spm 9550: Chaos, Randomness and Instability

Dr. ir. Igor Nikolic

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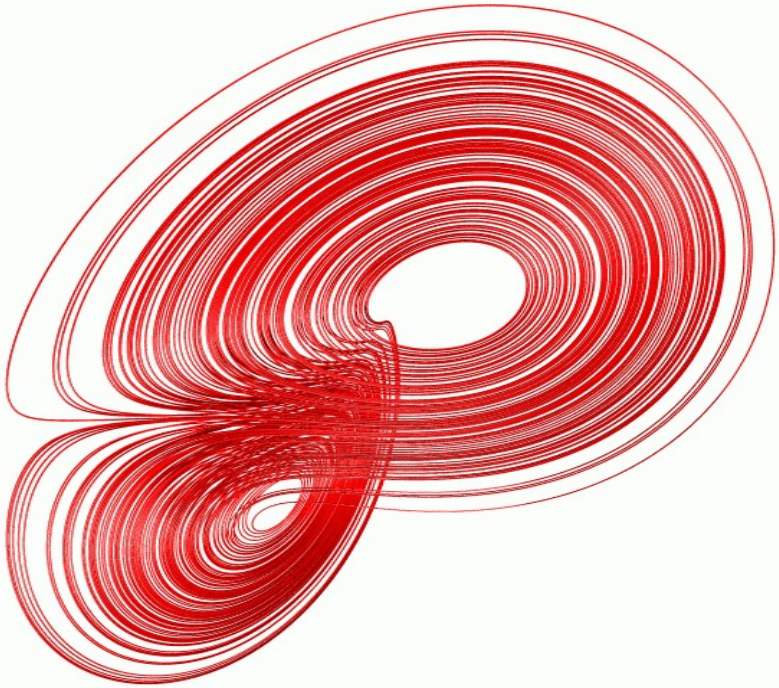
Lecture goals

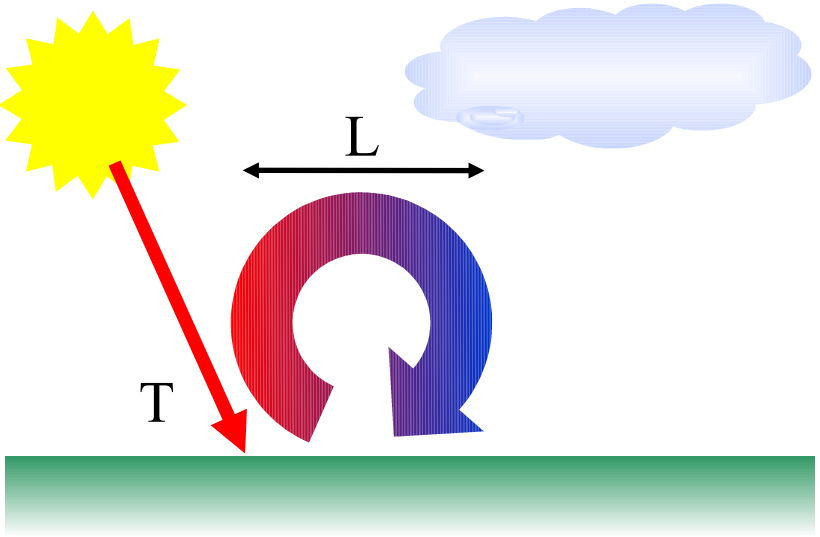
- Be able to define chaos and differentiate it from randomness
- Understand pseudo random numbers and their role of in Agent Based Models
- Understand model bifurcations
- Understand the notion of attractors and attractor maps.

