

CIE4801 Transportation and spatial modelling

Assignment: special topics

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Content

- System optimal assignment
- Public transport
 - Differences
 - Modelling approaches
 - Network representations
 - Algorithms
 - Example Barcelona

1.

System optimal assignment

Assignment: optimisation of objectives

- Objective DUE $\min_{q_a} \sum_a \int_{x=0}^{q_a} t_a(x) dx$
- Objective AON $\min_{q_a} Z = \sum_a t_a q_a$
- Similarity or difference?
- Both minimise the surface under function t_a , but.....
- DUE: additional traveller affects travel time of all travellers
AON: additional traveller doesn't affect other travellers' travel times

DSO assignment

Deterministic user-equilibrium (DUE):

Assignment in which each traveller minimizes his/her own travel time



Deterministic system optimum (DSO):

Assignment in which the total travel time for all travellers is minimized

DSO assignment: formulation

DUE

$$\min_{q_a} Z_1 = \sum_a \int_{x=0}^{q_a} t_a(x) dx$$

DSO

$$\min_{q_a} Z_2 = \sum_a t_a(q_a) \cdot q_a$$

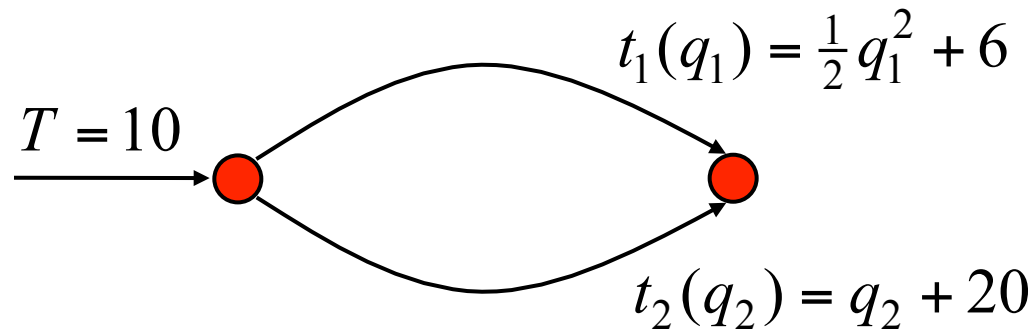
subject to:

$$\sum_r T_{ijr} = T_{ij} \quad \forall i, j$$

$$q_a = \sum_i \sum_j \sum_r \alpha_{ijr}^a T_{ijr} \quad \forall a$$

$$T_{ijr} \geq 0 \quad \forall i, j, r$$

Example DSO assignment



DUE assignment

$$q_1 = 6, t_1 = 24$$

$$q_2 = 4, t_2 = 24$$

total travel time:
240

DSO assignment

$$q_1 = 4.14, t_1 = 14.57$$

$$q_2 = 5.86, t_2 = 25.86$$

total travel time:
211.8

Analogy with Game Theory: “Prisoners’ dilemma”

Individuals X and Y committed a crime
They are in jail and are asked individually to confess

Depending on the confession of both X and Y, they face a prison time

		Y	
		confess	don't confess
X	confess	(5, 5)	(0, 20)
	don't confess	(20, 0)	(1, 1)

Nash-equilibrium
(# years in prison)

Better off if cooperating!

Pareto-optimum

How to solve for a DSO assignment?

We can use the same algorithm for solving for a DUE assignment by *replacing the travel time function!*

$$\min_{q_a} \sum_a t_a(q_a) \cdot q_a \quad \Rightarrow \quad \min_{q_a} \sum_a \int_{x=0}^{q_a} t_a^*(x) dx$$

What should we choose for $t_a^*(q_a)$?

$$f(x) = \int f'(x) dx$$

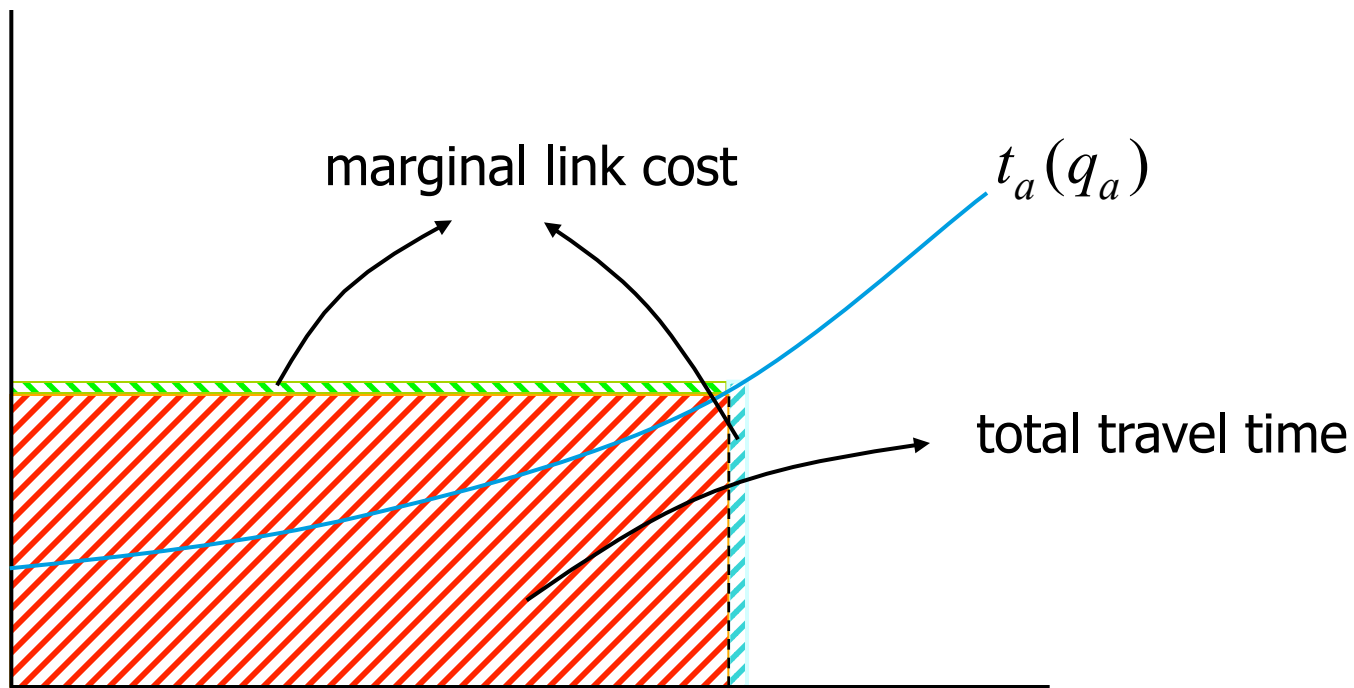
$$f(x) \rightarrow t_a(q_a) \cdot q_a$$

marginal link cost

$$f'(x) \rightarrow \frac{d}{dq_a} (t_a(q_a) \cdot q_a) = t_a(q_a) + \overbrace{t'_a(q_a) \cdot q_a}^{\text{marginal link cost}} \equiv t_a^*(q_a)$$

