Bed, Bank and Shoreline protection

Chapter 7: Waves, loads

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wave issues

1. wave generation

2. wave hydrodynamics

3. wave statistics
examples of wave loads
standing waves

When waves reflect against a vertical, the sum of the incident wave and reflected wave give a standing wave.

In a standing wave orbital movement is different from a progressive wave.
wave motion in periodic, unbroken wave
validity of wave theories

[Graph showing the transition from linear to non-breaking wave theories with different order theories and their respective breaking limits.]
application of linear wave theory
gradient in filter under breakwater
friction under waves

\[ c_r = \exp \left( -6.0 + 5.2 \left( \frac{a_b}{k_r} \right)^{-0.19} \right) \]
friction factor and $c_f$

$$\hat{\tau}_w = \frac{1}{2} \rho c_f \hat{u}_b^2$$

with: $\hat{u}_b = \omega a_b = \frac{\omega a}{\sinh kh}$

$$u = \hat{u}_b \sin \omega t$$

$$c_f = \exp\left[-6.0 + 5.2 \left(\frac{a_b}{k_r}\right)^{-0.19}\right] \quad \text{with: } c_{f_{\text{max}}} = 0.3$$
waves and currents (1)

\[ u_{c-t} = \frac{\sqrt{g}}{\kappa C} u_c \quad \text{and} \quad u_{b-t} = \frac{1}{\kappa} \sqrt{\frac{c_f}{2}} u_b \sin(\omega t) \]

\[ u_r = \sqrt{\frac{g}{\kappa^2 C^2} u_c^2 + \frac{c_f}{2 \kappa^2} u_b^2 \sin^2(\omega t) + 2 \frac{\sqrt{g}}{\kappa C} u_c \frac{1}{\kappa} \sqrt{\frac{c_f}{2}} u_b \sin(\omega t) \sin(\phi)} \]

\[ \tau_r = \rho \kappa^2 u_r^2 \]
current and waves (2)

$H = 3 \text{ m}$
$T = 7 \text{ sec}$
$u_b = 1.29 \text{ m/s}$
$c_f = 0.057$
$C = 50 \sqrt{\text{m/s}}$
Nearshore effects

- Shoaling
- Refraction
- Diffraction
- Breaking
- Reflection
breaking waves

\[ H_b = 0.142 \ L \ \tanh \left( \frac{2 \pi}{L} h \right) \]

\[ \frac{H_b}{h} \approx 0.78 \text{ (solitary wave)} \]

\[ \frac{H_s}{h} \approx 0.4 - 0.5 \]
the iribarren number (surf similarity parameter)

\[ \xi = \frac{\tan \alpha}{\sqrt{\frac{H}{L_0}}} \]

\( \tan \alpha \): slope of the shoreline/structure
\( H \): wave height
\( L_0 \): wave length at deep water
breaker types (1)

\[ \xi = \tan \alpha / \sqrt{H_s / L_0} \]

Definitions

Surging, \( \xi = 5 \)

Collapsing, \( \xi = 3 \)

Plunging, \( \xi = 1.5 \)

Plunging, \( \xi = 0.5 \)

Spilling, \( \xi = 0.2 \)
breaker types

- spilling $\xi < 0.5$
- plunging $0.5 < \xi < 3$
- collapsing $\xi = 3$
- surging $\xi > 3$
bore and hydraulic jump
waves on a foreshore

**energy loss in a bore (analogy to hydraulic jump)**

\[ D = \rho g \frac{H^3}{4Th} \]

\( H = \text{wave height}, \ T = \text{wave period}, \ h = \text{water depth} \)

**energy loss during the breaking process (Battjes/Janssen method)**

\[ D = \frac{1}{4} \frac{Q_b \rho g H_m^2}{T_p} \]

**introducing wave distribution**

\[ 1 - \frac{Q_b}{\ln Q_b} = - \left( \frac{H_{rms}}{H_m} \right)^2 \]

\( Q_b \) is the fraction of all waves broken
reflection

\[ K_r = \frac{H_R}{H_I} \approx 0.1 \xi^2 \]
absorption

plunging breaker

spilling breaker

plunging jet

energy dissipation
breakerdepth

experimental data

Miche
Sol. wave

\( \gamma_b \)

\( \xi_0 \)
change of distribution in shallow water
run up

\[ \frac{R_u}{H} = \xi \]

\( R_u \) - run-up
\( R_d \) - depth
\( \text{SWL} \) - sea level

Smooth and rough cases are shown.
2% run-up ($R_{2\%}$)

There is a linear relation between $H$ and $R_u$.
When we assume $H$ is Rayleigh distributed
then $R_u$ is also Rayleigh distributed

$$\frac{R_n}{R_{2\%}} = 0.71 \sqrt{-\frac{1}{2} \ln(n)}$$

$n = $ exceedance percentage (e.g. 0.01)

