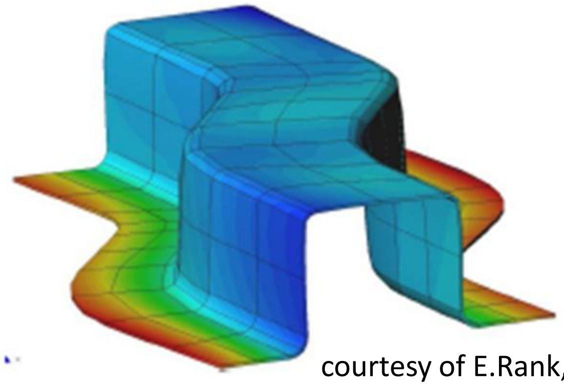
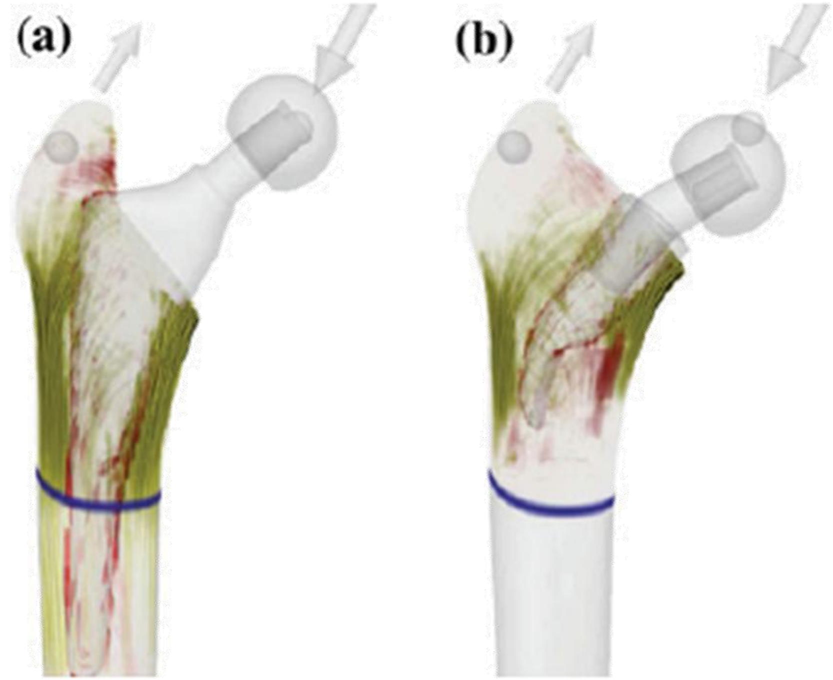
Contact Analysis

contact phenomena

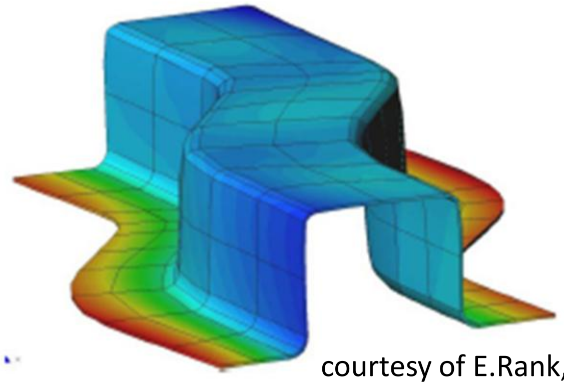


courtesy of E.Rank, TUM



- examples:
  - sheet metal forming
  - crash analysis
  - abrasive wear in engines or tyres
  - roller bearings in bridges
  - mantle friction of piles in soil mechanics
  - bone implants, e.g hip joint prosthesis
  - ...

contact phenomena



courtesy of E.Rank, TUM

- nonlinear problem – *changing boundary conditions* during analysis
- often contact region is unknown
  - contact search algorithms required in each time step/load step/iteration
  - !!! search process can easily dominate the analysis
    - sophisticated local, global search algorithm required

- task (static/quasi-static problems)

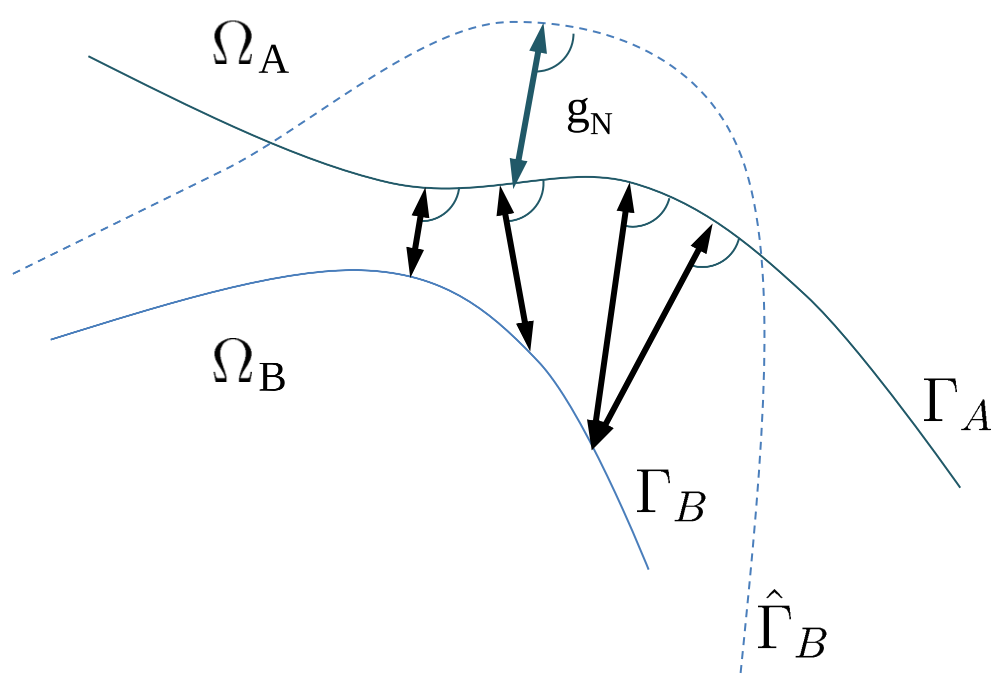
$minimize$	$\mathbf{\Pi}(\mathbf{u})$	total potential energy functional
$subject\ to$	$\mathbf{G}(\mathbf{u}) \geq \mathbf{0}$	contact constraints

normal contact

- *two* conditions to be satisfied :
  - non-penetration condition → geom. constraint
  - compressive stress condition → only compression in Γ_C

$$\Gamma_C = \Gamma_A \cap \Gamma_B$$

- t_N – contact pressure
- g_N – normal gap function



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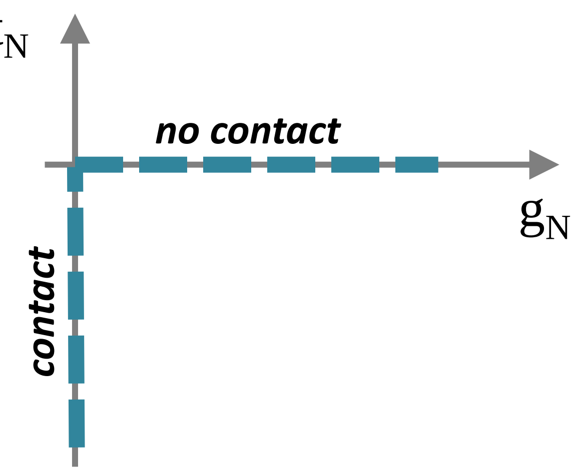
- related to ***Kuhn-Tucker*** conditions

  necessary conditions for optimal solution of a nonlinear optimization problem [...]