Chaos

- Complex behavior, arising in a deterministic non-linear dynamic system, which exhibits two special properties:
  - sensitive dependencies in initial conditions
  - characteristic structures



The Weather



- Edward Lorenz, 1961
- Meteorological calculations:
  - convection (turbulence)
- Simplified model:
  - 3 variables
  - 3 parameters

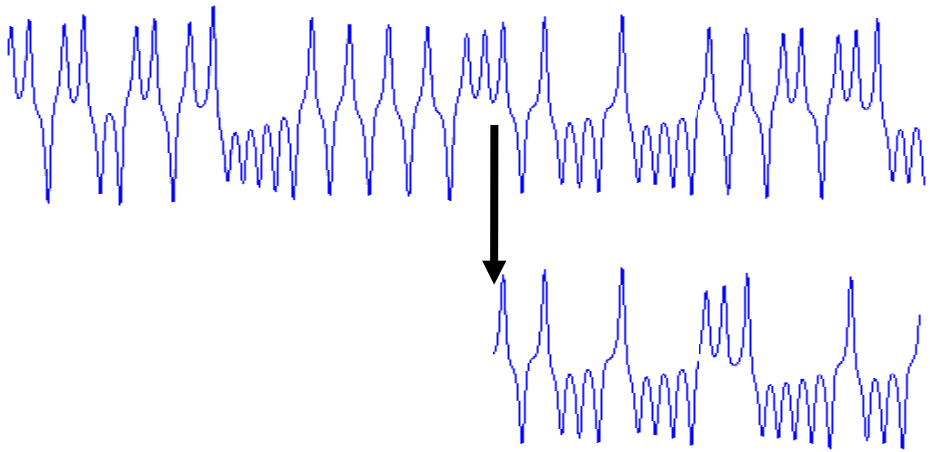
- $x' = \sigma (y - x)$
- $y' = rx - y - xz$
- $z' = xy - bz$

- x: size of the convection cells
- y: temperature difference in the air
- σ: material constant

etc...

Problem…

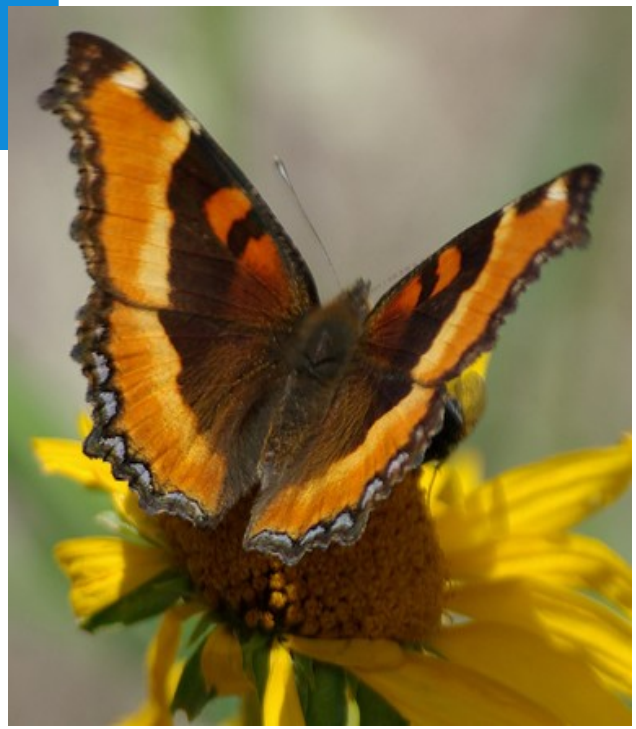
- Calculation interrupted
- Intermediate results entered by hand
  - Same results
  - … up to a certain simulated time…



Conclusions

- System is extremely sensitive to initial values
  - Lorenz copied only 3 of 6 decimals
- Non-linearity
  - $x' = \sigma (y - x)$
  - $y' = rx - y - xz$
  - $z' = xy - bz$
- Chaos

Butterfly & Tornado



?

Brazil

Texas

Chaos

- Chaos…
- …but 'deterministic':
  - **if** we knew the initial conditions completely:
  - predictable

- It has structure !
  - Weather patterns
  - B-Z reaction oscillation
  - Your heart beat