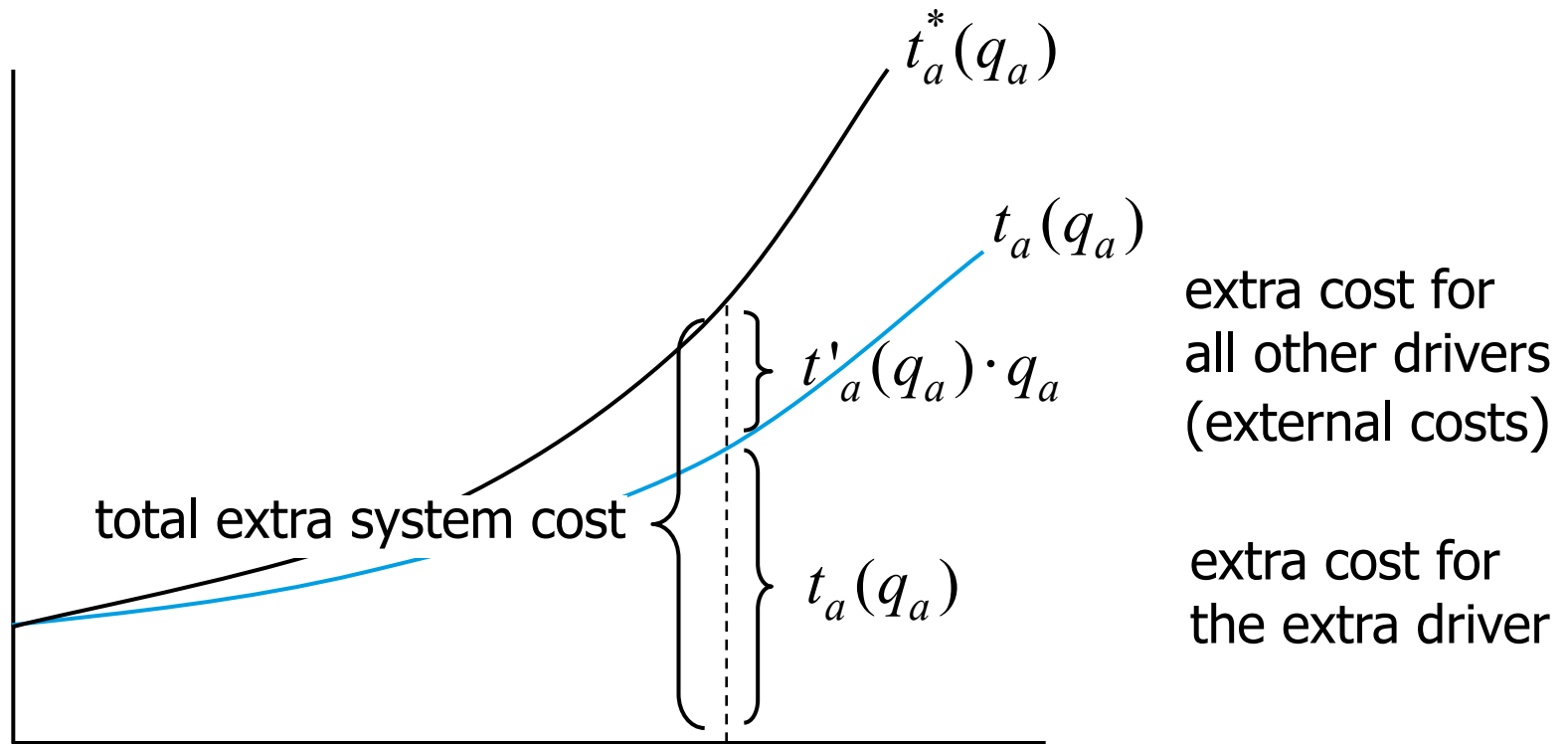
DSO assignment

Marginal link cost =
total extra cost of one extra vehicle on the link
(when there are already q_a vehicles)



Marginal link cost concept

Marginal link cost =
total extra cost of one extra vehicle on the link
(when there are already q_a vehicles)



DSO assignment: Example

Suppose the link travel times are described by the BPR function:

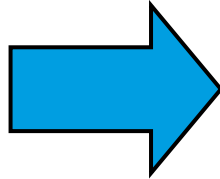
$$t_a(q_a) = t_a^0 \left[1 + \alpha \left(\frac{q_a}{C_a} \right)^\beta \right]$$

Which link travel time function should be used such that we can solve for a DSO assignment by using a DUE algorithm (such as Frank-Wolfe) ?

$$\begin{aligned} t_a(q_a) + t'_a(q_a) \cdot q_a &= t_a^0 \left[1 + \alpha \left(\frac{q_a}{C_a} \right)^\beta \right] + t_a^0 \frac{\alpha\beta}{C_a} \left(\frac{q_a}{C_a} \right)^{\beta-1} \cdot q_a \\ &= t_a^0 \left[1 + \alpha(\beta + 1) \left(\frac{q_a}{C_a} \right)^\beta \right] \end{aligned}$$

DSO assignment in practice?

How to reach a system optimal assignment in practice?

 **road pricing** ('rekeningrijden', 'kilometerheffing')

How much should the road pricing toll be on each link?

$$t_a^*(q_a) \equiv t_a(q_a) + \underbrace{t'_a(q_a) \cdot q_a}_{\approx \text{link toll}}$$

Public transport assignment

2.

Difference between car and public transport assignment

Differences between car & PT assignment

- Network itself
- Congestion effects
- Route travel costs

Network itself

Car: physical infrastructure network

PT: network of services

Space accessibility

- stops
- lines

Time accessibility

- frequencies
- operating hours

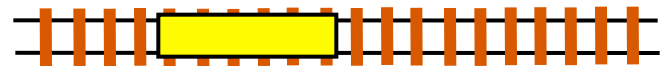
(access links, transfer links, in-vehicle links, ...)

Link travel times

Car: $t_a = t_a(q_a)$



PT: $t_a = t_a^0$



Bus travel times?

Crowding?

Route travel costs

$$\text{Car: } c_{ijr} = \sum_a \alpha_{ijr}^a c_a \quad (c_a = t_a)$$

Additive costs

$$\text{PT: } c_{ijr} = \dots$$

$$c_{ijr} = \overset{2.2}{\gamma_1} t_{ijr}^{\text{access}} + \overset{1.5}{\gamma_2} t_{ijr}^{\text{waiting}} + \overset{1.0}{\gamma_3} t_{ijr}^{\text{in-vehicle}} + \overset{1.1}{\gamma_4} t_{ijr}^{\text{egress}} + \\ + \overset{2.3}{\gamma_5} t_{ijr}^{\text{walking_int}} + \overset{1.3}{\gamma_6} t_{ijr}^{\text{waiting_int}} + \overset{5.7}{\gamma_7} N$$

Non-additive costs (?)

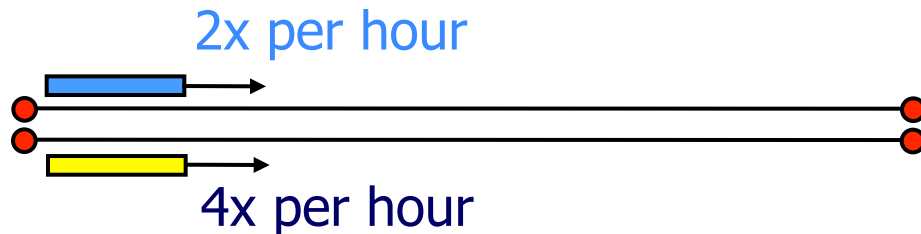
Consequences for assignment?

Non-additive costs would mean that **route-enumeration** is necessary.

Once the routes have been enumerated, the **route costs** can be computed.

How can frequencies, waiting times, and PT costs be determined for specific routes?

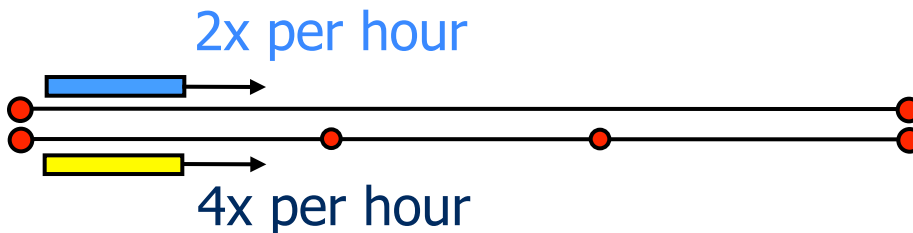
Common lines: frequencies



$$\max\{f_1, f_2\} \leq f \leq f_1 + f_2$$

What is the frequency along this route?

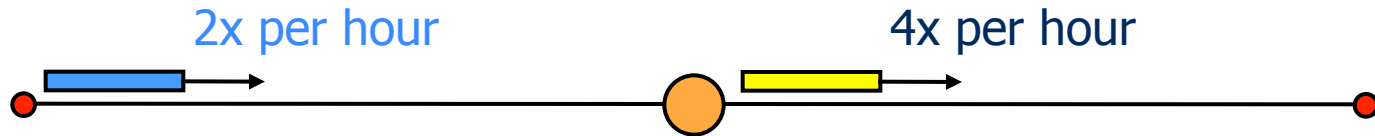
$$4 \leq f \leq 4 + 2$$



What is the frequency along this route?

???

Examples waiting times and costs



What is the **waiting time** at the transfer node?

- based on frequency of the yellow train: $\tau^{\text{waiting}} \approx 7.5 \text{ min.}$
- assuming the lines are synchronized: $\tau^{\text{waiting}} \approx 0.0 \text{ min.}$
- assuming the time schedules are known: ...

How much are the **PT costs**?

Depends on:

- Zone based, distance based, boarding penalty
- single trip or round trip
- combined ticket (e.g. 'strippenkaart')
- discount pass
- timing of the trip

3.