$$g_N \geq 0, \qquad t_N \leq 0, \qquad g_N \, t_N = 0$$



- tensile forces from adhesion possible

  → *van der Waals* attraction may develop in the contact zone

  → ongoing research, in general of negligible effect

normal contact

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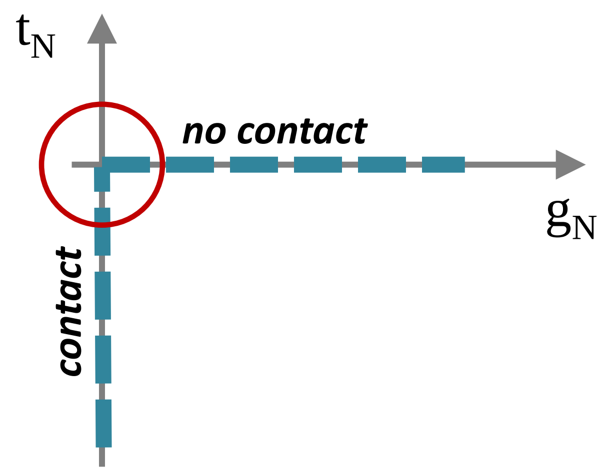


- g_N is non-unique & discontinuous → non-differentiable

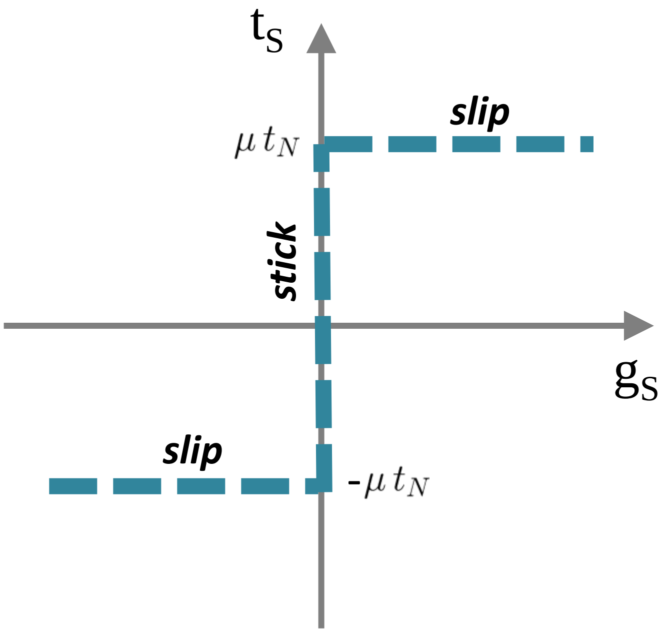
tangential contact

- friction due to relative tangential movement of bodies
- conversion of kinetic energy into heat
- *Coulomb* friction reduces complexity → only one parameter μ
- t_S – tangential traction
- g_S – tangential motion
- related to ***Kuhn-Tucker*** conditions

$$|t_S| \;\leq\; \mu\, t_N$$

$$|t_S| \;<\; \mu\, t_N \qquad \text{stick condition}$$

$$|t_S| \;=\; \mu\, t_N \qquad \text{slip condition}$$



solution methods — weak formulation

... must satisfy the Kuhn Tucker conditions

- Lagrange multiplier method
- penalty method
- mortar methods (weak enforcement of constraints)
- ...

Lagrange multiplier method

$$\int_{\Gamma_C} \left(t_N\,\delta g_N + \mathbf{t}_S^T\,\delta\mathbf{g}_T\right)\,da$$

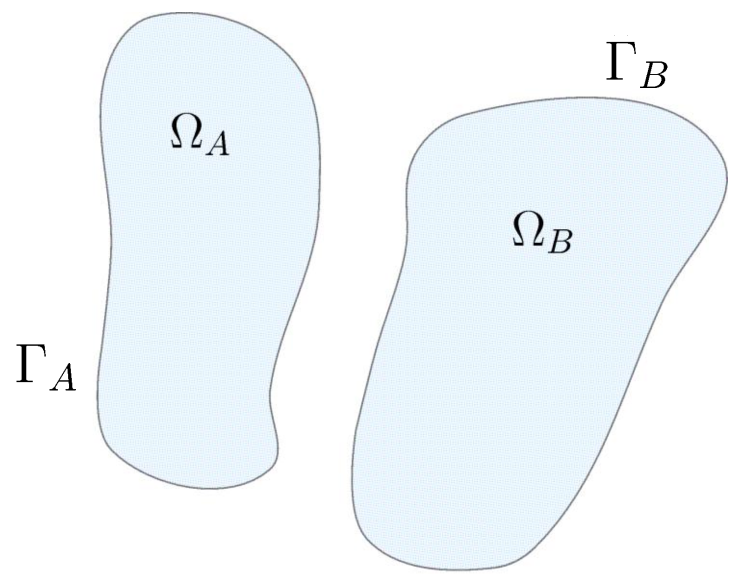
penalty method

$$\int_{\Gamma_C} \left(\epsilon_N\,g_N\,\delta g_N + \epsilon_S\,\mathbf{g}_S^T\delta\mathbf{g}_S\right)\,da, \qquad \epsilon_N,\epsilon_S > 0$$ stick condition