Why 2% and not 3% ?????
Old Delft Formula

\[ R_{2\%} = 8H_s \tan \alpha \]

- Valid for \( \tan \alpha < 1/3 \) and relatively smooth slopes
- Valid for “normal” wave steepness (between 4 and 5 %)
Run-up calculation

\[ \frac{R_u}{H} = \xi \]

Hunt’s formula

\[ R_{u2\%} = 1.5 \gamma_r \gamma_\beta \gamma_B \gamma_f H_s \xi_p \]

\[ (R_{u2\% \ max} = 3H_s) \]

correction factors:

• \( \gamma_r \) roughness
• \( \gamma_\beta \) approach angle
• \( \gamma_B \) berm reduction
• \( \gamma_f \) foreshore reduction
EurOtop (TAW, Van der Meer)

\[
\frac{R_{2\%}}{H_s} = 1.65 \xi_0 \\
\frac{R_{2\%}}{H_s} = 4.0 - \frac{1.5}{\sqrt{\xi_0}}
\]

for \( \xi_0 \leq 1.6 \)

for \( 1.6 < \xi_0 < 10 \)

\( H_s = \) significant wave height
\( \xi_0 = \) breaker parameter based on \( T_{m-1.0} \)

Dutch and English version downloadable from [www.ENWinfo.nl](http://www.ENWinfo.nl)

See also: [www.overtopping-manual.com](http://www.overtopping-manual.com)
EurOtop formula
Example

Given: \( H_s = 2.5 \text{ m} \quad T_p = 8 \text{ s} \quad \text{smooth slope 1:4} \)

How much is Run-up ??

Old Delft Formula: \( R = 8 \times 2.5 / 4 = 5 \text{ m} \)

New Delft Formula:
\[
T_{m-1,0} = 0.9 \times T_p = 7.2 \text{ s} \\
\xi = 0.25 / \sqrt{(2.5/(1.56 \times 7.2^2))} = 1.42 \\
R = 1.65 \times 2.5 \times 1.42 = 5.86
\]
# friction values

<table>
<thead>
<tr>
<th>$\gamma_r$</th>
<th>Type of revetment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>Asphalt, concrete, smooth blocks, grass, Sand-asphalt</td>
</tr>
<tr>
<td>0.95</td>
<td>Blocks in asphalt or concrete matrix, blocks with grass</td>
</tr>
<tr>
<td>0.90</td>
<td>Basalt, Basalton, Hydroblock, Haringman, Fixstone, Armorflex</td>
</tr>
<tr>
<td>0.85</td>
<td>Lessinische and Vilvoordse, small roughness blocks</td>
</tr>
<tr>
<td>0.80</td>
<td>riprap penetrated with asphalt</td>
</tr>
<tr>
<td>0.70</td>
<td>Single layer of riprap</td>
</tr>
<tr>
<td>0.55</td>
<td>Double layer of riprap</td>
</tr>
</tbody>
</table>
Run up on Elastocoast and Haringman
cross section of the location
Waves and waterlevels on 12/10/2009
<table>
<thead>
<tr>
<th>case</th>
<th>Runup on slope</th>
<th>Runup above msl</th>
<th>Runup above HW</th>
<th>$f \times 8H\tan\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Har high</td>
<td>$0.5 \times 9/4 = 1.25$</td>
<td>3.25</td>
<td>3.25 - 2.20 = 1.05</td>
<td>1.0</td>
</tr>
<tr>
<td>Har low</td>
<td>$0.5 \times 8/4 = 1.00$</td>
<td>3.00</td>
<td>3.00 - 1.90 = 1.10</td>
<td>1.0</td>
</tr>
<tr>
<td>Elas high</td>
<td>$0.5 \times 4/4 = 1.00$</td>
<td>3.00</td>
<td>3.00 - 2.20 = 0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>Elas low</td>
<td>$0.5 \times 3/4 = 0.75$</td>
<td>2.75</td>
<td>2.75 - 1.90 = 0.85</td>
<td>0.80</td>
</tr>
</tbody>
</table>
Very rough slopes
Example with Cress

run demo Cress
Relative run-up measurements
deep water, smooth slope

from TAW-report
Run-up and Overtopping
may 2002
Short crested and long crested

Old equations assumed regular waves
New equation assumes shortcrested waves

Important for oblique wave attack:
Van der Meer: \( \gamma_b = 1 - 0.0022 \beta \)
Old Equation: \( \gamma_b = \cos (\beta - 10^\circ) \) for \( \beta < 65^\circ \)
shortcrested and longcrested

\[ \gamma_b \]

\[ 1.05\beta \]

\[ 1 - 0.0022\beta \]

\[ \cos(\beta - 10^\circ) \]