Modelling approaches

Criteria for modelling approach

- Network
 - complexity and size
- Algorithm
 - complexity
- Level of detail of results
 - modal split
 - average or maximum line occupancies
 - impact of ITS
- Time dimension
 - long term
 - short term

Algorithms

- Shortest path
- Multi routing (i.e. between pairs of stops)
- Multiple routing (between origin and destination)
- Feasible strategy
- Enumeration of route sets

Representations & algorithms

Representations

- Line based
 - Trunk lines
 - Line specific
- Route sections

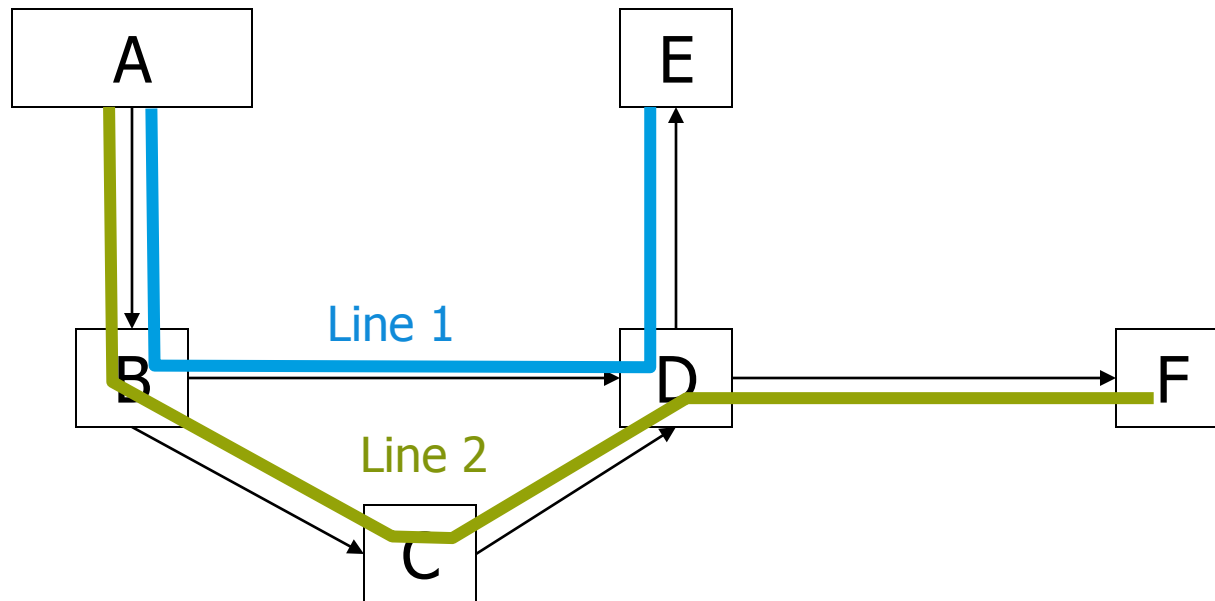
Algorithms

- Floyd-Warshall
- Strategies
 - Frequency based
 - Sequential choices

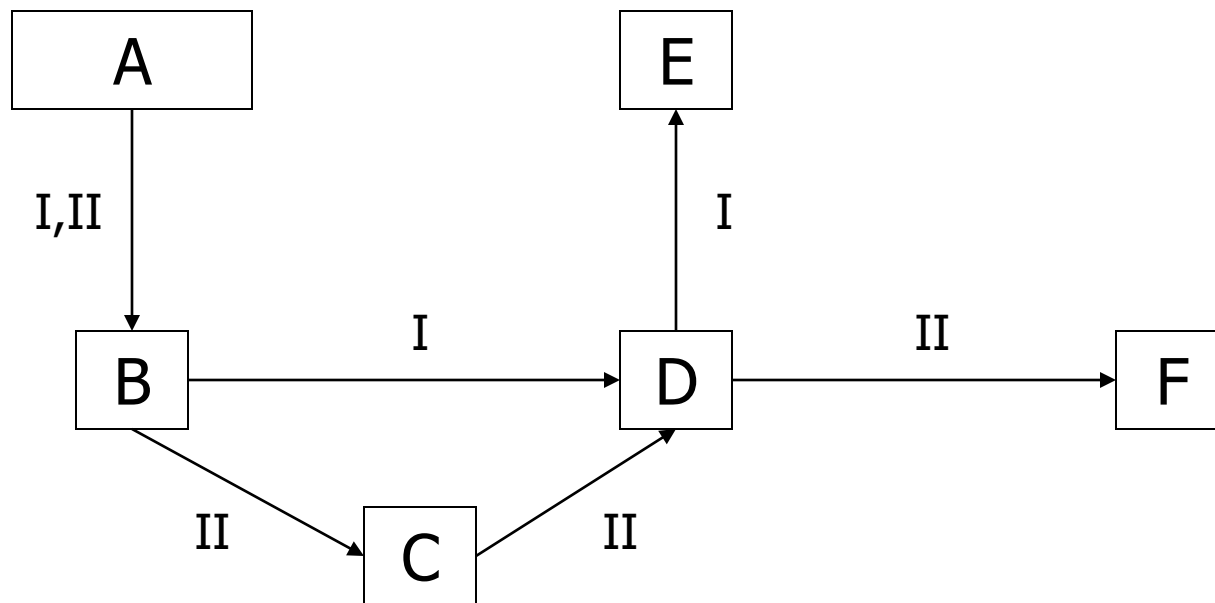
4.