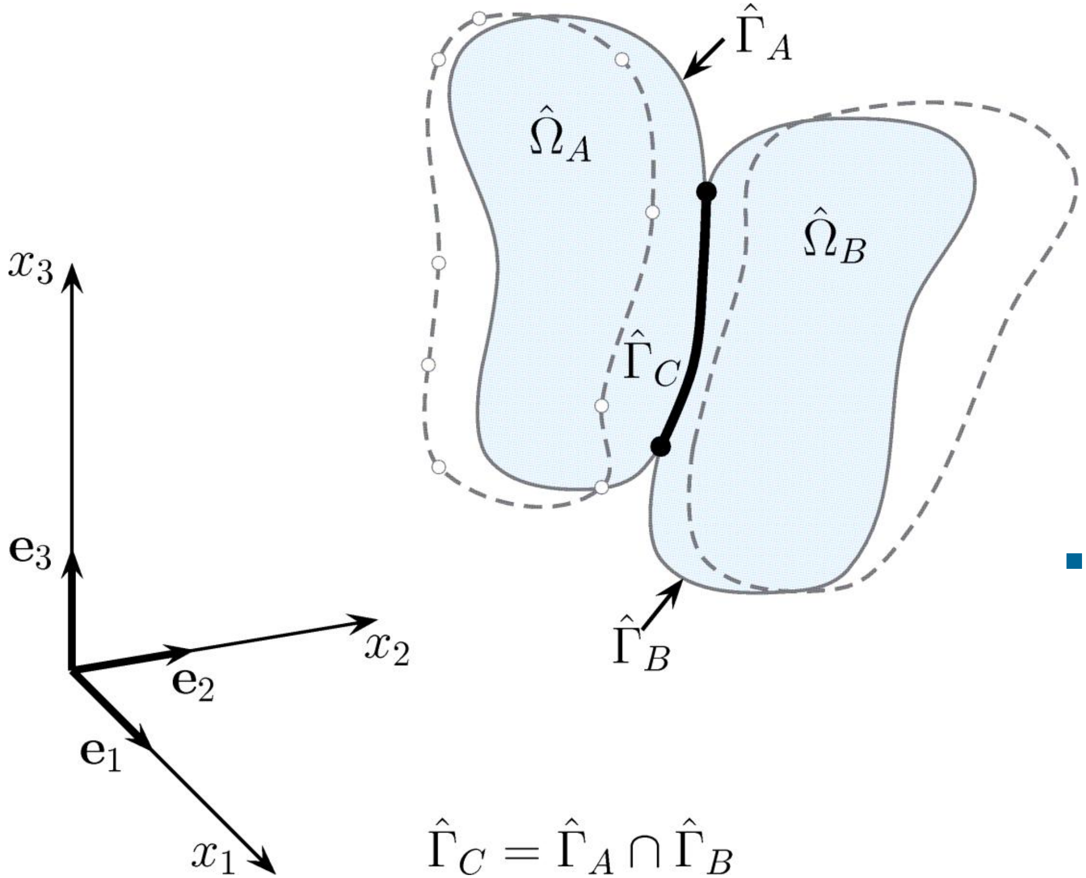
$$\int_{\Gamma_C} \left(\epsilon_N\,g_N\,\delta g_N + \mathbf{t}_S^T\delta\mathbf{g}_S\right)\,da, \qquad \epsilon_N > 0$$ slip condition

terminology – time t=0

- following a *master-slave concept*
- freely chosen: body A → *master*
  body B → *slave*
- body surfaces Γ_A & Γ_B
- contact interface $\Gamma_C = \Gamma_A \cap \Gamma_B$



contact – time t≠0



- contact forces $\hat{f}_i$ on Γ_C

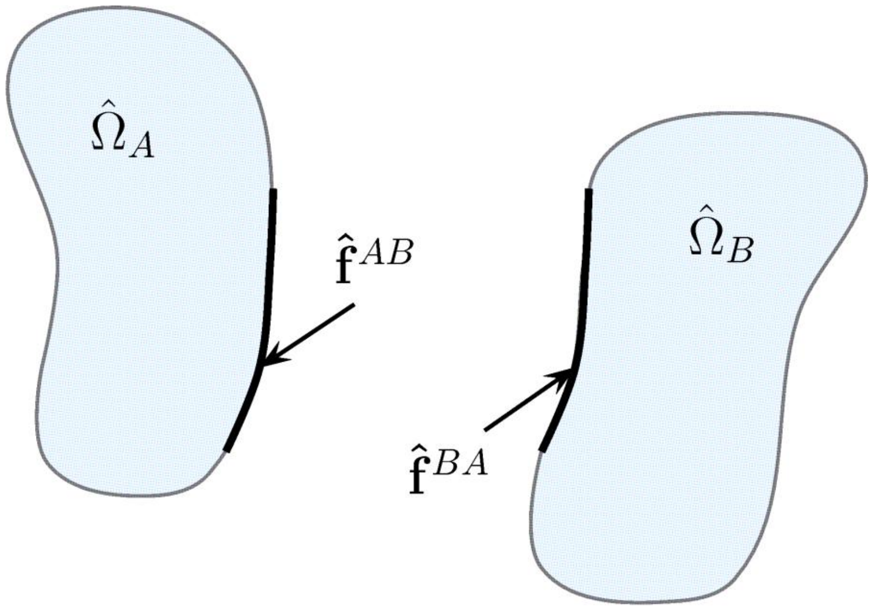
$$\hat{\Gamma}_C = \hat{\Gamma}_A \cap \hat{\Gamma}_B$$

Principle of virtual work at $t \neq 0$

$$\sum_{m=1}^{M} \left\{ \int_{\hat{\Omega}_m} \delta(\hat{\epsilon}_{ik})\, \hat{s}_{ik}\, d\hat{v} \right\} \;=\; \sum_{m=1}^{M} \left\{ \int_{\hat{\Omega}_m} \delta\hat{u}_i\, \hat{q}_i\, d\hat{v} + \int_{\hat{\Gamma}_{tm}} \delta\hat{u}_i\, \hat{p}_i\, d\hat{a} \right\} +$$

$$\sum_{m=1}^{M} \left\{ \int_{\hat{\Gamma}_{cm}} \delta\hat{u}_i\, \hat{f}_i\, d\hat{a} \right\}$$

contact forces



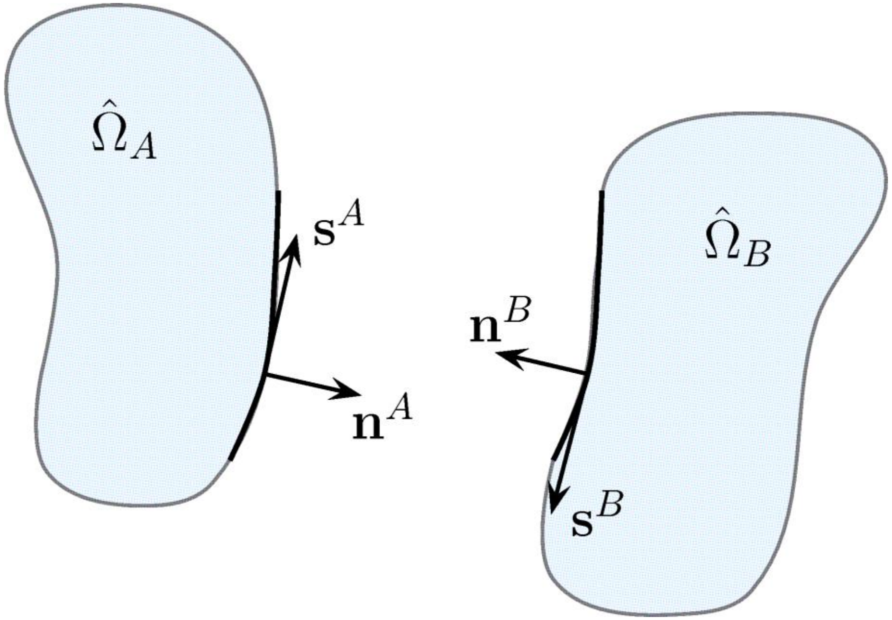
$$\hat{\mathbf{f}}^{AB} = -\hat{\mathbf{f}}^{BA}$$