- Test results shortcrested
- Test results longcrested

Angle of incidence \( \beta \)
berm effect

\[
\gamma_B = 1 - \frac{B_B}{L_B} \left( 0.5 + 0.5 \cos \left[ \pi \frac{d_h}{x} \right] \right)
\]

\[
x = z_{2\%} \quad \text{if} \quad z_{2\%} > -d_h > 0 \quad \text{(aboveSWL)}
\]

\[
x = 2H_{m0} \quad \text{if} \quad 2H_{m0} > d_h > 0 \quad \text{(belowSWL)}
\]
berm effect (2)
Shallow foreshore

geometry of test set-up

locations of wave measurements

bottom of flume

SWL

foreshore 1:100

slope of dike 1:4

height from bottom of flume (m)

distance from wave generator (m)

wave spectra measured in location

- A: relatively deep water
- B: half way the foreshore
- C: near the toe of the dike

energy density (m^2/Hz)

frequency (Hz)
Shallow foreshore

- Parameter $T_{m-1,0}$ is a good descriptor
- Use SwanOne to calculate $T_{m-1,0}$
run-down

\[ R_d = R_u \left( 1 - 0.4 \xi \right) \]
\[ = H \left( 1 - 0.4 \xi \right) \xi \]

\[ R_{d\ 2\%} = -0.33 H_s \xi_p \]
\[ (R_{d\ 2\%\ max} = -1.5 H_s) \]
Overtopping (1)

- Basically same type of equations as for run-up.
- Usually wave overtopping is expressed in a discharge $q$.

However, this is a time-averaged discharge.

$$Q = a \exp\left( b \frac{R}{\gamma} \right)$$

*for breaking (plunging)*  \( a = 0.067, \ b = 4.75, \ \sigma_b = 0.5 \)

*for non-breaking*  \( a = 0.2, \ b = 2.6, \ \sigma_b = 0.35 \)

$Q$ is dimensionless overtopping,
$R$ is dimensionless freeboard.
Overtopping (2)

\[ Q = \frac{q}{\sqrt{gH_s^3 \tan \alpha}} \sqrt{\frac{h/L_0}{1}} \]

\[ R = \frac{h_k}{H_s} \frac{1}{\xi} \]

In which:

- \( q \) = average overtopping (\( m^3/s \) per meter)
- \( h_k \) = crest freeboard (m)

In case of non-breaking, this root should be 1.
Measured overtopping (breaking)

\[ q = \frac{S_0}{gH_{m0}^{3/2}} \tan \alpha \]

- \( q \) is the dimensionless overtopping discharge
- \( S_0 \) is the dimensionless overtopping discharge
- \( g \) is the acceleration due to gravity
- \( H_{m0} \) is the mean wave height
- \( \tan \alpha \) is the slope angle

\[ \frac{R_c}{H_{m0}} \sqrt{\frac{S_0}{\tan \alpha}} = \frac{1}{\gamma_b \gamma_f / \gamma_v} \]

- \( R_c \) is the dimensionless crest height
- \( H_{m0} \) is the mean wave height
- \( \gamma_b \), \( \gamma_f \), and \( \gamma_v \) are density factors

Examples:
- \( H_{m0} = 1 \text{ m}; \ cot \alpha = 4; S_0 = 0.03 \)
- \( H_{m0} = 2.5 \text{ m}; \ cot \alpha = 4; S_0 = 0.03 \)
- \( \xi_0 < 2 \)
Measured overtopping (non-breaking)
Overtopping (3)

- Reduction coefficients are equal to Run-up
- However:
correction for angle of incidence:

\[ \gamma_b = 1 - 0.0033 \beta \]

run demo Cress
Overtopping vs. Run-up

- For design inner slope overtopping is more relevant than run-up
- In the past overtopping could not be computed
- In modern design, apply overtopping rules
Allowable overtopping

• Dutch dikes:
  • any slope \( q < 0.1 \text{ l/s} \)
  • normal slope \( q < 1.0 \text{ l/s} \)
  • high quality slope \( q < 10 \text{ l/s} \)
• For breakwaters much higher values can be applied
• For safe passages of cars \( q < 0.001 \text{ l/s} \)
• For safe passage of pedestrians \( q < 0.005 \text{ l/s} \)
• For no damage to buildings \( q < 0.001 \text{ l/s} \)
• For acceptable damage to buildings \( q < 0.02 \text{ l/s} \)
Overtopping per wave