Network representations

Example with 2 transit lines

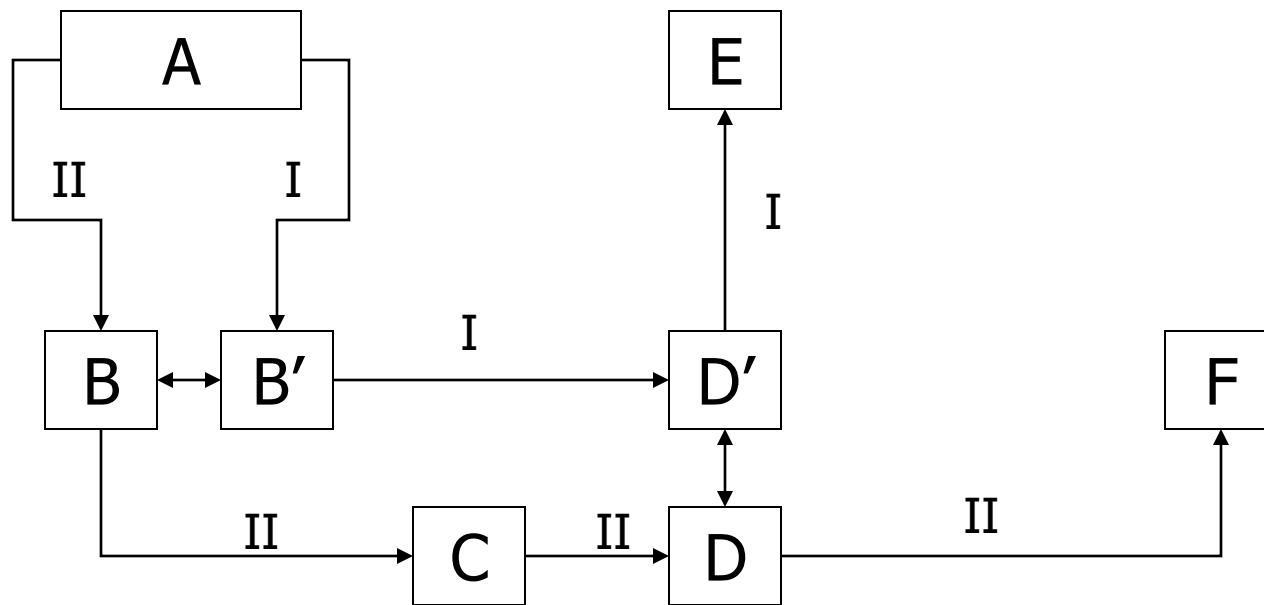


Trunk line links



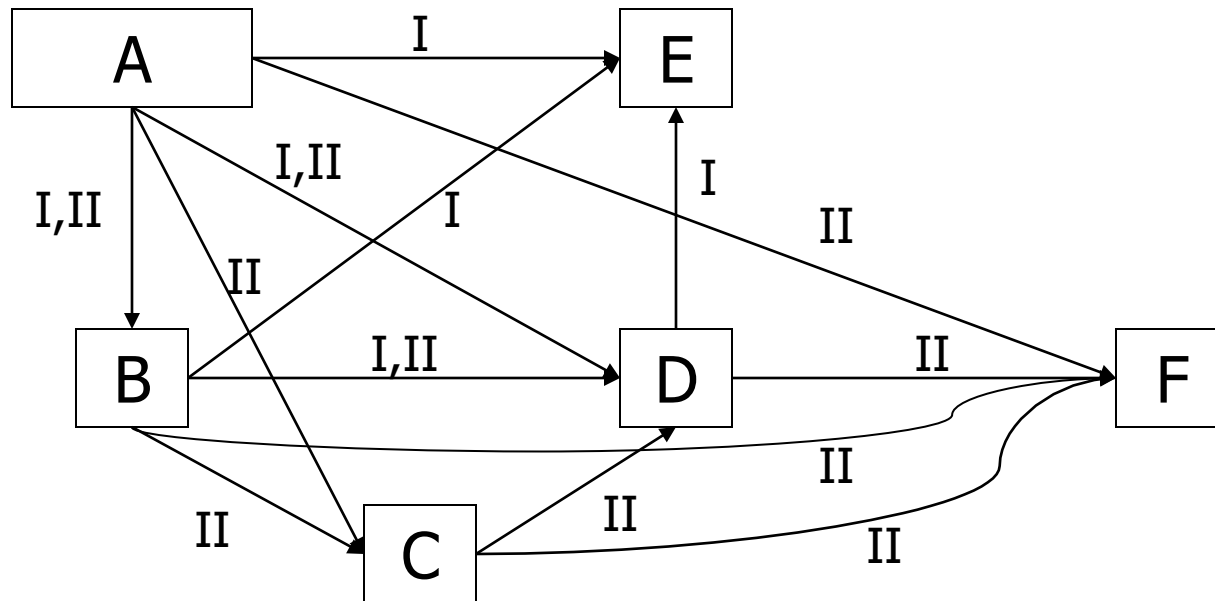
Transfer in algorithm

Line specific network



Transfer links

Route Sections

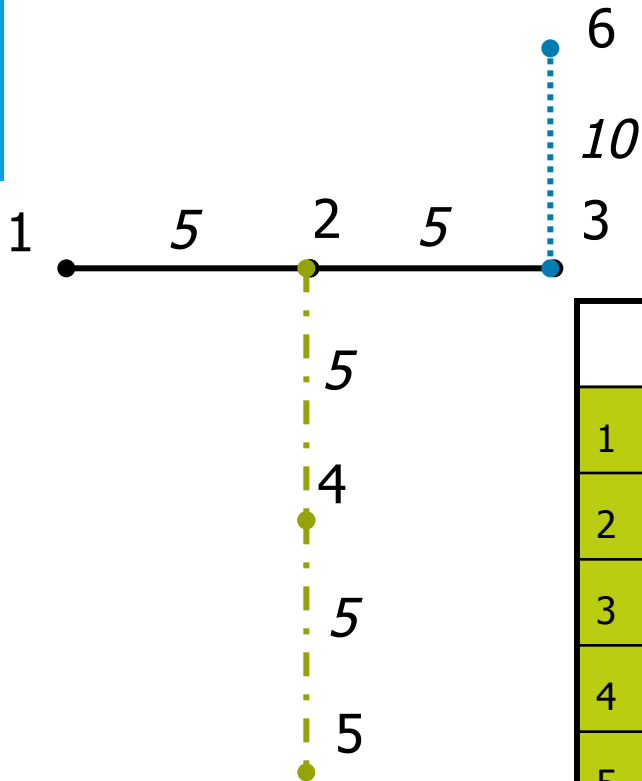


“Non-transfer links”

5.

Algorithms

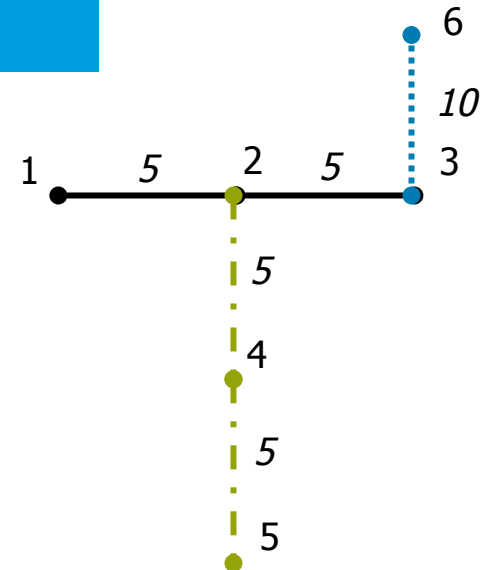
Floyd



	1	2	3	4	5	6
1	X	5 † 1	10 † 1	∞	∞	∞
2	5 † 2	X	5 † 2	5 † 2	10 † 2	∞
3	10 † 3	5 † 3	X	∞	∞	10 † 3
4	∞	5 † 4	∞	X	5 † 4	∞
5	∞	10 † 5	∞	5 † 5	X	∞
6	∞	∞	10 † 6	∞	∞	X

Time | back node

Floyd



	1	2	3	4	5	6
1	X	5 1	10 1	10+Pt 2	15+Pt 2	20+Pt 3
2	5 2	X	5 2	5 2	10 2	∞
3	10 3	5 3	X	∞	∞	10 3
4	∞	5 4	∞	X	5 4	∞
5	∞	10 5	∞	5 5	X	∞
6	∞	∞	10 6	∞	∞	X

For $i = 1, N$

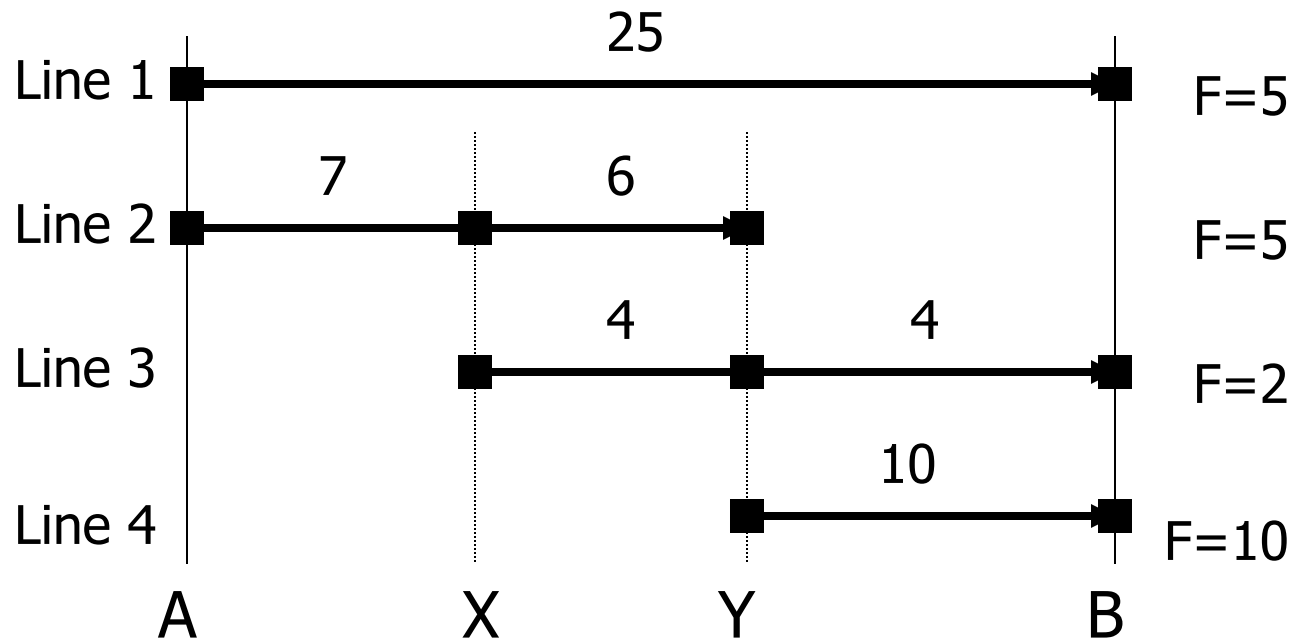
for $j = 1, N$

for $k=1, N$

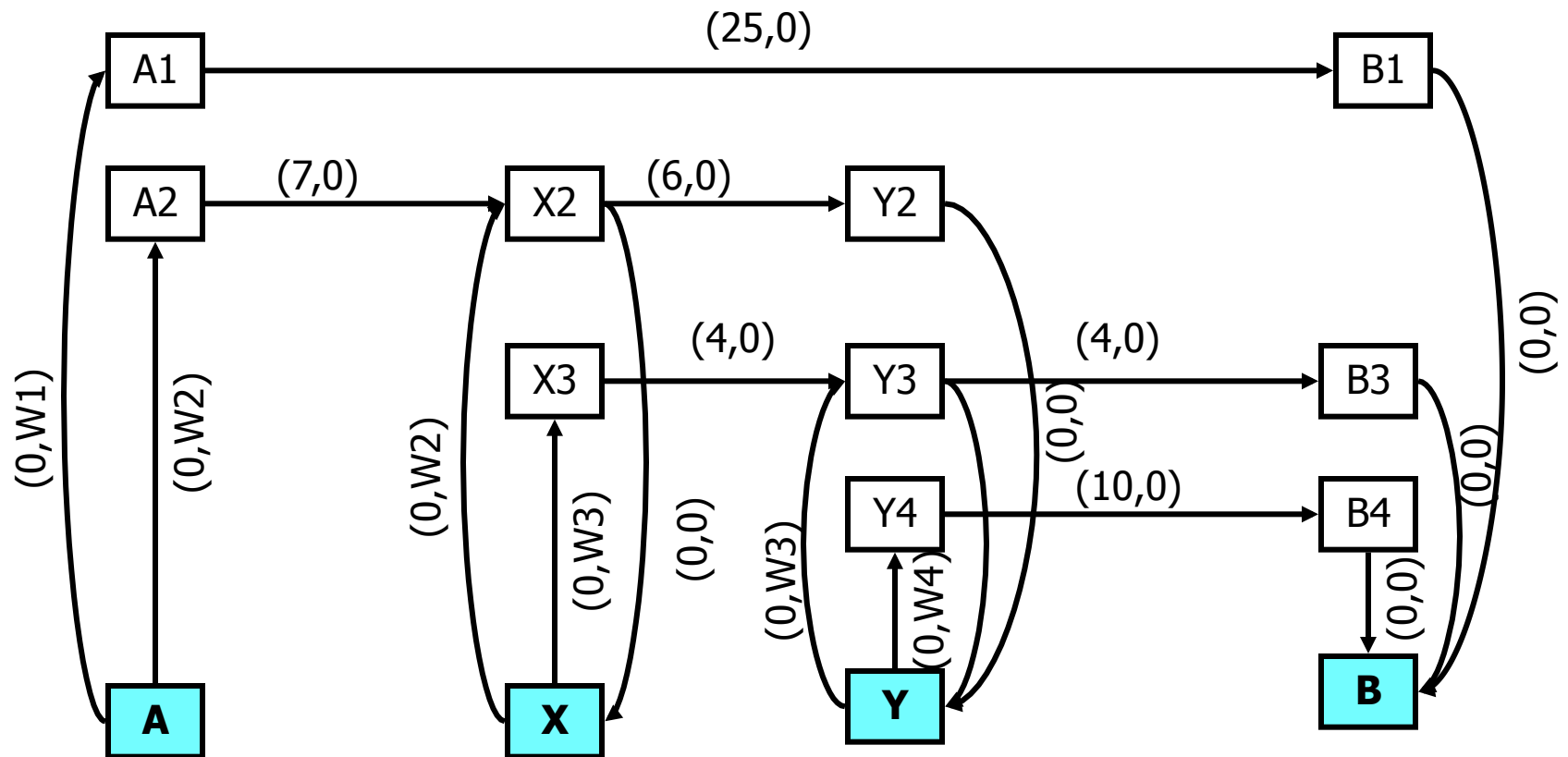
If $Z_{ikj} = Z_{ik} + P_t + Z_{kj} < Z_{ij}$