$$\hat{\mathbf{f}}^{(AB)} = \hat{t}_N\,\mathbf{n} + \hat{t}_S\,\mathbf{s}$$

$$\hat{t}_N = (\hat{\mathbf{f}}^{(AB)})^T\mathbf{n}$$

$$\hat{t}_S = (\hat{\mathbf{f}}^{(AB)})^T\mathbf{s}$$

geometry description



$$\hat{\mathbf{f}}^{(AB)} = \hat{t}_N\,\mathbf{n} + \hat{t}_S\,\mathbf{s}$$

$$\hat{t}_N = (\hat{\mathbf{f}}^{(AB)})^T\mathbf{n}$$

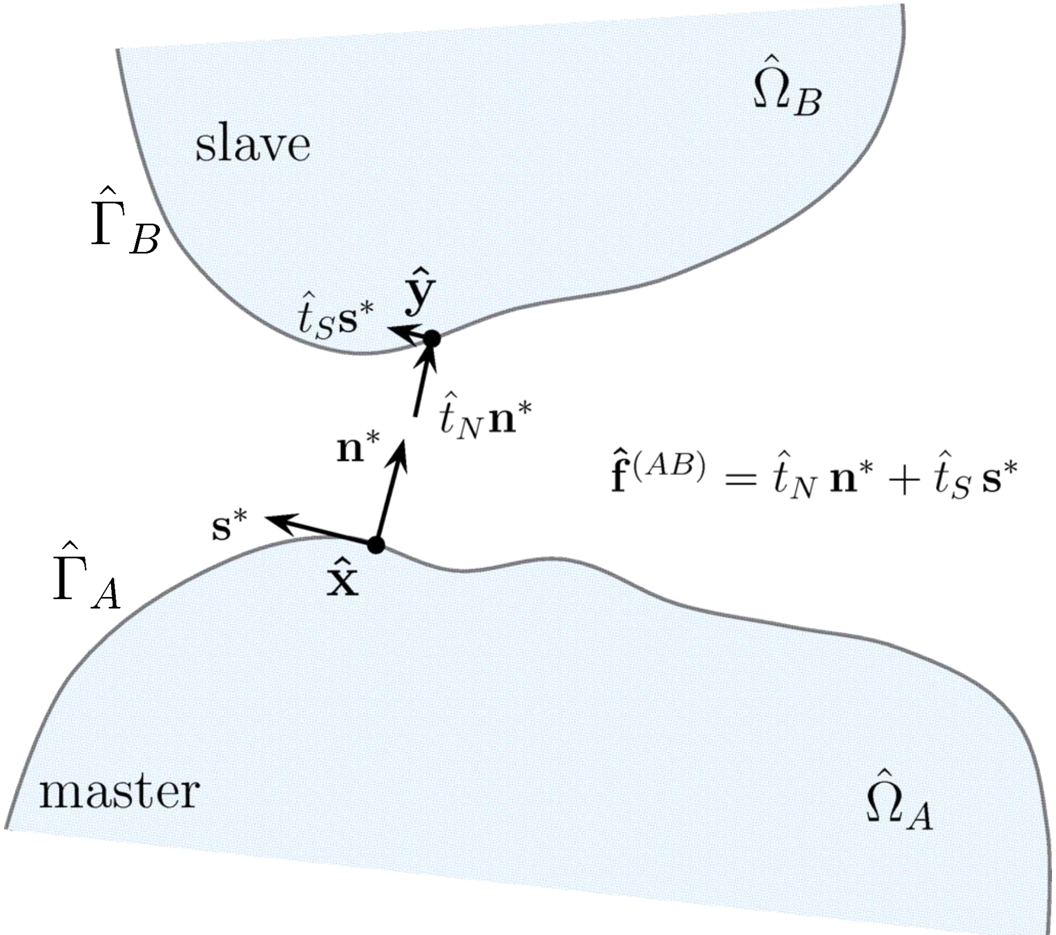
$$\hat{t}_S = (\hat{\mathbf{f}}^{(AB)})^T\mathbf{s}$$

- **n** outward pointing normal (positive)
- **s** tangent (forming a right handed coordinate system)
- $\hat{t}_N$ & $\hat{t}_S$ normal and tangential stress components

- work expression of the virtual relative displacements with contact stresses

$$\int_{\Gamma_C} \delta\hat{u}_i^{(AB)}\,\hat{f}_i^{(AB)}\,da$$

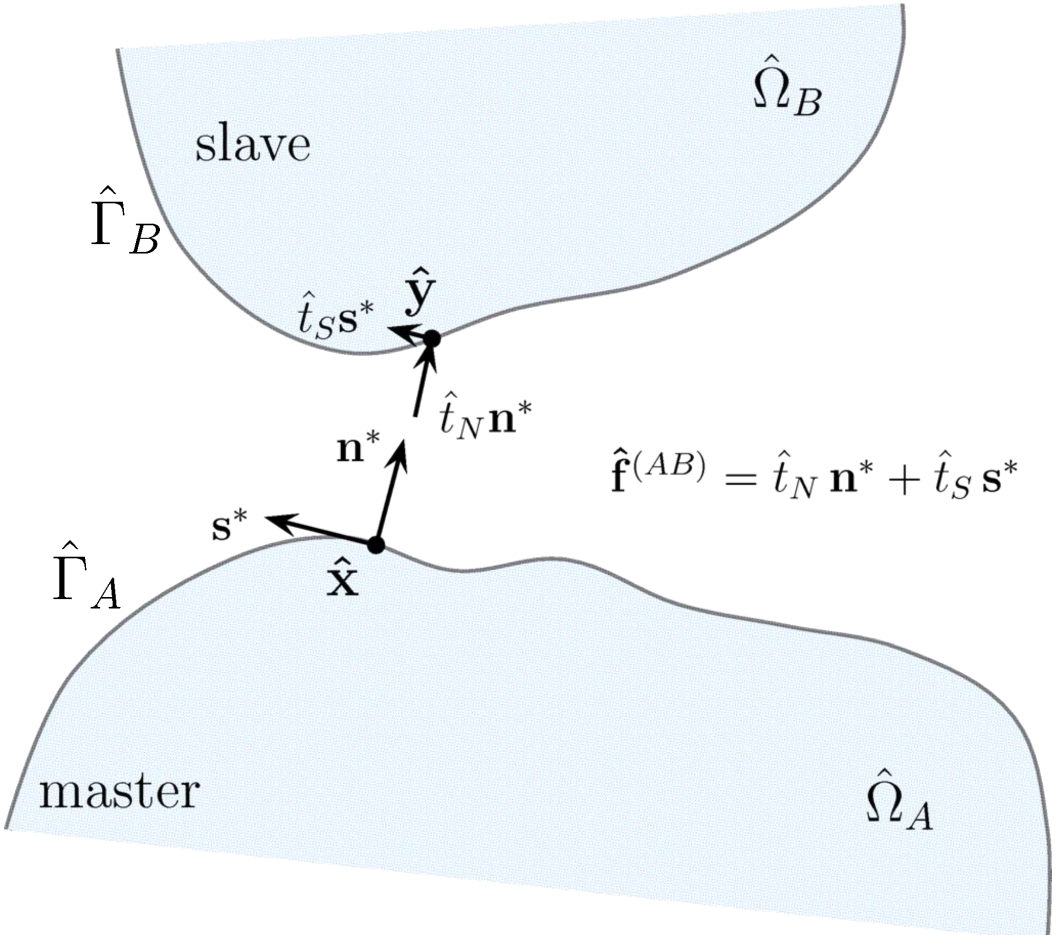
kinematics – contact definiton



$$\hat{\mathbf{f}}^{(AB)} = \hat{t}_N\,\mathbf{n}^* + \hat{t}_S\,\mathbf{s}^*$$

* indicates components of the master surface

- contact force at $\mathbf{x}$ referred to point $\mathbf{y}$
- *master* surface → defines set of test points
- *slave* surface → set of surface points checked for penetration with master surface



