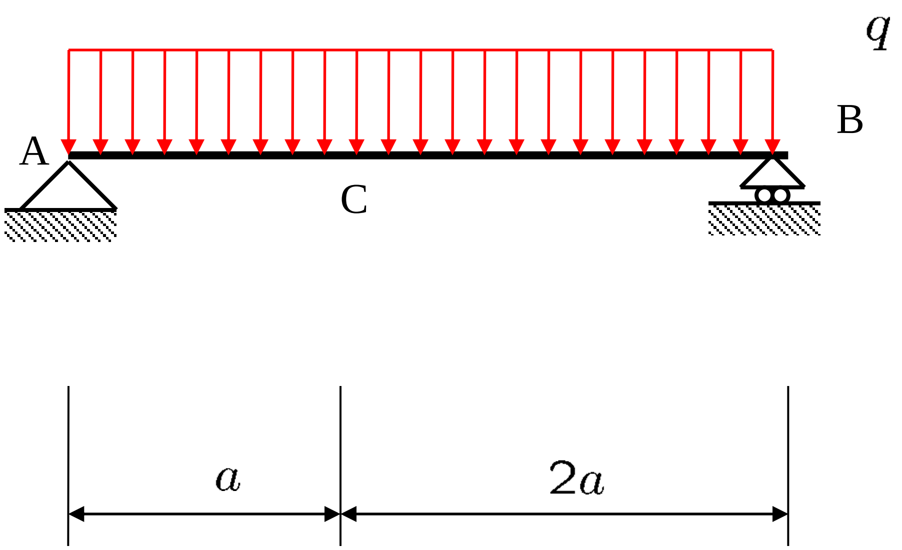
Calculate the internal moment in C



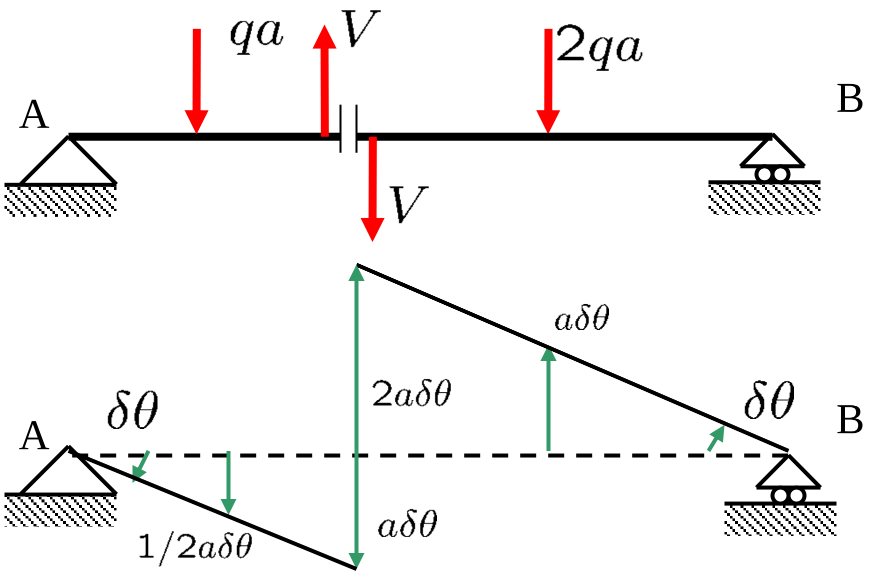
Calculate the internal moment in C

Internal moment

- **Hinge**: only different rotation angles on either side of the cut.
- Reaction moments *M*