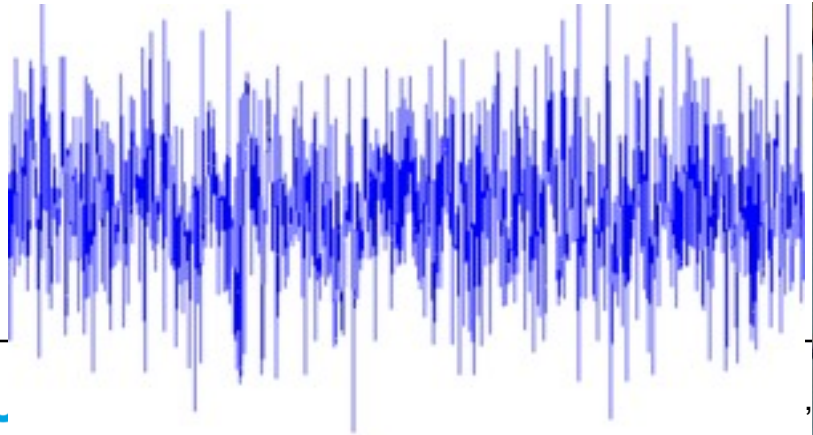
Chaotic vs. Random

- State of a dynamic system changes over time, according to some rule or procedure (linear or not). Such a system is called deterministic.
- Fractals are deterministic systems that are chaotic. That means that their dynamic is very sensitive so small variations in initial conditions of the parameters.
- They are, however, NOT random.
- Randomness cannot be produced by any system (model).
- The only (suspected) true random process known to man is the decay of radioactive atoms.

Randomness

- True Randomness has NO cause !
- Your computer CANNOT make a random number...
- Get your random numbers here: http://www.fourmilab.ch/hotbits
- Randomness contains NO information !
- Drives mutation in nature !

$$^{137}\mathrm{Cs} \xrightarrow{30.17\mathrm{y}} {}^{137\mathrm{m}}\mathrm{Ba} + \beta^- + \overline{\nu_e} \xrightarrow{156\mathrm{s}} {}^{137}\mathrm{Ba} + \gamma$$



Pseudo Random Numbers

- You computer is a deterministic Turing machine
- It *can not* generate a random number
- Instead it uses:
  - a chaotic function (Mersenne twister)
  - the clock in milliseconds as a seed
  - to give you a pseudo-random number

- This function can use a fixed *random seed* in order to produce exactly the same series of pseudo-random numbers