- Graph showing the relationship between maximum volume in overtopping wave (l/m) and average overtopping discharge (l/m/s).
- Two curves are shown for different wave heights: $H_m = 2.5\, \text{m}$ and $H_m = 1\, \text{m}$.
- Relations for $\tan \alpha = 0.25$, $s_0 = 0.04$ during 1 hour.
wave impacts

\[ p_{\text{max}} 50\% \approx 8 \rho_w g H_s \tan \alpha \]

\[ p_{\text{max}} 0.1\% \approx 16 \rho_w g H_s \tan \alpha \]
load reduction - pile screens

\[ \frac{1}{8} \rho g H_i^2 = \frac{1}{8} \rho g H_T^2 + \frac{1}{8} \rho g H_R^2 + \text{absorption} \]

\[ K_T = \sqrt{\frac{F_T}{F_I}} = \frac{H_T}{H_I} \approx \left( \frac{H_T}{H_I} \right)_{1/3} \]

\[ H_T = \sqrt{(1-W) H_I^2} \]

\[ \rightarrow K_T = \sqrt{1 - W^2} \]

\[ K_T = \sqrt{1 - W^2} \]
wave transmission over low breakwaters
floating breakwaters

![Graph showing floating breakwaters](image-url)
Example dike height determination

• Dike height is:
  • design water level
  • + freeboard

• Design water level is:
  • astronomical tide (high water)
  • + storm surge (wind set-up)
  • + margin for seiches and gust bumps
  • + surcharge for
    • sea level rise
    • increase of tidal amplitude
  • + settlement of subsoil

for text of examples, see “blackboard”

• freeboard is:
  • 2% Run-up height
  • height determined by critical overtopping (e.g. 1 l/s)
  • at least 50 cm

but
Dike height determination (2)

- design water level
- mean high water
- normal water level
- mean low water level
- extreme loading
- acceptable loading

I II III IV
Sea dike design (input)

Design a seadike for 1/100 and 1/1000, using Hook of Holland boundary conditions.

<table>
<thead>
<tr>
<th></th>
<th>1/100</th>
<th>1/1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waterlevel</td>
<td>3.5 + MSL</td>
<td>4.25 + MSL</td>
</tr>
<tr>
<td>Wind speed</td>
<td>22 m/s</td>
<td>25 m/s</td>
</tr>
<tr>
<td>Wave height</td>
<td>6.5 m</td>
<td>7.5 m</td>
</tr>
</tbody>
</table>

Depth in front of dike: 5 m below MSL
Slope of dike 1:4
Berm at SWL, berm width 10 m
Design life is 50 years
Sea dike design (levels)

Sea level rise now 20 cm/century
Rise tidal amplitude 10 cm
So, \( \frac{20+10}{2} = 15 \text{ cm surcharge} \)

And what to do with accelerated sea level rise (climate change) ??

Summary table:

<table>
<thead>
<tr>
<th></th>
<th>1/100</th>
<th>1/1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>1/100</td>
<td>1/1000</td>
</tr>
<tr>
<td>Design level</td>
<td>3.50 m+MSL</td>
<td>4.25 m+MSL</td>
</tr>
<tr>
<td>Gust bump</td>
<td>0.50 m</td>
<td>0.50 m</td>
</tr>
<tr>
<td>Seiches</td>
<td>0.00 m</td>
<td>0.00 m</td>
</tr>
<tr>
<td>Sealevel rise</td>
<td>0.15 m</td>
<td>0.15 m</td>
</tr>
<tr>
<td>Design water level</td>
<td>4.15 m+MSL</td>
<td>4.90 m +MSL</td>
</tr>
</tbody>
</table>
Sea dike design (waves)

Design wave height is depth limited, in this situation a breaker index $\gamma = 0.5$ may be used

<table>
<thead>
<tr>
<th>Frequency</th>
<th>1/100</th>
<th>1/1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water depth</td>
<td>5+4.15 = 9.15 m</td>
<td>5+4.90 = 9.90 m</td>
</tr>
<tr>
<td>Design wave</td>
<td>4.45 m</td>
<td>4.95 m</td>
</tr>
</tbody>
</table>
## Sea dike design (freeboard)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>1/100</th>
<th>1/1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total run-up</td>
<td>6.92 m</td>
<td>8.28 m</td>
</tr>
<tr>
<td>Freeboard (q= 1 l/s)</td>
<td>7.88 m</td>
<td>8.54 m</td>
</tr>
</tbody>
</table>
Sea dike design (crest level)

Design crest level is 
design water level + freeboard + settlement

assume good quality subsoil, 50 cm settlement 
during design life

<table>
<thead>
<tr>
<th>frequency</th>
<th>Ultimate crest level</th>
<th>Construction crest level, above MSL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/100</td>
<td>4.15+6.92=11.10 m</td>
<td>11.6 m</td>
</tr>
<tr>
<td>1/1000</td>
<td>4.90+8.28=13.20 m</td>
<td>13.7 m</td>
</tr>
</tbody>
</table>
berm and slope optimization (1)
berm and slope optimization (2)

Given:  
wave height 3 m  
wave period 8 seconds  
SWL = berm height  
berm at 1:20  
slope with Basalton (1:3, 1:5) or grass (1:8)  
deepth of dike below SWL = 6.5 m  
crest width = 2 m  
inner slope 1:2
berm and slope optimization (3)