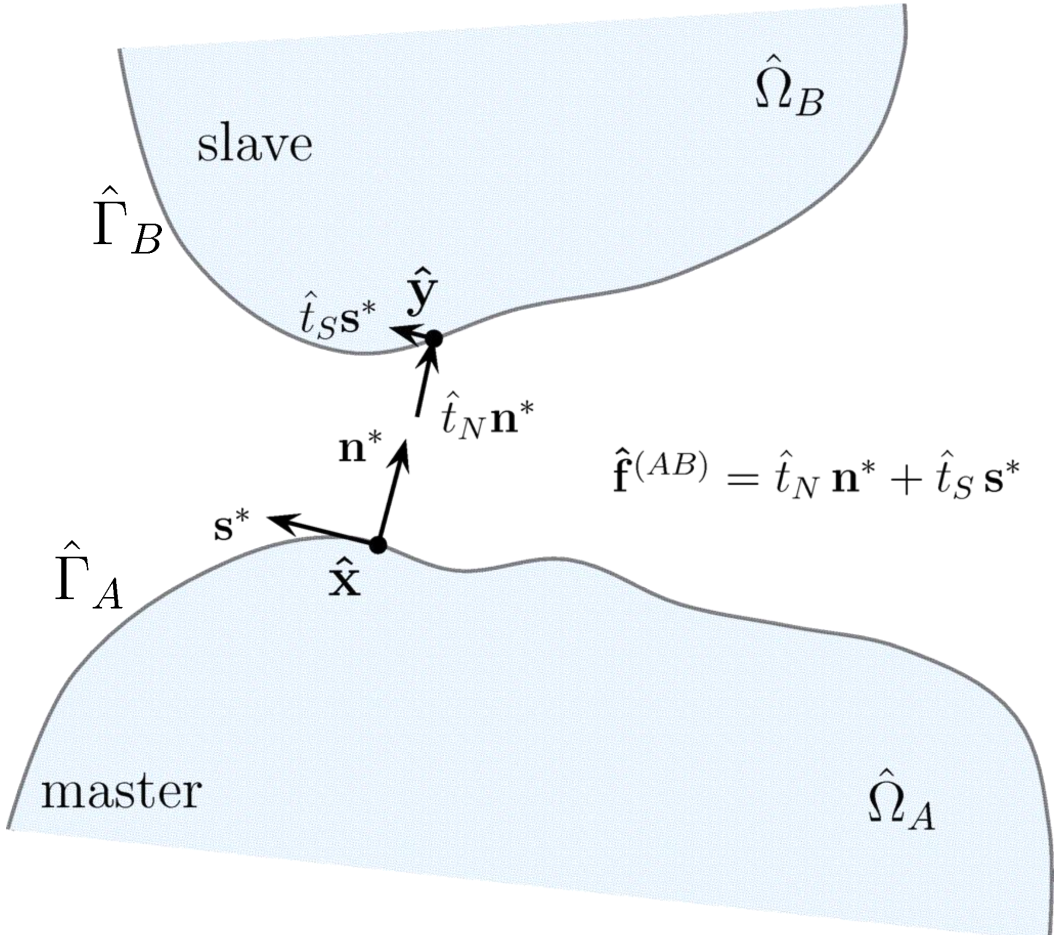
$$\hat{\mathbf{f}}^{(AB)} = \hat{t}_N \, \mathbf{n}^* + \hat{t}_S \, \mathbf{s}^*$$

* indicates components of the master surface

master surface parameterization $\hat{\mathbf{x}} = \hat{\mathbf{x}}(\xi_1, \xi_2)$

tangent on master surface $\mathbf{s} = \hat{\mathbf{x}}_{,\alpha}(\xi_1, \xi_2) \qquad \alpha = 1, 2$

kinematics – contact definiton



minimal distance function → *closest projection point*

$$\|\hat{\mathbf{y}} - \hat{\mathbf{x}}^*\| \;=\; \min_{\hat{\mathbf{x}} \subseteq \hat{\Gamma}_A} \|\hat{\mathbf{y}} - \hat{\mathbf{x}}(\xi_1, \xi_2)\|$$

minimal distance function → *closest projection point*

$$\|\hat{\mathbf{y}} - \hat{\mathbf{x}}^*\| \;=\; \min_{\hat{\mathbf{x}} \subseteq \hat{\Gamma}_A} \|\hat{\mathbf{y}} - \hat{\mathbf{x}}(\xi_1, \xi_2)\|$$

check condition

$$\frac{d}{\xi^\alpha}\|\hat{\mathbf{y}} - \hat{\mathbf{x}}(\xi_1, \xi_2)\| \;=\; \frac{\hat{\mathbf{y}} - \hat{\mathbf{x}}(\xi_1, \xi_2)}{\|\hat{\mathbf{y}} - \hat{\mathbf{x}}(\xi_1, \xi_2)\|} \cdot \hat{\mathbf{x}}_{,\alpha}(\xi_1, \xi_2) \;=\; 0$$

which satisfies the minimization problem.

minimal distance function → *closest projection point*

$$\|\hat{\mathbf{y}} - \hat{\mathbf{x}}^*\| = \min_{\hat{\mathbf{x}} \subseteq \hat{\Gamma}_A} \|\hat{\mathbf{y}} - \hat{\mathbf{x}}(\xi_1, \xi_2)\|$$

condition

$$\frac{d}{\xi^\alpha}\|\hat{\mathbf{y}} - \hat{\mathbf{x}}(\xi_1, \xi_2)\| = \frac{\hat{\mathbf{y}} - \hat{\mathbf{x}}(\xi_1, \xi_2)}{\|\hat{\mathbf{y}} - \hat{\mathbf{x}}(\xi_1, \xi_2)\|} \cdot \hat{\mathbf{x}}_{,\alpha}(\xi_1, \xi_2) = 0$$

satisfies the minimization problem.