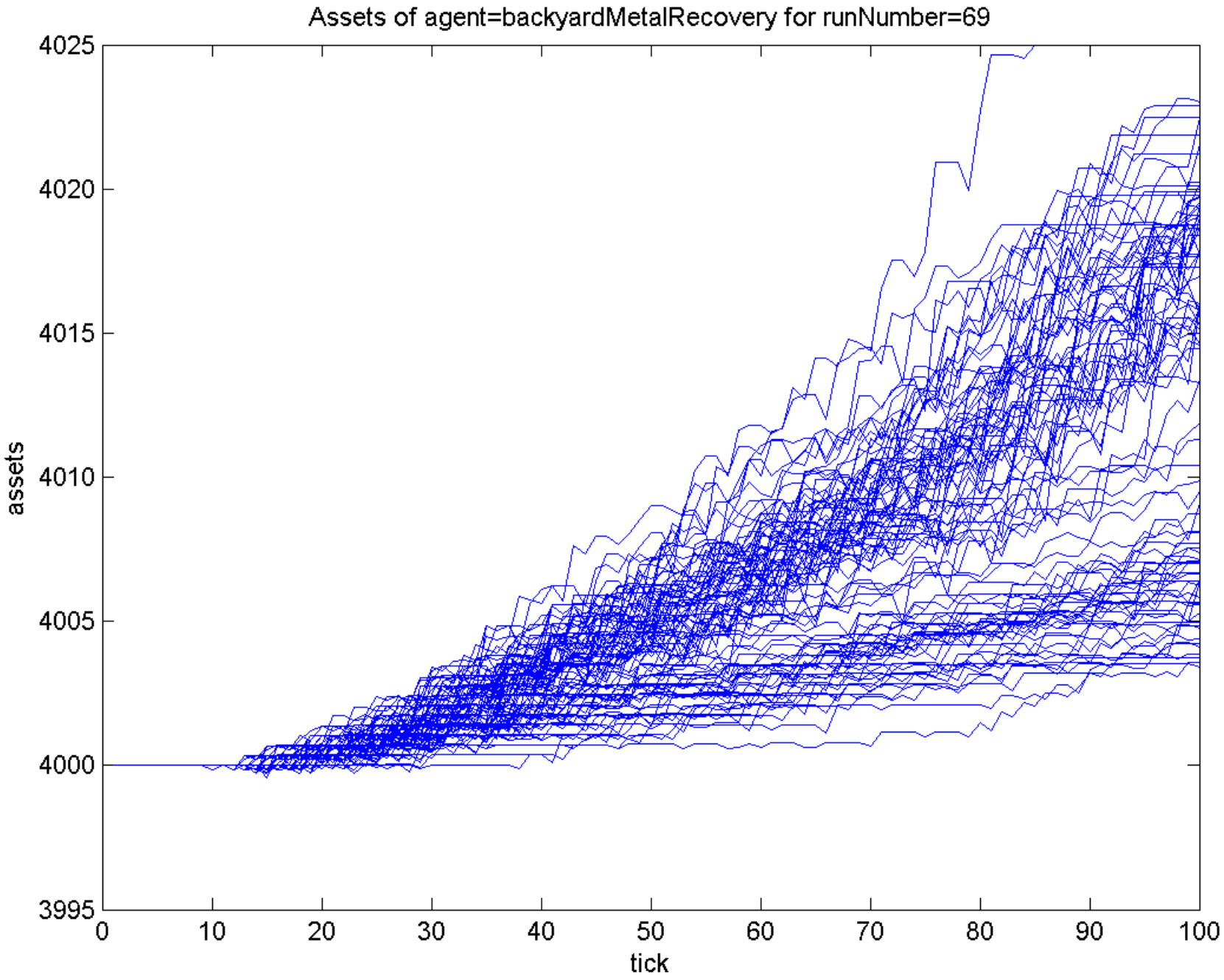
Relevance for ABM

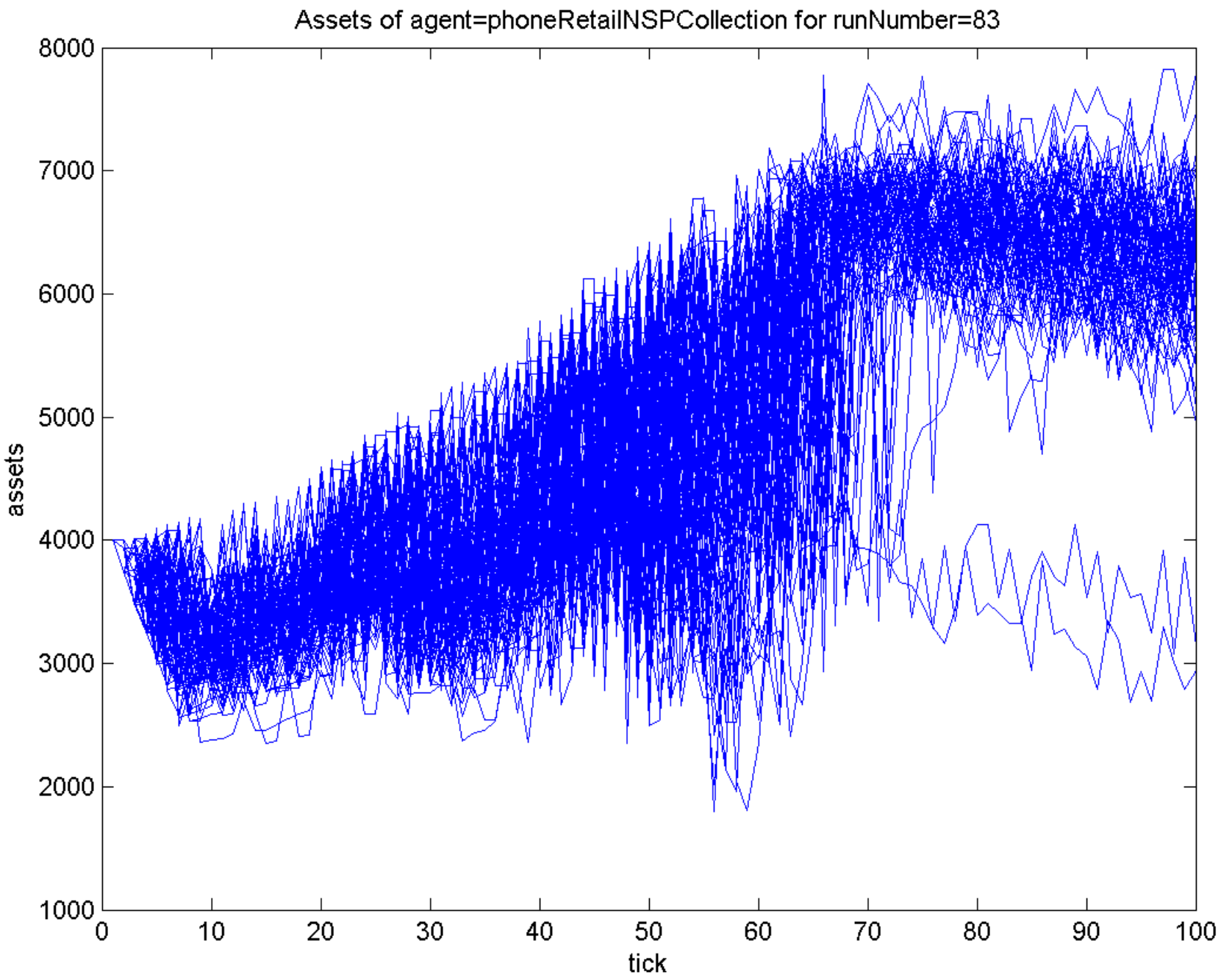
- CAS are path dependent
- The pseudo-random generator governs the interaction order etc.
- Two repeats of the same model will not be exactly the same
- It some cases, the difference in Agent iteration order can be enough to bifurcate the model
- You can thus ***NEVER trust a single run*** of an ABM
- Many repetitions at the same parameter point are necessary to establish model bahavior