<table>
<thead>
<tr>
<th>Slope</th>
<th>1 : 3</th>
<th>1 : 5</th>
<th>1 : 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berm width</td>
<td>Overtopping</td>
<td>Berm width</td>
<td>Overtopping</td>
</tr>
<tr>
<td>0 m</td>
<td>0.1 l/m/s</td>
<td>0.1 l/m/s</td>
<td>0.1 l/m/s</td>
</tr>
<tr>
<td></td>
<td>1.0 l/m/s</td>
<td>1.0 l/m/s</td>
<td>1.0 l/m/s</td>
</tr>
<tr>
<td></td>
<td>10 l/m/s</td>
<td>10 l/m/s</td>
<td>10 l/m/s</td>
</tr>
<tr>
<td>5 m</td>
<td>0.1 l/m/s</td>
<td>0.1 l/m/s</td>
<td>0.1 l/m/s</td>
</tr>
<tr>
<td></td>
<td>1.0 l/m/s</td>
<td>1.0 l/m/s</td>
<td>1.0 l/m/s</td>
</tr>
<tr>
<td></td>
<td>10 l/m/s</td>
<td>10 l/m/s</td>
<td>10 l/m/s</td>
</tr>
<tr>
<td>10 m</td>
<td>0.1 l/m/s</td>
<td>0.1 l/m/s</td>
<td>0.1 l/m/s</td>
</tr>
<tr>
<td></td>
<td>1.0 l/m/s</td>
<td>1.0 l/m/s</td>
<td>1.0 l/m/s</td>
</tr>
<tr>
<td></td>
<td>10 l/m/s</td>
<td>10 l/m/s</td>
<td>10 l/m/s</td>
</tr>
<tr>
<td>15 m</td>
<td>0.1 l/m/s</td>
<td>0.1 l/m/s</td>
<td>0.1 l/m/s</td>
</tr>
<tr>
<td></td>
<td>1.0 l/m/s</td>
<td>1.0 l/m/s</td>
<td>1.0 l/m/s</td>
</tr>
<tr>
<td></td>
<td>10 l/m/s</td>
<td>10 l/m/s</td>
<td>10 l/m/s</td>
</tr>
</tbody>
</table>

required crest level for a given berm, a given slope and a given discharge
berm and slope optimization (4)

<table>
<thead>
<tr>
<th>Slope</th>
<th>1 : 3</th>
<th>1 : 5</th>
<th>1 : 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overtopping Berm width</td>
<td>0,1 l/m/s</td>
<td>1,0 l/m/s</td>
<td>10 l/m/s</td>
</tr>
<tr>
<td>0 m</td>
<td>997 €14955</td>
<td>742 €11134</td>
<td>525 €8750</td>
</tr>
<tr>
<td>5 m</td>
<td>803 €12051</td>
<td>623 €9345</td>
<td>467 €6999</td>
</tr>
<tr>
<td>10 m</td>
<td>707 €10606</td>
<td>567 €8497</td>
<td>437 €6548</td>
</tr>
<tr>
<td>15 m</td>
<td>663 €9945</td>
<td>546 €8192</td>
<td>443 €6642</td>
</tr>
</tbody>
</table>

volume of the dike, using simple geometry; earth moving cost calculated using €15/m³
berm and slope optimization (5)

<table>
<thead>
<tr>
<th>Overtopping Berm width</th>
<th>Slope 1 : 3</th>
<th></th>
<th>Slope 1 : 5</th>
<th></th>
<th>Slope 1 : 8</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0 m</td>
<td>0,1 l/m/s</td>
<td>1,0 l/m/s</td>
<td>10 l/m/s</td>
<td>0,1 l/m/s</td>
<td>1,0 l/m/s</td>
<td>10 l/m/s</td>
</tr>
<tr>
<td>5 m</td>
<td>109</td>
<td>94</td>
<td>79</td>
<td>108</td>
<td>96</td>
<td>84</td>
</tr>
<tr>
<td></td>
<td>€ 820</td>
<td>€ 708</td>
<td>€ 596</td>
<td>€ 807</td>
<td>€ 717</td>
<td>€ 627</td>
</tr>
<tr>
<td>10 m</td>
<td>101</td>
<td>89</td>
<td>77</td>
<td>104</td>
<td>96</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>€ 759</td>
<td>€ 669</td>
<td>€ 579</td>
<td>€ 778</td>
<td>€ 718</td>
<td>€ 640</td>
</tr>
<tr>
<td>15 m</td>
<td>97</td>
<td>88</td>
<td>77</td>
<td>107</td>
<td>98</td>
<td>88</td>
</tr>
<tr>
<td></td>
<td>€ 734</td>
<td>€ 657</td>
<td>€ 576</td>
<td>€ 804</td>
<td>€ 732</td>
<td>€ 660</td>
</tr>
</tbody>
</table>

purchase of land needed for the construction of the dike, values in m$^2$; cost using € 7.50/ m$^2$
berm and slope optimization (6)