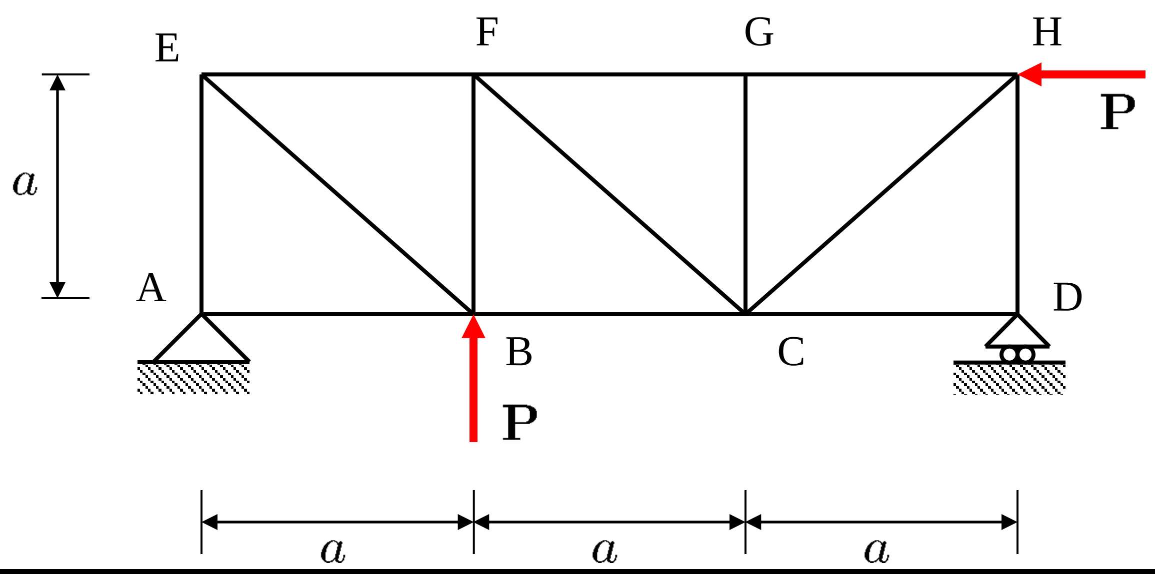
Calculate the shear force in C



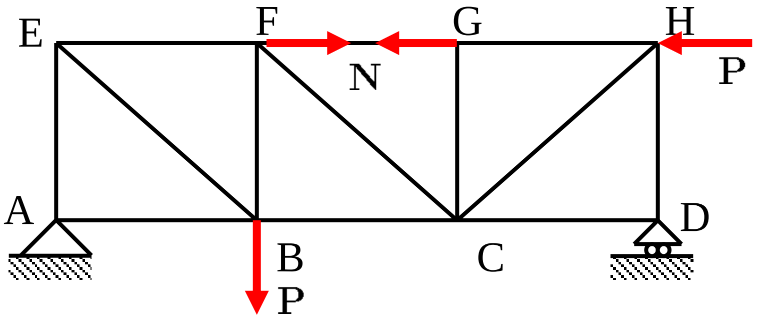
Calculate the shear force in C

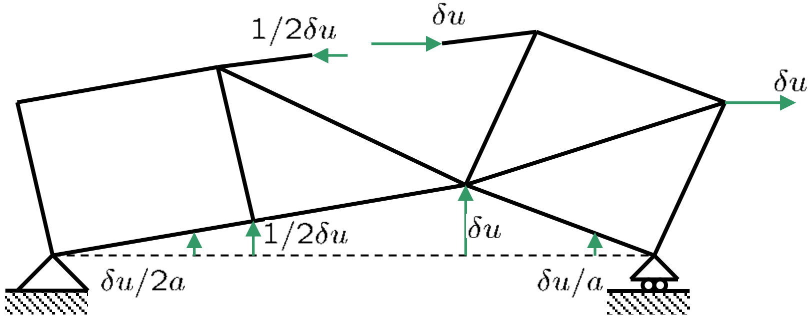
Shear force

- **Shear hinge**: only different displacements perpendicular to the member on either side of the cut.
- Reaction forces *V*



Calculate the normal force in member FG of this truss structure.





Calculate the normal force in member FG of this truss structure.

Normal force

- **Telescope hinge**: only different displacements parallel to the member on either side of the cut
- Reaction forces *N*