Attractor maps

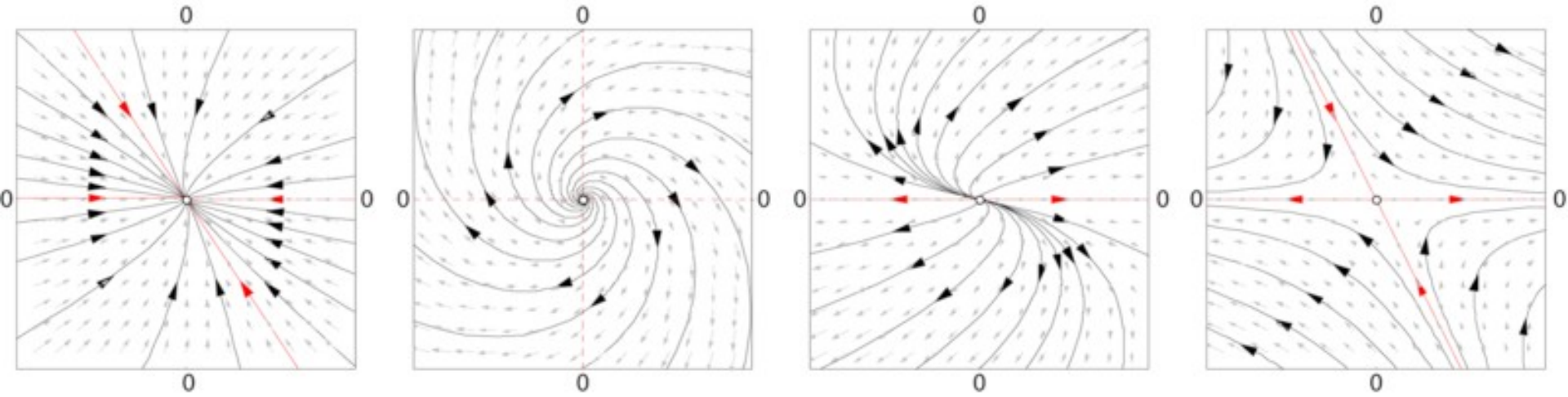
- At a single parameter point
- Across a parameter space