<table>
<thead>
<tr>
<th>Slope Berm width</th>
<th>1 : 3</th>
<th>1 : 5</th>
<th>1 : 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 m</td>
<td>8.3 m, 22.6 m² (€ 2713)</td>
<td>5.0 m, 28.0 m² (€ 2244)</td>
<td>3.0 m, 0 m² (€ 250)</td>
</tr>
<tr>
<td>5 m</td>
<td>6.7 m, 19.8 m² (€ 2630)</td>
<td>4.4 m, 26.3 m² (€ 2351)</td>
<td>2.8 m, 0 m² (€ 250)</td>
</tr>
<tr>
<td>10 m</td>
<td>5.6 m, 17.8 m² (€ 2641)</td>
<td>4.0 m, 25.0 m² (€ 2500)</td>
<td>2.6 m, 0 m² (€ 250)</td>
</tr>
<tr>
<td>15 m</td>
<td>5.0 m, 16.6 m² (€ 2747)</td>
<td>3.6 m, 23.7 m² (€ 2648)</td>
<td>2.5 m, 0 m² (€ 250)</td>
</tr>
</tbody>
</table>

**Cost of the revetment**
- 1:3 € 120/m²
- 1:5 € 80/m²
- 1:8 no cost (grass)

**Cost of the berm**
- € 50/m²

June 3, 2012
# berm and slope optimization (7)

<table>
<thead>
<tr>
<th>Slope</th>
<th>1 : 3</th>
<th>1 : 5</th>
<th>1 : 8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1 l/m/s</td>
<td>1.0 l/m/s</td>
<td>10 l/m/s</td>
</tr>
<tr>
<td></td>
<td>0 m</td>
<td>5 m</td>
<td>10 m</td>
</tr>
<tr>
<td>Berm width</td>
<td>Overtopping</td>
<td>Berm width</td>
<td>Overtopping</td>
</tr>
<tr>
<td></td>
<td>€18,489 (100%)</td>
<td>€15,439 (84%)</td>
<td>€13,980 (76%)</td>
</tr>
<tr>
<td></td>
<td>€14,555 (75%)</td>
<td>€12,643 (55%)</td>
<td>€13,980 (53%)</td>
</tr>
<tr>
<td></td>
<td>€13,901 (62%)</td>
<td>€12,765 (55%)</td>
<td>€13,132 (62%)</td>
</tr>
<tr>
<td></td>
<td>€11,525 (62%)</td>
<td>€11,283 (61%)</td>
<td>€13,132 (53%)</td>
</tr>
<tr>
<td></td>
<td>€9,419 (51%)</td>
<td>€9,536 (52%)</td>
<td>€9,835 (53%)</td>
</tr>
<tr>
<td></td>
<td>€10,108 (55%)</td>
<td>€10,521 (57%)</td>
<td>€10,940 (59%)</td>
</tr>
<tr>
<td></td>
<td>€8,529 (46%)</td>
<td>€8,972 (49%)</td>
<td>€9,583 (52%)</td>
</tr>
<tr>
<td></td>
<td>€7,084 (38%)</td>
<td>€7,707 (42%)</td>
<td>€8,331 (45%)</td>
</tr>
</tbody>
</table>

**Summary of all the costs**

Realise that for 10 l/s the cost of the inner slope will be higher than for a 0.1 l/s slope!!
berm and slope optimization (8)
berm and slope optimization (9)
berm and slope optimization (8)