$Z_{ij} = Z_{ikj}$, back node(Z_{ij}) back node (Z_{kj})

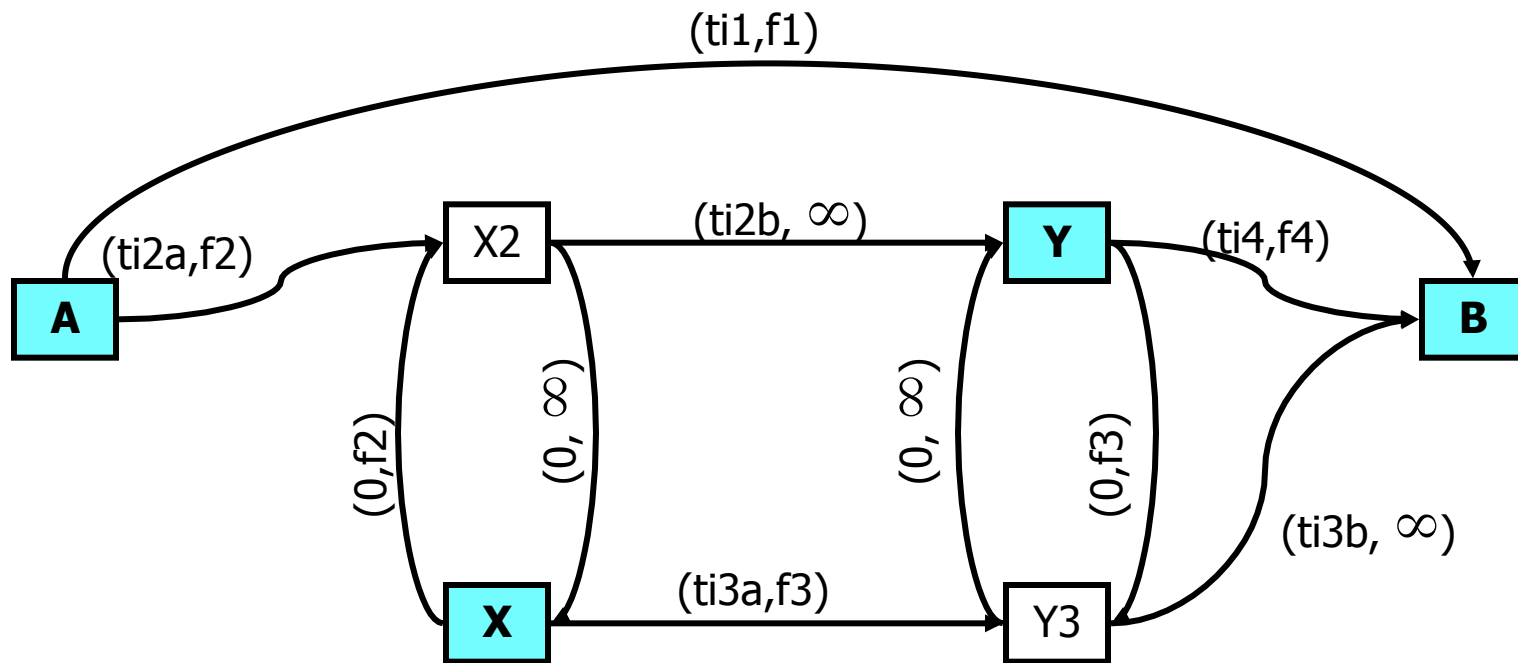
Strategies: network example



Strategies: extended network



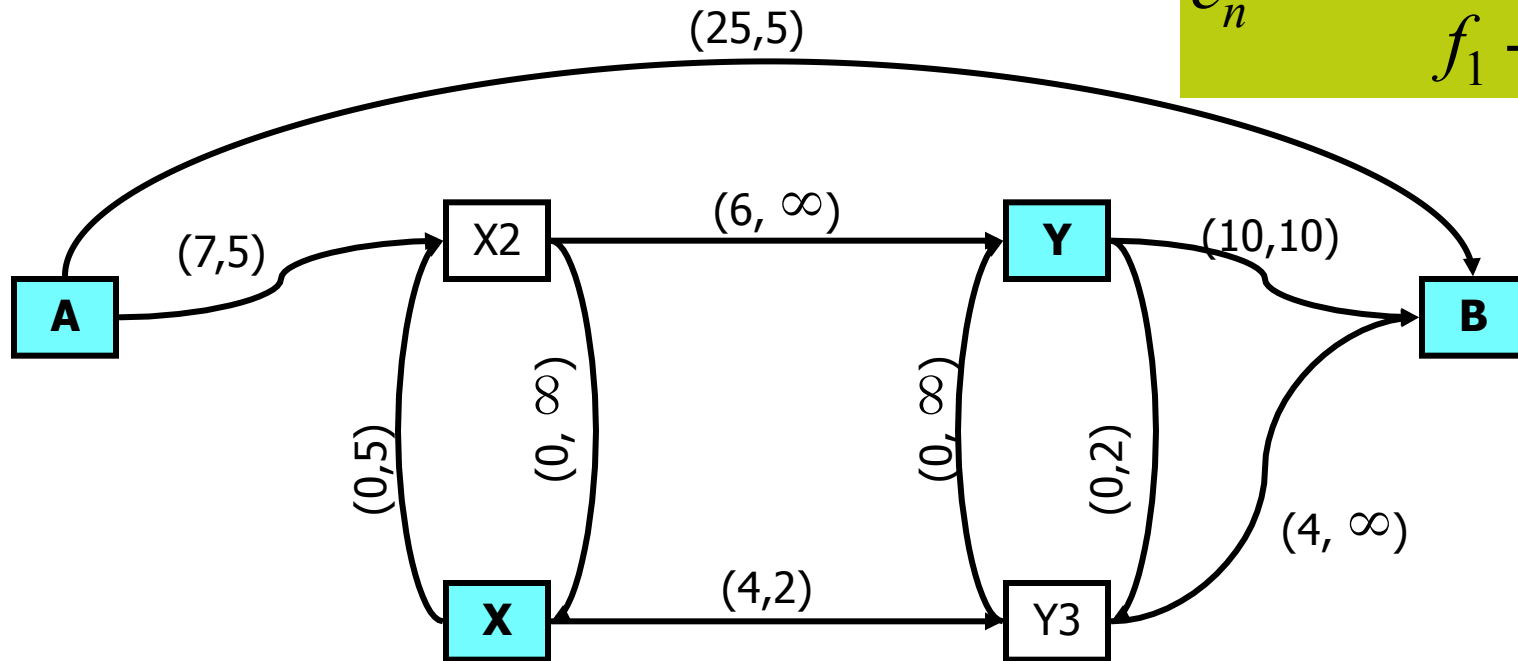
Strategies: reduced version



Strategies: reduced version

(time, freq)