Bifurcation

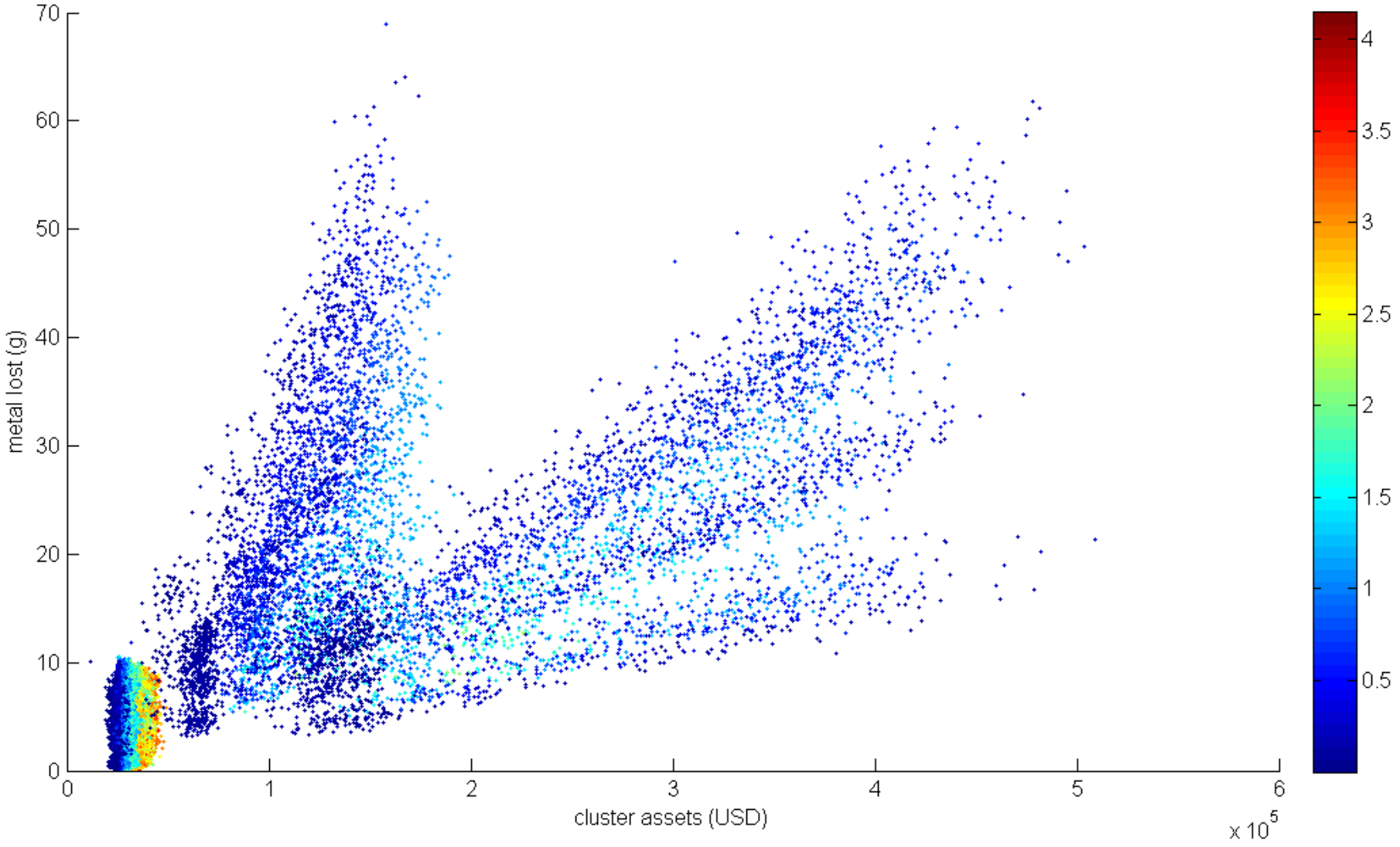

Assets of agent=backyardMetalRecovery for runNumber=69
assets
tick
4025
4020
4015
4010
4005
4000
3995
0
10
20
30
40
50
60
70
80
90
100