NOTE: $\mathbf{s} = \hat{\mathbf{x}}_{,\alpha}(\xi_1, \xi_2)$ $\quad \alpha = 1, 2$

= tangent to the master surface

- **s** is normal to minimum distance vector
- minimum distance vector is normal to master surface
- *distance function not always unique!*

$$\bar{\hat{\mathbf{x}}}(\xi_1, \xi_2) \;=\; \hat{\mathbf{x}}(\bar{\xi}_1, \bar{\xi}_2)$$

point on the master surface with minimum distance to on the slave surface

- definition of a *gap function* → ***normal*** *contact*

$$g_N \;=\; \left(\hat{\mathbf{y}} - \hat{\mathbf{x}}(\bar{\xi}_1, \bar{\xi}_2)\right)^T \cdot \bar{\mathbf{n}}(\bar{\xi}_1, \bar{\xi}_2)$$

- constraints define states of the contact zone

$$\begin{aligned} g_N &> 0 \quad \text{no contact} \\ g_N &= 0 \quad \text{perfect contact} \\ g_N &< 0 \quad \text{penetration} \end{aligned}$$

- contact constraints, eliminating penetration

$$g_N \;=\; \left(\hat{\mathbf{y}} - \hat{\mathbf{x}}(\bar{\xi}_1, \bar{\xi}_2)\right)^T \cdot \bar{\mathbf{n}}(\bar{\xi}_1, \bar{\xi}_2) \;\geq\; 0$$

$$\bar{\hat{\mathbf{x}}}(\xi_1, \xi_2) \;=\; \hat{\mathbf{x}}(\bar{\xi}_1, \bar{\xi}_2)$$

point on the master surface with minimum distance to on the slave surface

- definition of a *gap function* (N – normal)

$$g_N \;=\; \left(\hat{\mathbf{y}} - \hat{\mathbf{x}}(\bar{\xi}_1, \bar{\xi}_2)\right)^T \cdot \bar{\mathbf{n}}(\bar{\xi}_1, \bar{\xi}_2)$$

- constraints define states of the contact zone

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how does $\hat{t}_N$ behave?

- contact constraints, eliminating penetration

$$g_N \;=\; \left(\hat{\mathbf{y}} - \hat{\mathbf{x}}(\bar{\xi}_1, \bar{\xi}_2)\right)^T \cdot \bar{\mathbf{n}}(\bar{\xi}_1, \bar{\xi}_2) \;\geq\; 0$$

consideration of friction

assumptions:

- *Coulomb* friction valid pointwise on the surface
- friction coefficient μ
- introduction of a dimensionless variable $\tau = \dfrac{t_S}{\mu\, t_N}$
  with $(\mu\, t_N)$ = *frictional resistance*
- relative tangential velocity $\dot{u}(\hat{\mathbf{x}}, t) = \left(\dot{\mathbf{u}}^B|_{\hat{\mathbf{y}}(\hat{\mathbf{x}},t)} - \dot{\mathbf{u}}^A|_{(\hat{\mathbf{x}},t)}\right)^T \mathbf{s}$
  of $\hat{\mathbf{y}}$ relative to $\hat{\mathbf{x}}$ at time t

$$|\tau| \leq 1$$
$$|\tau| = 0 \ \text{implies} \ sgn(\dot{u}) = sgn(\tau)$$
$$|\tau| < 0 \ \text{implies} \ \dot{u} = 0$$

constitutive equations in the contact zone

→ **normal contact**

- *classical* approach
  - leads to a *non-penetration* constraint & *compressive stress* constraint for displacements normal to the contact surface

$$g_N \geq 0, \qquad \hat{t}_N \leq 0, \qquad g_N \, \hat{t}_N = 0$$

  - *= Kuhn-Tucker conditions*

- *alternative:* micromechanical approach
  - introduction of constitutive equations on the contact surface

    → describes the spatial approximation of the bodies A & B
  - captures, in general, only essential phenomena of micromechanical constitutive relations → higher complexity

constitutive equations in the contact zone

→ **tangential contact**

- adhesion OR friction
  - requires even more complex micromechanical considerations as
    - *temperature*
    - *surface roughness*
    - *normal pressure on the surface*
    - *tangential velocity*
  - often simplified by use of a *Coulomb friction model*
    - requires only a single parameter → friction parameter μ

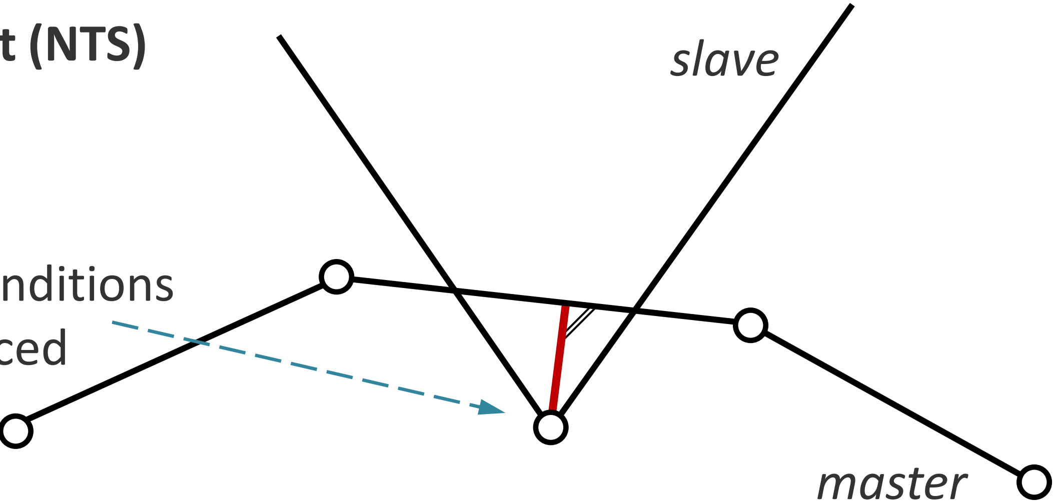
$$|\hat{t}_S| \leq \mu \, \hat{t}_N$$

$$|\hat{t}_S| < \mu \, \hat{t}_N \qquad \text{stick condition}$$

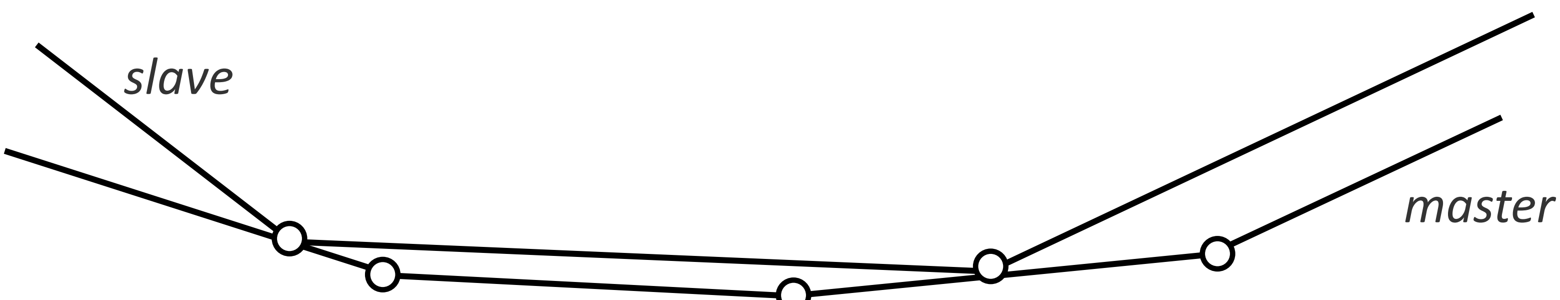
$$|\hat{t}_S| = \mu \, \hat{t}_N \qquad \text{slip condition}$$