- q = 0.1 l/m/s
- q = 1 l/m/s
- q = 10 l/m/s

given berm width 10 m

- b = 0
- b = 5 m
- b = 10 m
- b = 15 m

given discharge 0.1 l/s
Simulator for highschools

user interface

spreadsheet
recapitulation of short wave theory
definitions and behaviour of hyperbolic functions
standing wave
<table>
<thead>
<tr>
<th>Relative depth</th>
<th>Shallow Water</th>
<th>Transitional water depth</th>
<th>Deep Water</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h &lt; L &lt; 20$</td>
<td>$1 &lt; h &lt; L &lt; 2$</td>
<td>$h &gt; L &gt; 2$</td>
</tr>
<tr>
<td>Wave Celerity</td>
<td>$c = \frac{L}{T} = \sqrt{gh}$</td>
<td>$c = \frac{L}{T} = \frac{gT}{2\pi} \tanh kh$</td>
<td>$c = \frac{L}{T} = \frac{gT}{2\pi}$</td>
</tr>
<tr>
<td>Wave Length</td>
<td>$L = T\sqrt{gh}$</td>
<td>$L = \frac{gT}{2\pi} \tanh kh$</td>
<td>$L = \frac{gT}{2\pi}$</td>
</tr>
<tr>
<td>Group Velocity</td>
<td>$c_g = c = \sqrt{gh}$</td>
<td>$c_g = n c = \frac{1}{2} \cosh 2kh \sinh 2kh$</td>
<td>$c_g = \frac{1}{4\pi} \frac{gT}{c_0}$</td>
</tr>
<tr>
<td>Energy Flux</td>
<td>$F = E c_g = \frac{1}{2} \rho g a^2 \sqrt{gh}$</td>
<td>$F = E c_g = \frac{1}{2} \rho g a^2 n c$</td>
<td>$F = \frac{T}{8\pi} \rho g a^2$</td>
</tr>
<tr>
<td>Particle velocity</td>
<td>$u = \frac{a}{\sqrt{h}} \sin \theta$</td>
<td>$u = \omega a \frac{\cosh k(h+z)}{\sinh kh} \sin \theta$</td>
<td>$a = \omega a e^{ki} \sin \theta$</td>
</tr>
<tr>
<td>Horizontal</td>
<td>$w = \omega a e^{kz} \cos \theta$</td>
<td>$w = \omega a \frac{\sinh k(h+z)}{\sinh kh} \cos \theta$</td>
<td>$w = \omega a \sin \theta$</td>
</tr>
<tr>
<td>Vertical</td>
<td>$\xi = -\frac{a}{\omega} \sqrt{\frac{g}{h}} \cos \theta$</td>
<td>$\xi = -\frac{a}{\omega} \frac{\cosh k(n+z)}{\sinh kh} \cos \theta$</td>
<td>$\zeta = a e^{zi} \sin \theta$</td>
</tr>
<tr>
<td>Particle displacement</td>
<td>$p = -\rho g z + \rho g a \sin \theta$</td>
<td>$p = -\rho g z + \rho g a \frac{\cosh k(h+z)}{\cosh kh} \sin \theta$</td>
<td>$p = -\rho g z + \rho g a \cos \theta$</td>
</tr>
<tr>
<td>Horizontal</td>
<td>$\zeta = a e^{zi} \sin \theta$</td>
<td>$\zeta = a e^{zi} \sin \theta$</td>
<td>$\zeta = a e^{zi} \sin \theta$</td>
</tr>
<tr>
<td>Vertical</td>
<td>$\zeta = a e^{zi} \sin \theta$</td>
<td>$\zeta = a e^{zi} \sin \theta$</td>
<td>$\zeta = a e^{zi} \sin \theta$</td>
</tr>
</tbody>
</table>

For a linear wave theory, the basic equations are:

$$a = \frac{H}{2}, \quad \omega = \frac{2\pi}{T}, \quad k = \frac{2\pi}{L}, \quad \theta = \omega t - kx$$
parameters in linear wave theory
definition of $H$ and $T$
wave definitions and wave height distribution

---

**Diagram:**

- **Diagram a:**
  - $\eta$: Wave height
  - $H$: Wave height
  - $T$: Wave period

- **Diagram b:**
  - $H/H_s$: Wave height normalized by significant wave height
  - $H_{rms}$: Root mean square wave height
  - $H_{av}$: Average wave height

- Plot showing the relationship between $P(H > H)$ and $H/H_s$.
Rayleigh distribution

\[ P\{H > H\} = \exp \left[ -\left( \frac{H}{H_{rms}} \right)^2 \right] = \exp \left[ -2 \left( \frac{H}{H_s} \right)^2 \right] \]

\[ H_s \equiv H_{visual} \equiv H_{1/3} \equiv H_{13.5\%} \equiv H_{m0} \approx 4\sqrt{m_0} \]
wave height and wave period
wave registration in the North Sea

Spectral moments: $m_0 = \text{surface of energy density spectrum}$
$m_{-1} = \text{first negative moment of spectrum}$
$T_{m-1,0} = m_{-1}/m_0 = \text{spectral wave period} \approx 0.9 T_p$
$m_n = \int_0^\infty f^n S(f) \, df$
Example wave record

28 waves, $H_s = "13\%\ wave", H_s = \text{wave nr 4}, H_s \approx 3.8$

28 waves in 150 seconds, so $T_m = 5.3\ s$
composition of the record

\[ \begin{align*}
H_1 &= 0.63 \text{ m} & T_1 &= 4 \text{ sec} \\
H_2 &= 1.80 \text{ m} & T_2 &= 5 \text{ sec} \\
H_3 &= 1.55 \text{ m} & T_3 &= 6.67 \text{ sec} \\
H_4 &= 0.90 \text{ m} & T_4 &= 10 \text{ sec} & T_m &= 5.3 \text{ sec}
\end{align*} \]
Spectrum

\[ \frac{1}{2} a^2 = S \cdot \Delta f \]

\[ H = \sqrt{8S \cdot \Delta f} \]

\[ S = \frac{H^2}{8\Delta f} = \frac{1.55^2}{8 \cdot 0.05} = 6 \text{[m}^2\text{s]} \]
Calculation of $m_0$