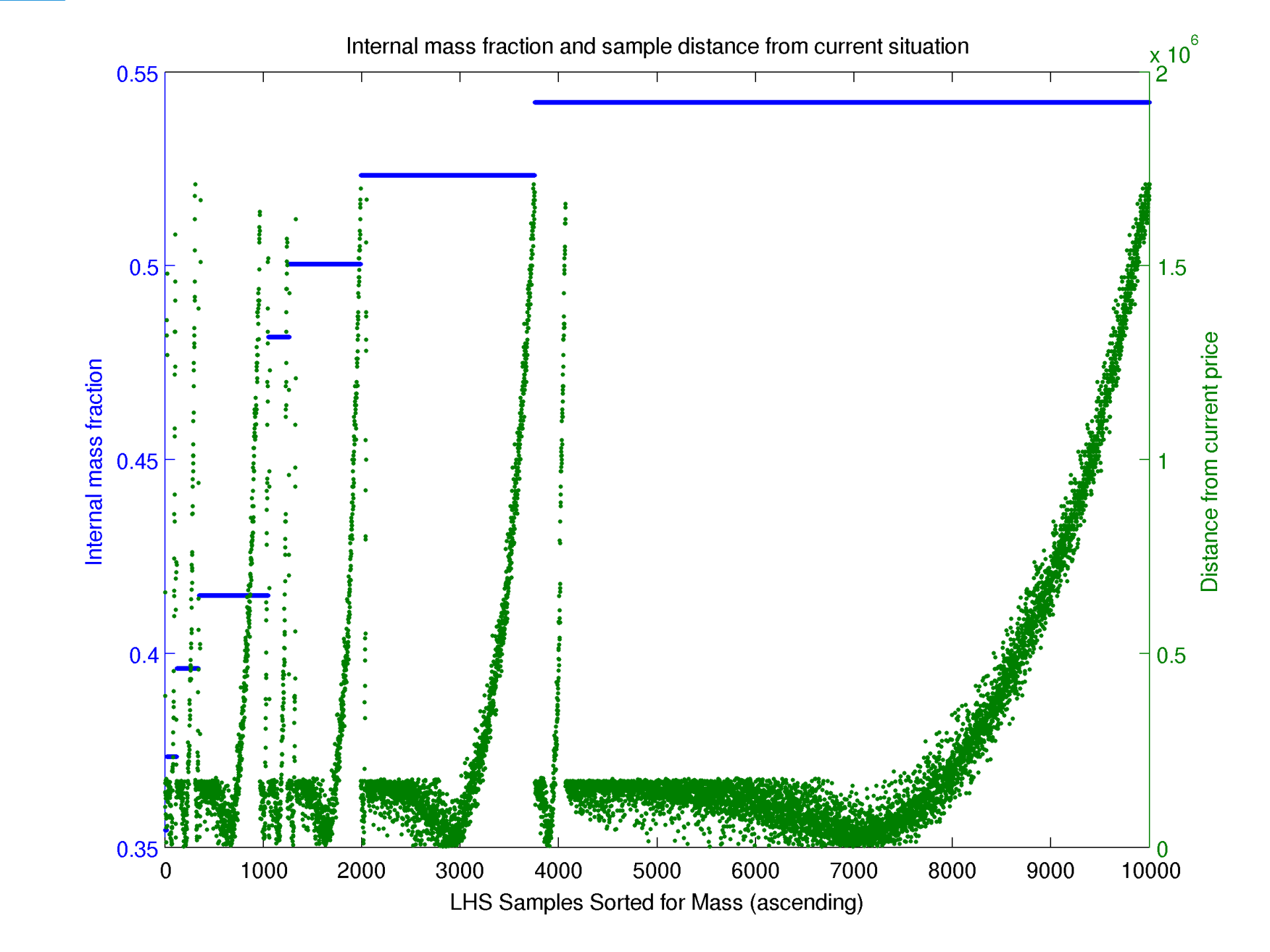

Assets of agent=phoneRetailNSPCollection for runNumber=83
assets
tick
8000
7000
6000
5000
4000
3000
2000
1000
0
10
20
30
40
50
60
70
80
90
100

Attractor maps



Cluster assets vs. metal lost (vs. fraction of phones reused)




Internal mass fraction and sample distance from current situation
x 10^6
0.55
0.5
0.45
0.4
0.35
2
1.5
1
0.5
0
Internal mass fraction
Distance from current price
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
LHS Samples Sorted for Mass (ascending)