node-to-segment (NTS)

non-penetration conditions
must be enforced

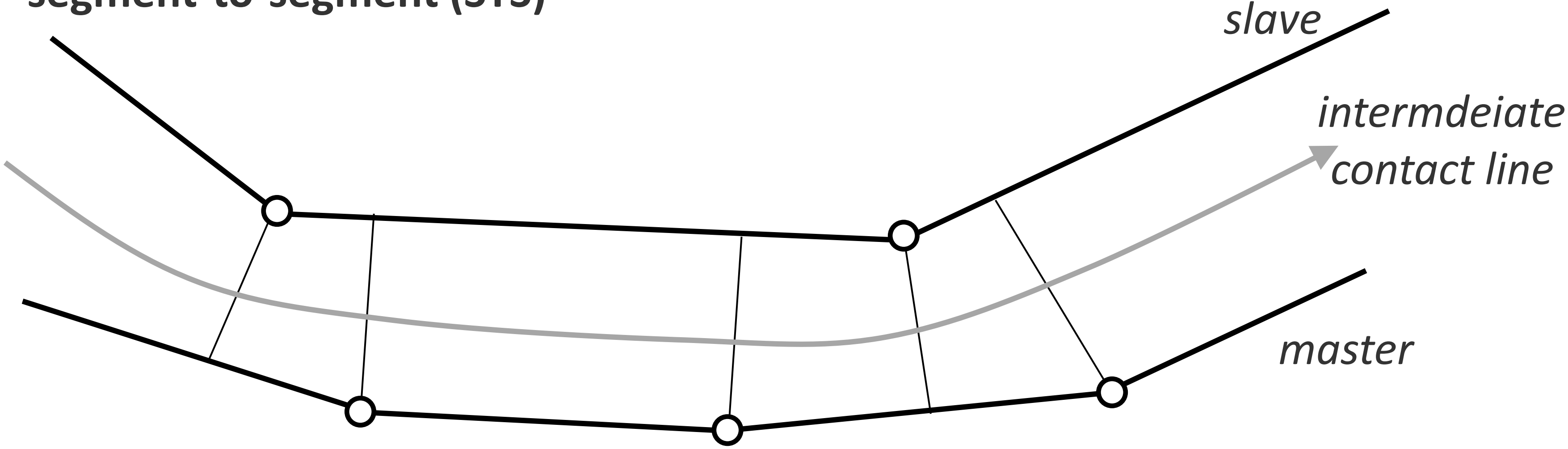


- nodes of the slave is not allowed to penetrate the master surface
- contact conditions are satisfied for the slave nodes
- non-penetration condition satisfied only pointwise on master surface
- problem of non-physical solution close to the interface region



contact interface discretization

segment-to-segment (STS)



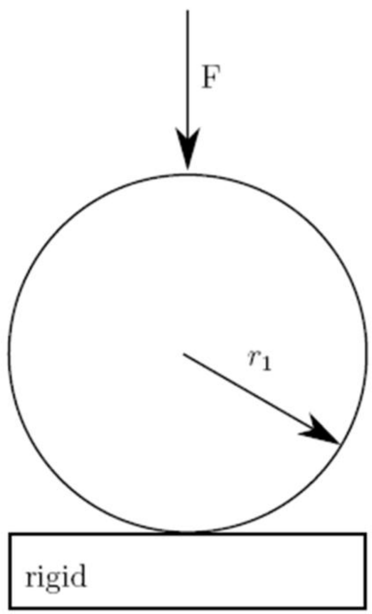
- definition of a C^1-continuous intermediate contact line
- arbitrary placement of the contact line between the surfaces
- non-penetration condition is satisfied in the mean
- gaps & penetrations along the interrface can occur

other alternatives:

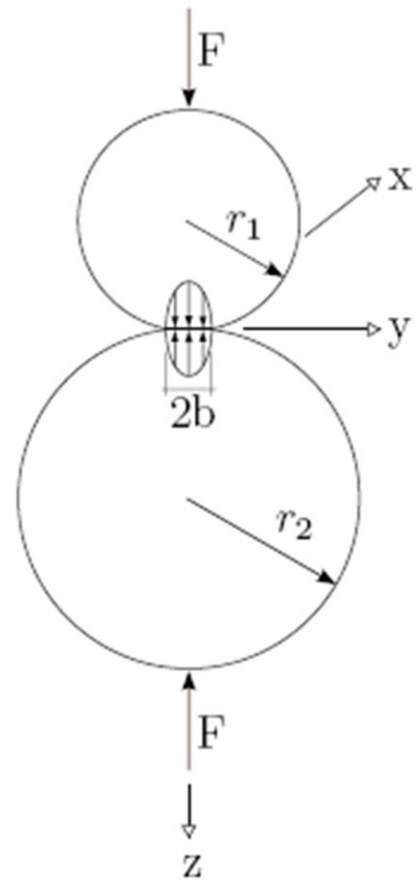
- isogeometric analysis OR p-version FEM with blending functions

assumptions

- bodies are considered as elastic half spaces
- homogeneous & isotropic properties
- frictionless contact
- contact area small compared to the bodies



unilateral contact



bilateral contact

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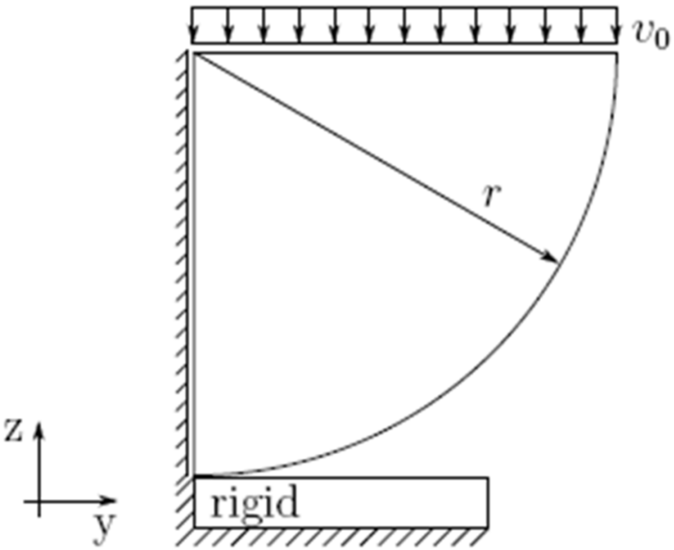
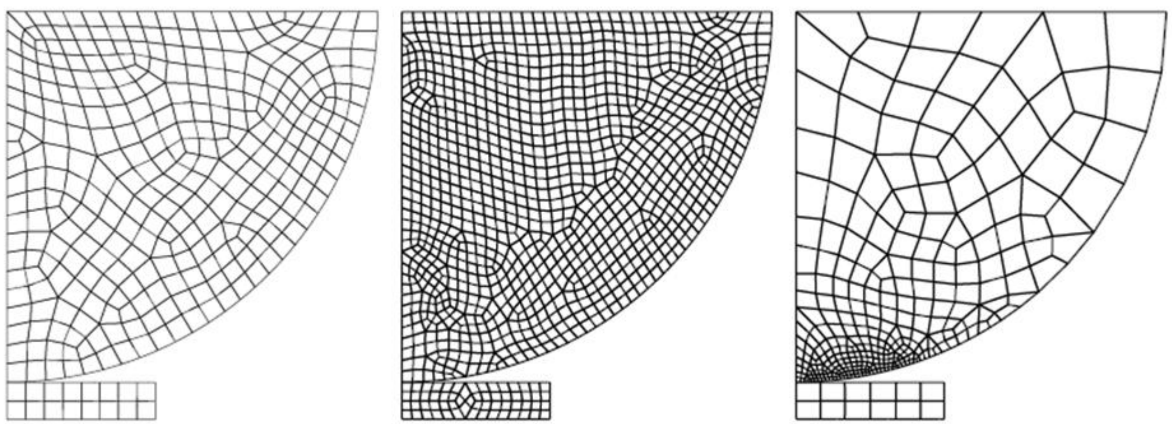
numerical model