\[0.05 \times 2 = 0.10\]
\[0.05 \times 6 = 0.30\]
\[0.05 \times 3 = 0.15\]
\[0.05 \times 1 = 0.05\]

\[\sum = 0.60\]

\[4 \sqrt{m_0} = 3.1 \, m\]

\[m_n = \int_0^\infty f^n S(f) \, df\]
Calculation of $m_1$

$$\text{dist} \times S \Delta f$$

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Energy Density ($m^2s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>0.15</td>
<td>0.30</td>
</tr>
<tr>
<td>0.20</td>
<td>0.15</td>
</tr>
<tr>
<td>0.25</td>
<td>0.05</td>
</tr>
</tbody>
</table>

\[
m_n = \int_0^\infty f^n S(f) \, df
\]
Calculation of $m_2$

\[ \text{dist}^2 \ast S \Delta f \]

\[
\begin{array}{ccc}
0.10^2 \ast 0.10 & 1.00 \times 10^{-3} \\
0.15^2 \ast 0.30 & 6.75 \times 10^{-3} \\
0.20^2 \ast 0.15 & 6.00 \times 10^{-3} \\
0.25^2 \ast 0.05 & 3.12 \times 10^{-3}
\end{array}
\]

\[
m_n = \int_{0}^{\infty} f^n S(f) df
\]

\[
T = \sqrt{\frac{m_0}{m_2}} = 10 \sqrt{\frac{0.60}{1.69}} = 5.69 \text{ sec}
\]
Calculation of $m_{-1}$

\[ T_{m-1,0} = \frac{m_{-1}}{m_0} = \frac{3.95}{0.60} = 6.58 \text{ sec} \]

\[ m_n = \int_0^\infty f^n S(f) df \]
Overview

• $H_{m0} = 3.1 \text{ m}$
  \[(1.55+1.10+0.90+0.63=4.18)\]
• $T_{m0} = 5.69 \text{ sec}$
• $T_{m-1,0} = 6.58 \text{ sec}$
• $T_{\text{peak}} = 6.67 \text{ sec}$
• $T_m = 5.35 \text{ sec}$

\[
\frac{T_{m-1,0}}{T_{m0}} = \frac{6.58}{5.69} = 1.16
\]

For standard spectra:

- Goda: $T_p = 1.1 T_{1/3}$
- PM: $T_p = 1.15 T_{1/3}$
- Jonswap: $T_p = 1.07 T_{1/3}$
- TAW (vdMeer): $T_p = 1.1 T_{m-1,0}$
- Old Test (vdMeer): $T_p = 1.04 T_{m-1,0}$

Also:

$T_{m-1,0} = 1.064 T_{1/3}$
spectrum types

<table>
<thead>
<tr>
<th></th>
<th>Record</th>
<th>Spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>sine</td>
<td>![sine waveform]</td>
<td>![sine spectrum]</td>
</tr>
<tr>
<td>wind wave</td>
<td>![wind wave waveform]</td>
<td>![wind wave spectrum]</td>
</tr>
<tr>
<td>noise</td>
<td>![noise waveform]</td>
<td>![noise spectrum]</td>
</tr>
<tr>
<td>sea and swell</td>
<td>![sea and swell waveform]</td>
<td>![sea and swell spectrum]</td>
</tr>
</tbody>
</table>
two types of spectra
wave height distribution in shallow water
shallow water distributions

\[
\Pr\{H \leq H\} = \begin{cases} 
F_1(H) = 1 - \exp\left[-\left(\frac{H}{H_1}\right)^2\right] & H \leq H_{tr} \\
F_2(H) = 1 - \exp\left[-\left(\frac{H}{H_2}\right)^{3.6}\right] & H > H_{tr}
\end{cases}
\]
wave spectra across shallow bar
wave generation

\[
\frac{gH_s}{u_w^2} = 0.283 \tanh \left[ 0.578 \left( \frac{gh}{u_w^2} \right)^{0.75} \right] \tanh \left[ \frac{gF}{u_w^2} \right]^{0.42}
\]

\[
\frac{gT_s}{2\pi u_w} = 1.20 \tanh \left[ 0.833 \left( \frac{gh}{u_w^2} \right)^{0.375} \right] \tanh \left[ \frac{gF}{u_w^2} \right]^{0.25}
\]
wave height as function of wind, depth and fetch

![Graph showing wave height as function of wind speed, depth, and fetch distance.](image)
wave period as function of wind, depth and fetch
wave height as function of duration, fetch and wind velocity

- $F=100\ \text{km, } u_w=30\ \text{m/s}$
- $F=10\ \text{km, } u_w=30\ \text{m/s}$
- $F=100\ \text{km, } u_w=10\ \text{m/s}$
- $F=10\ \text{km, } u_w=10\ \text{m/s}$