$$f_n = f_1 + f_2$$
$$c_n = \frac{c_1 \cdot f_1 + c_2 \cdot f_2}{f_1 + f_2}$$

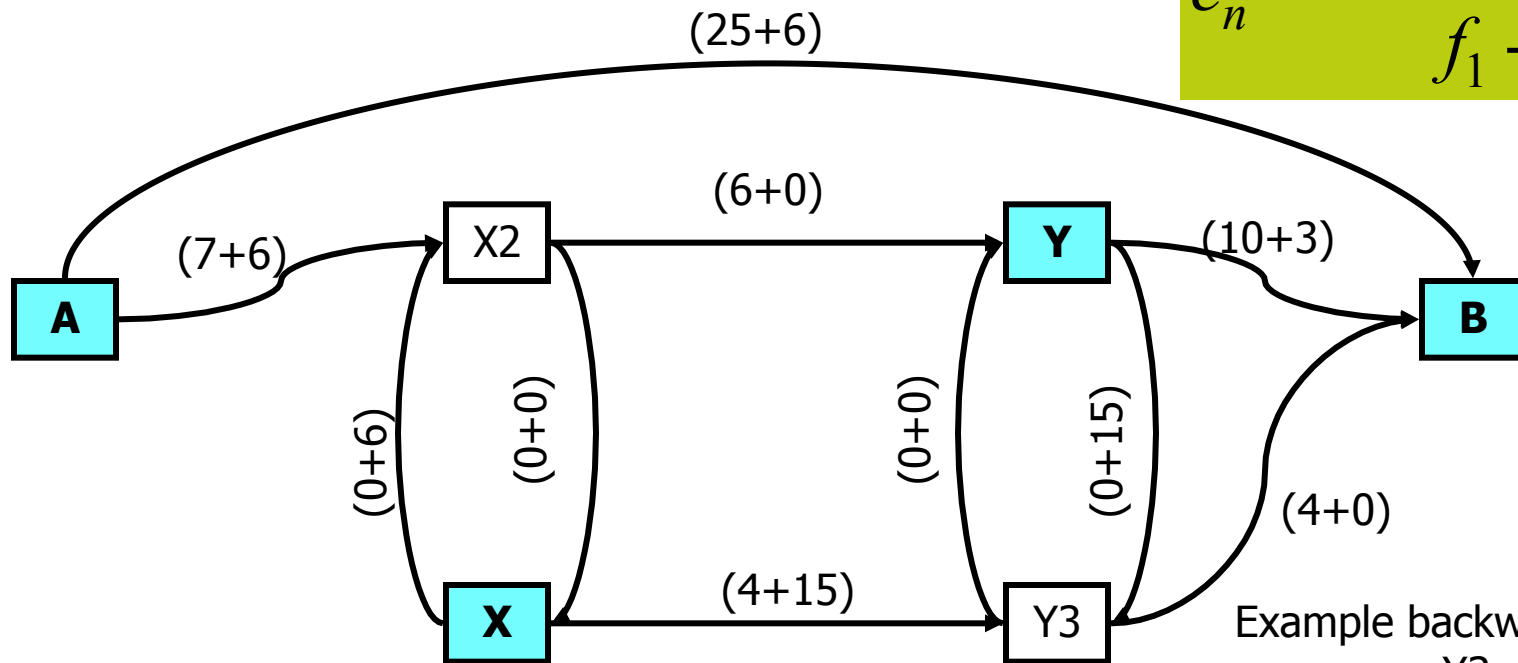


Strategies step 1: Backward search

(time + wait)

$$f_n = f_1 + f_2$$

$$c_n = \frac{c_1 \cdot f_1 + c_2 \cdot f_2}{f_1 + f_2}$$



Example backward search

Y3->B: 4+0=4

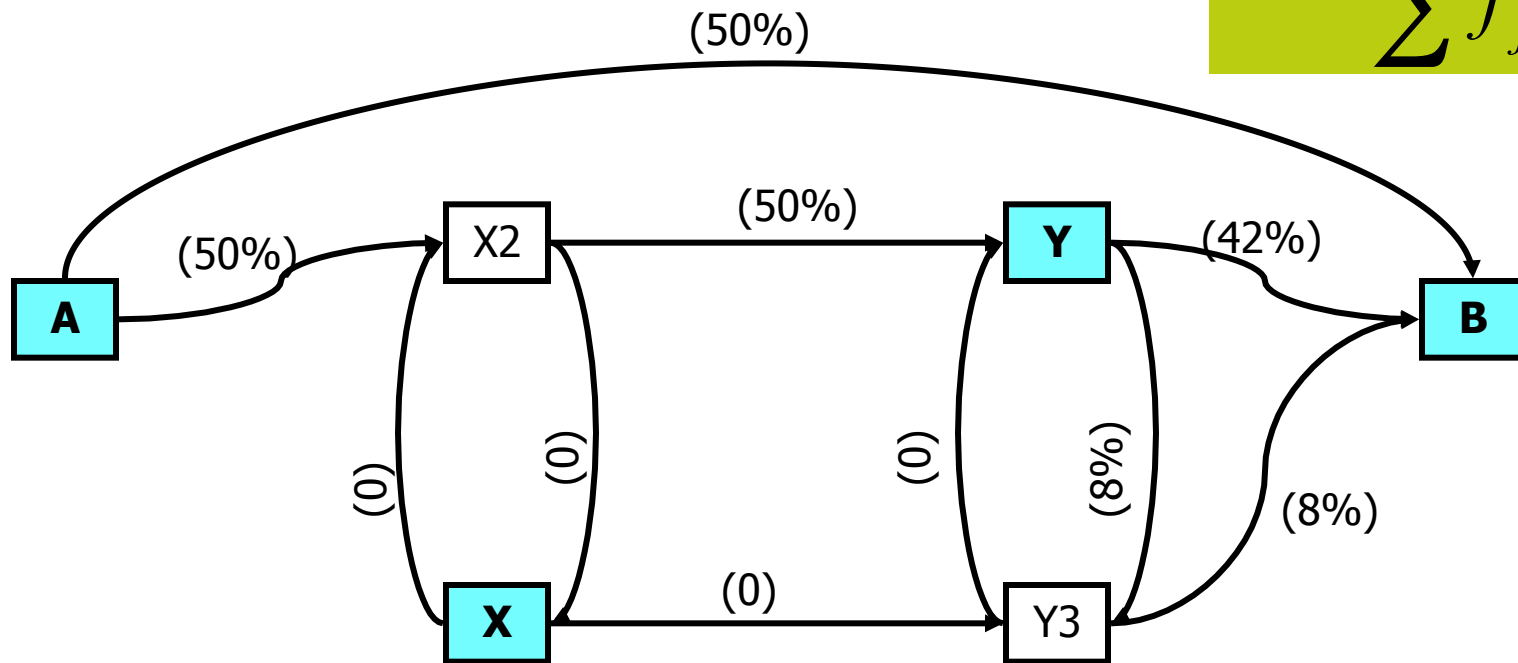
Y->B: 10+3=13

Y->B: 4+15=19

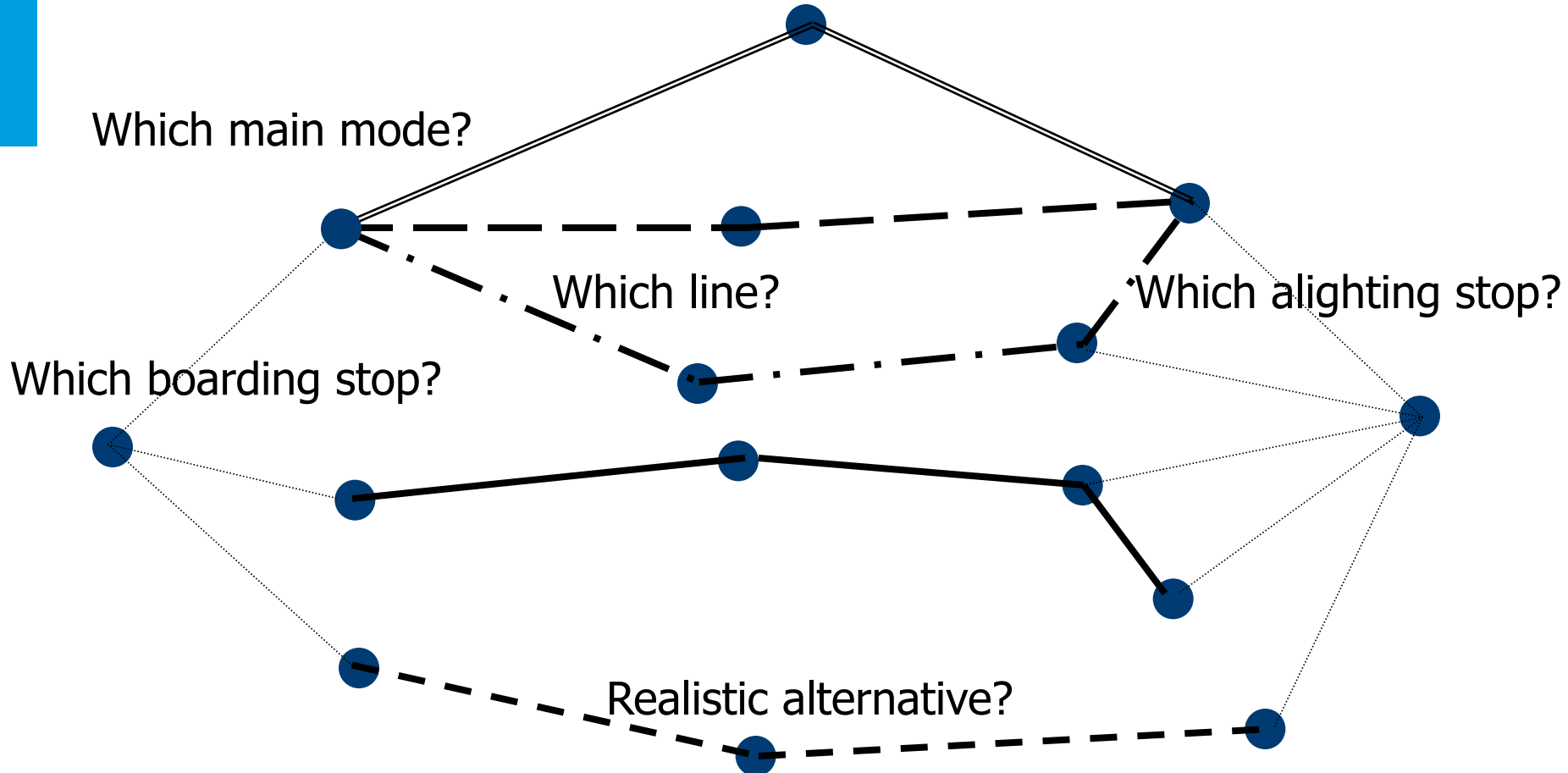
Y->B: $f_n = 10 + 2$, $c_n = (10 \cdot 10 + 2 \cdot 4) / 12 = 9 \Rightarrow 9 + 2.5 = 11.5 < \mathbf{13!}$