- plane strain assumptions
- bilinear finite elements



335 elements *995 elements* *315 elements*

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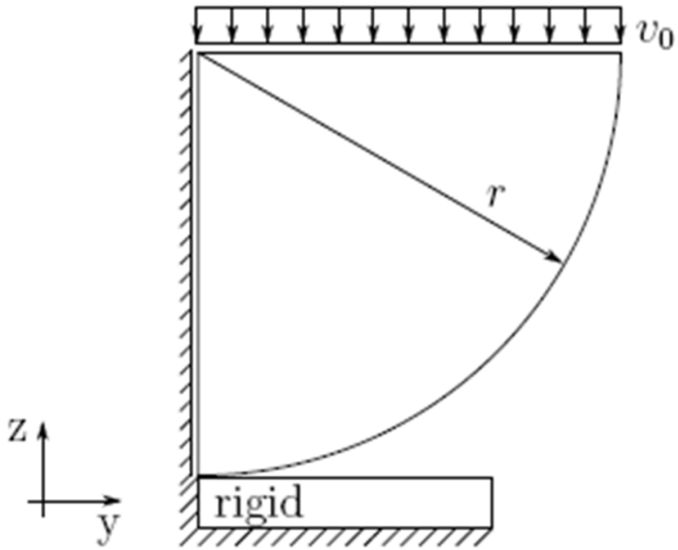
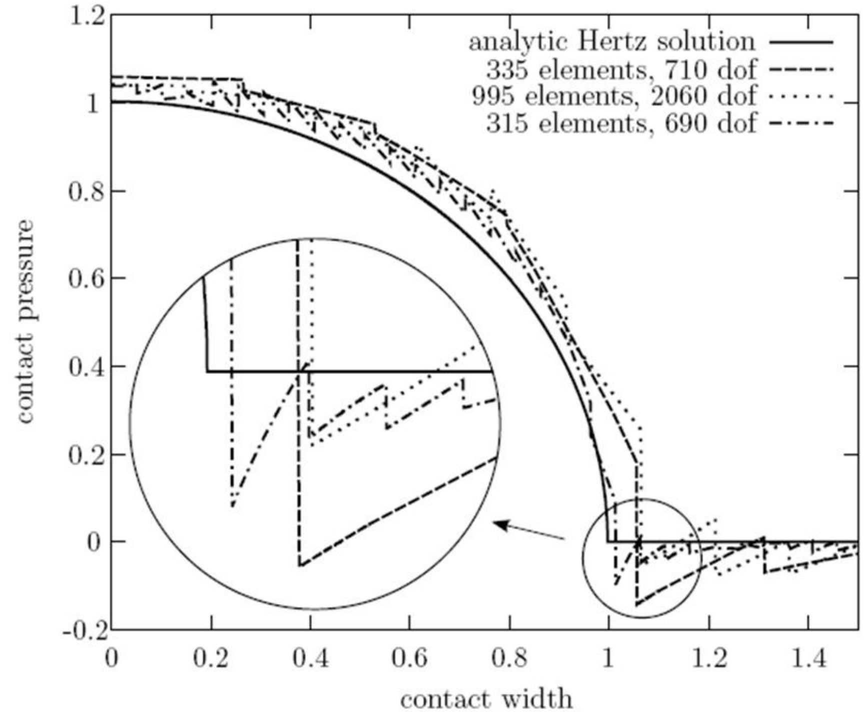
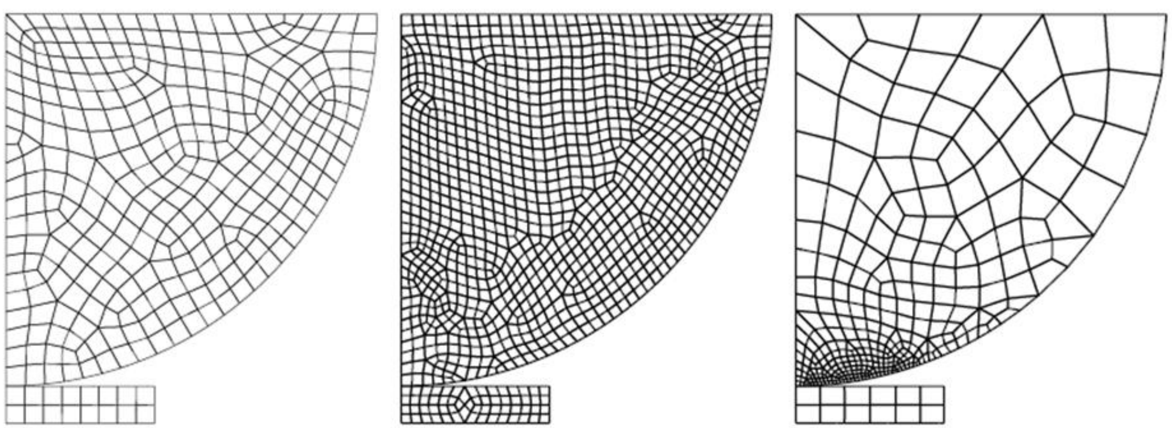
Hertz contact

numerical model







335 elements *995 elements* *315 elements*

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to be continued …

Principle of virtual work at $t \neq 0$ referred to the reference configuration

$$\sum_{m=1}^{M} \left\{ \int_{\Omega_m} \delta e_{ik}\, s_{ik}\, dv \right\} = \sum_{m=1}^{M} \left\{ \int_{\Omega_m} \delta u_i\, q_i\, dv + \int_{\Gamma_{tm}} \delta u_i\, p_{i0}\, da \right\} + \sum_{m=1}^{M} \left\{ \int_{\Gamma_{cm}} \delta u_i\, f_i\, da \right\}$$

- strains are nonlinear functions of the derivatives (Green-Lagrange)
- stresses (2^{nd} PK) are referred to base vectors of reference & instant config.
- conservative loads are assumed → independent of the displacements
- additional contact forces are considered