Strategies step 2: Forward assignment

$$P_i = \frac{f_i}{\sum f_j} P$$



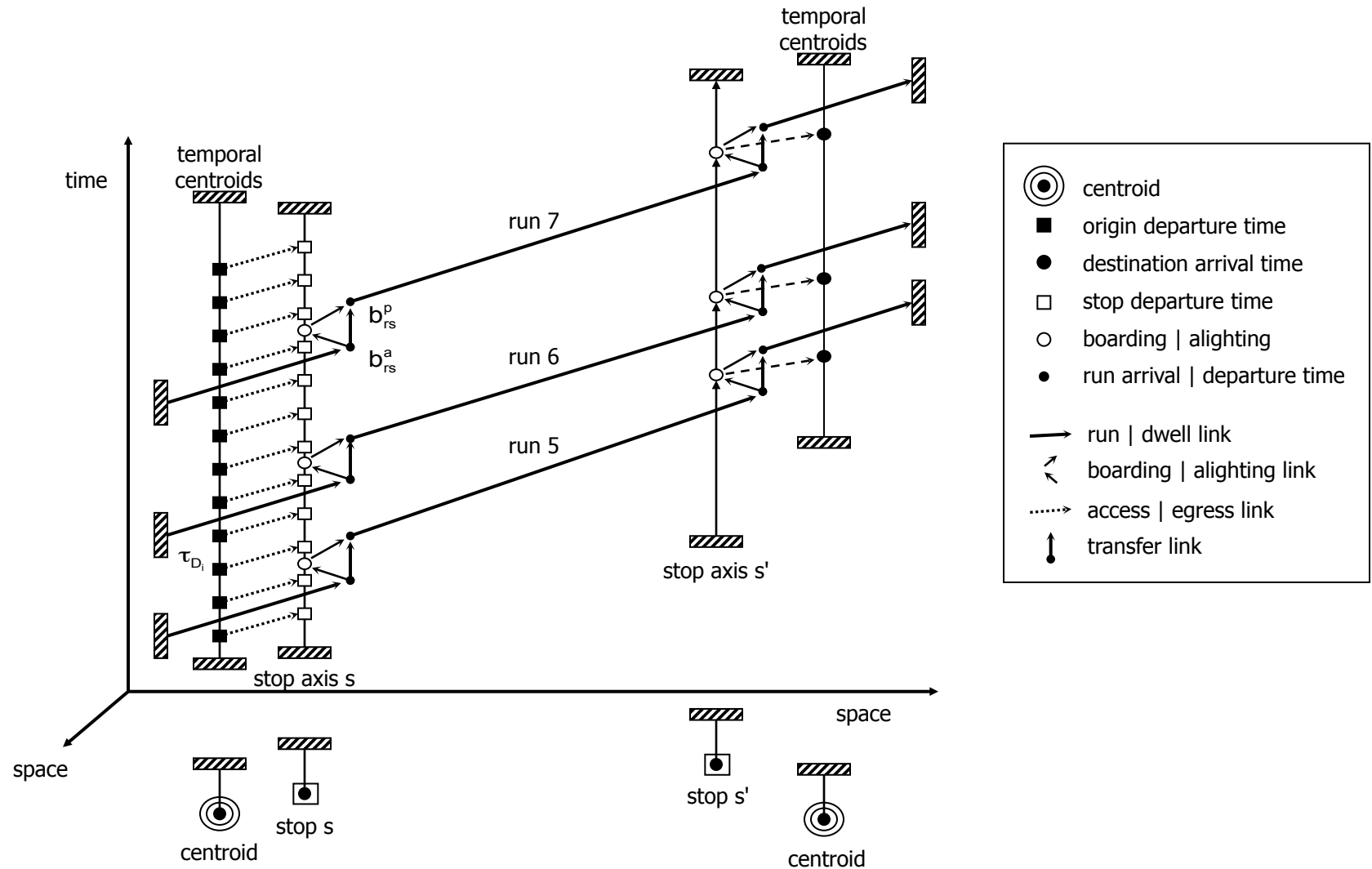
Sequential choices: network example



Capacity and (un)reliability

- Higher subjective in-vehicle time ('BPR')
- Operational frequency
- Perceived frequency
- Full vehicles

Run based ($\Rightarrow$ dynamic assignment)

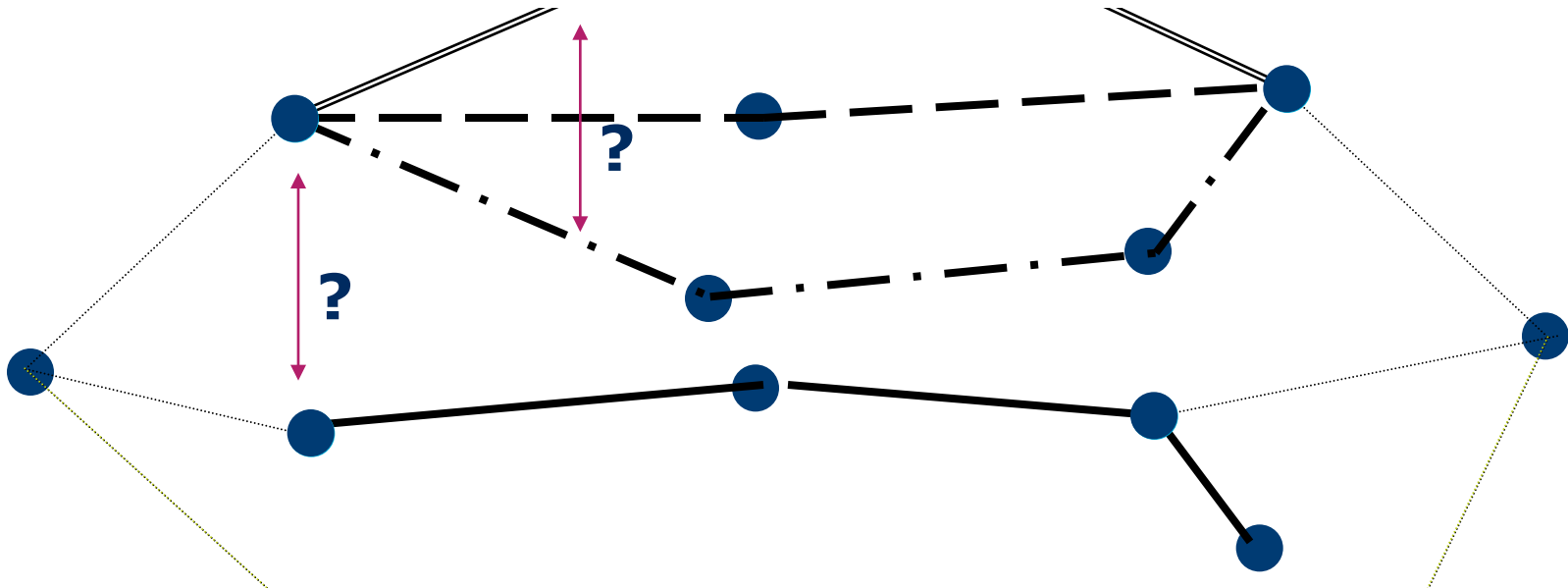


Omnitrans: Route search

- Start at destination
- Determine alighting stop(s) (maximum walking distance)
- Determine boarding stops without transfer
 - Determine stops that can be reached with one transfer
 - Determine stops that can be reached with two transfers
 -
 - Determine stops that can be reached with $N_{\max}$ transfers
 - Check logic of routes (detour)

Omnitrans: Sequential choices

$P(\text{service}) = f(\text{penalty, frequency, (aggregated) generalised costs of service(s)})$



$P(\text{stop}) = f(\text{access, penalties, frequencies, (aggregated) generalised costs of available service(s)})$

6.

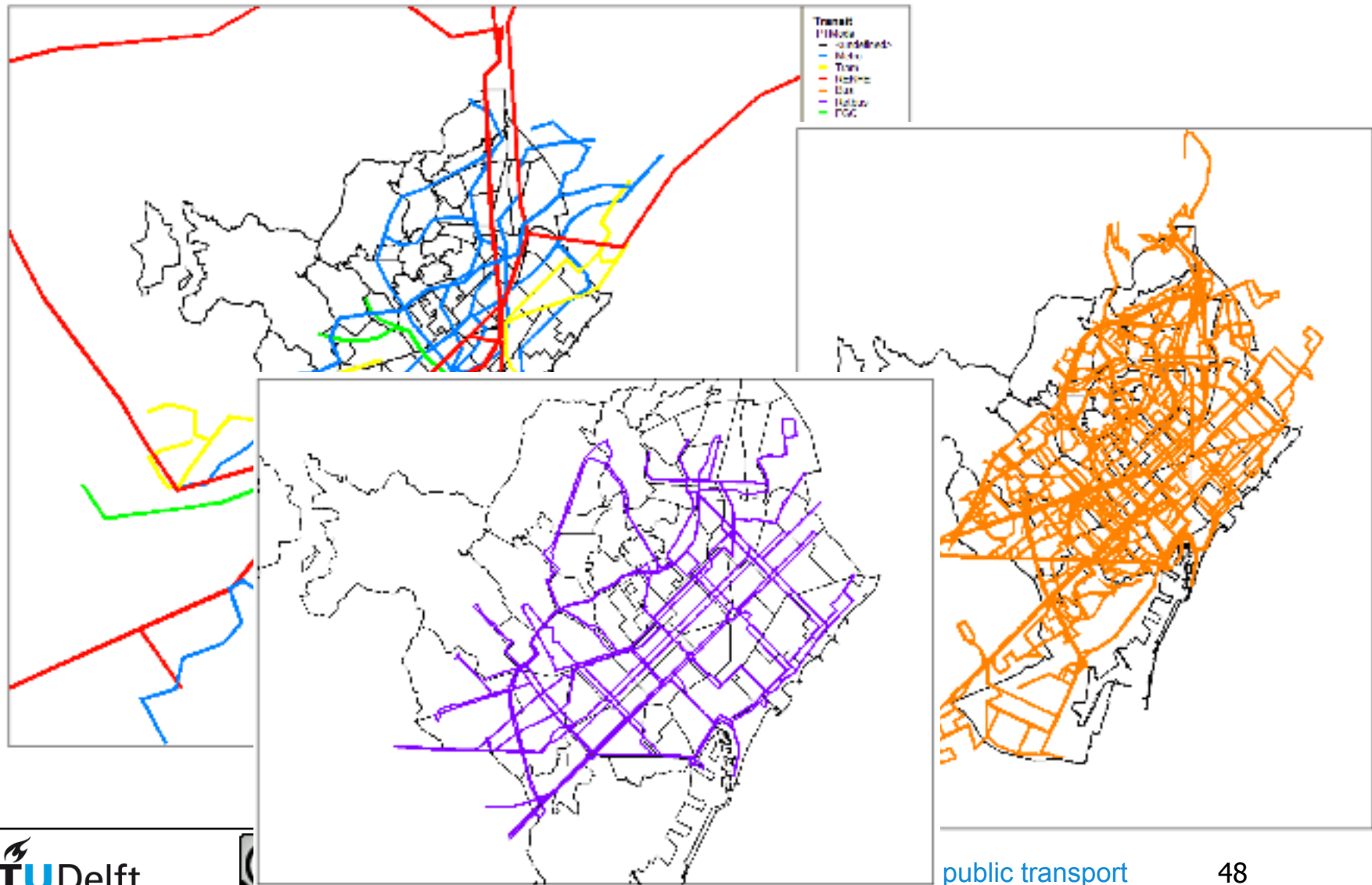
Case: Modelling PT in Barcelona

Case: RET-Bus network Barcelona



MSc Thesis Bernat Goñi

Existing PT network



How to model the RET-Bus system?

- Input
 - OD matrices car, bus and metro (based on surveys)
 - Networks car and PT
- Mode choice
 - Car and PT or Car, metro and bus?

- Assignment

$$V_{PT} = \sum_n \left[(2.0 \cdot t_a + 2.5 \cdot t_w + t_v + BP_m) \cdot VoT \right] + \sum_{n-1} (TP_m \cdot VoT) + 2.0 \cdot t_e \cdot VoT + Fare$$

- Difference metro and bus: boarding penalty and transfer penalty
 - Philosophy: metro is preferred over bus, metro stations are more fuzz => what's the net effect?
- RET-bus is either bus, metro or in between

Validation PT assignment

	Model 1	Model 2	Model 3	Model 4
Parameter ϕ	3,0	2,0	3,0	2,0
Parameter BP_{bus}	10,0	10,0	7,0	7,0
Parameter BP_{metro}	5,0	5,0	5,0	5,0
Parameter TP_{bus}	5,0	5,0	3,0	3,0
Parameter TP_{metro}	0,0	0,0	0,0	0,0
Difference in bus trip legs 2007 (predicted-observed) (%)	-20,2	-15,0	+16,5	+23,0
Difference in metro trip legs 2007 (predicted-observed) (%)	+50,0	+48,0	+26,9	+25,2

NB: Data is in trips, Omnitrans computes legs!

Impact on mode choice

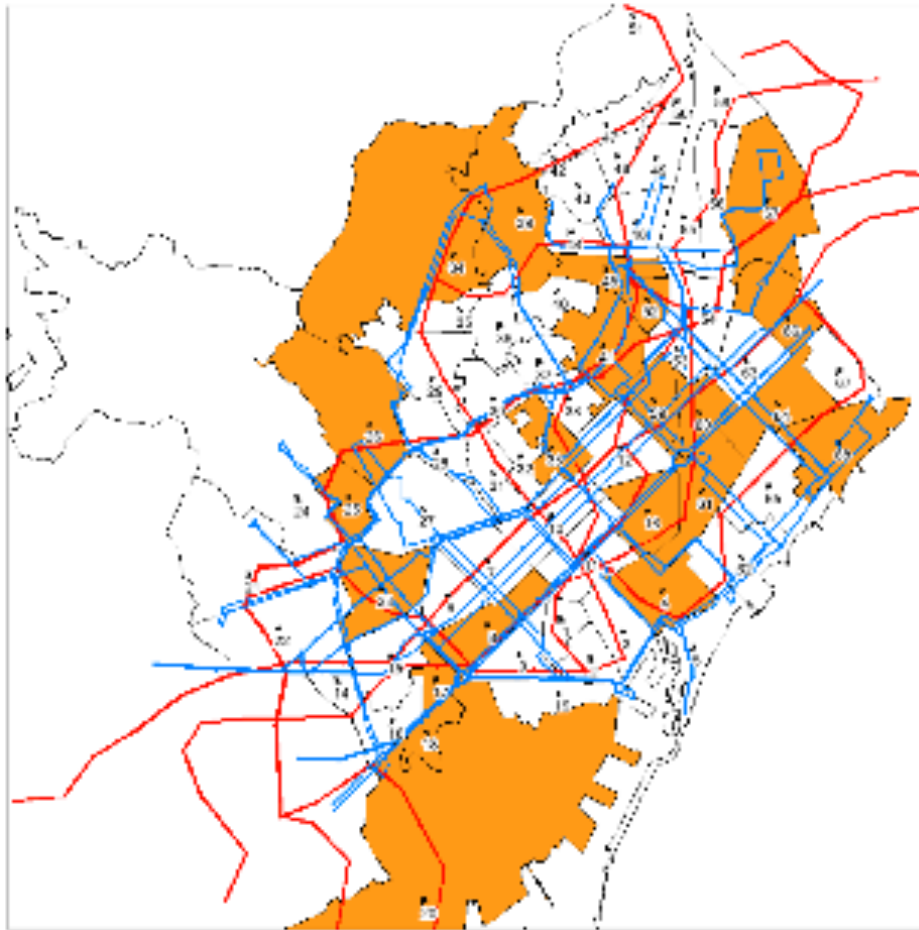


Figure 5-7: Zones that produce more than +1,50% additional transit trips in Alternative 1 (TMB plan) compared with the base case in 2016 (in orange). The metro and BRT networks are shown in red and blue, respectively.

Impact on ridership metro and bus

