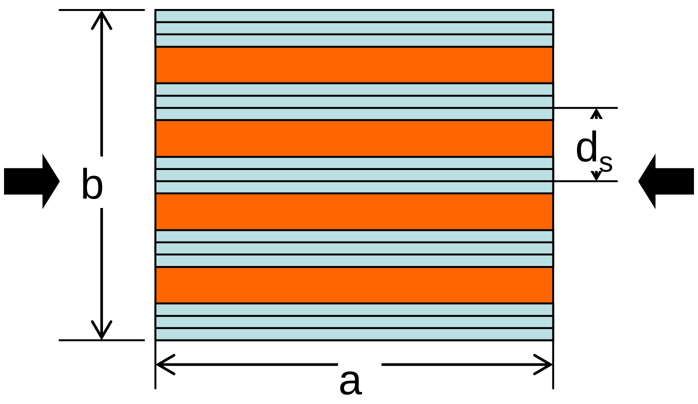
Panel buckling=Bay buckling

- bay (skin between stiffeners) buckles when

$$N_{xskin} = \frac{\pi^2}{a^2}\left[(D_{11})_{skin} k^2 + 2\left[(D_{12})_{skin} + 2(D_{66})_{skin}\right](\overline{AR})^2 + (D_{22})_{skin}\frac{(\overline{AR})^4}{k^2}\right] \qquad (5.4.2.3)$$

$$(\overline{AR}) = \frac{a}{d_s}$$

equation from before for buckling of ss plate under compression



Panel Buckling = Bay Buckling

- panel as a whole buckles when

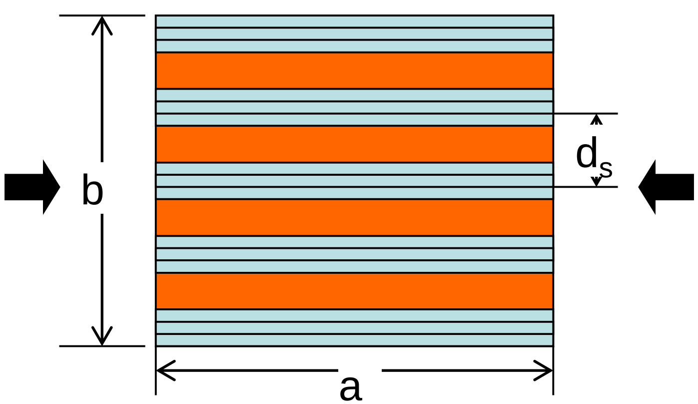
$$N_x = \frac{\pi^2}{a^2}\left\{\left[(D_{11})_{skin} + \frac{(EI)_{stif}}{d_s}\right]m^2 + 2\left[(D_{12})_{skin} + 2(D_{66})_{skin}\right](AR)^2 + (D_{22})_{skin}\frac{(AR)^4}{m^2}\right\} \qquad (5.4.2.4)$$

$$(AR) = \frac{a}{b}$$

stiffener contribution

$$N_x = \frac{F_{TOT}}{b} \qquad (5.4.2.5)$$

it is assumed the stiffener has an open cross-section and, therefore, GJ≈0



Panel Buckling = Bay Buckling

- combining (5.4.2.2) with (5.4.2.3) and dropping the subscript “skin” from D_{ij}:

$$F_{TOT} = b\frac{\left(A_{11} + \frac{EA}{d_s}\right)}{A_{11}}\frac{\pi^2}{a^2}\left[(D_{11})k^2 + 2[(D_{12}) + 2(D_{66})](\overline{AR})^2 + (D_{22})\frac{(\overline{AR})^4}{k^2}\right] \quad (5.4.2.6)$$

- combining (5.4.2.4) with (5.4.2.5) and dropping the subscript “skin” from Dij :

$$F_{TOT} = b\frac{\pi^2}{a^2}\left\{\left[(D_{11}) + \frac{(EI)_{stif}}{d_s}\right]m^2 + 2[(D_{12}) + 2(D_{66})](AR)^2 + (D_{22})\frac{(AR)^4}{m^2}\right\} \quad (5.4.2.7)$$

Panel Buckling = Bay Buckling

- combining (5.4.2.6) with (5.4.2.7) and rearranging a bit

$$\frac{\left(A_{11}+\frac{EA}{d_s}\right)}{A_{11}}\left[D_{11}k^2+2\left[D_{12}+2D_{66}\right](\overline{AR})^2+D_{22}\frac{(\overline{AR})^4}{k^2}\right]=\left[D_{11}+\frac{(EI)_{stif}}{d_s}\right]m^2+2\left[D_{12}+2D_{66}\right](AR)^2+D_{22}\frac{(AR)^4}{m^2}$$

(5.4.2.8)

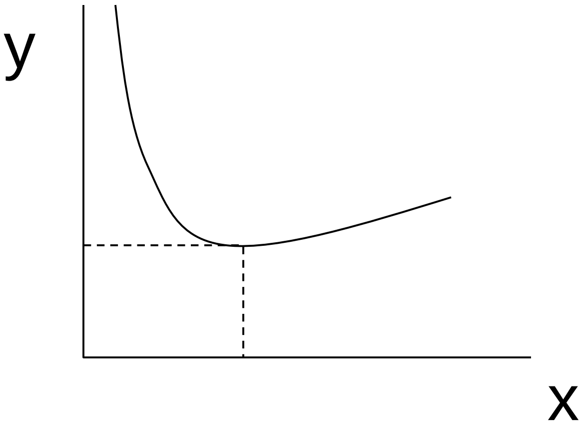
- to proceed, we need to decide on the values of k and m

- recall that k and m are integers that minimize the respective buckling loads in eqs (5.4.2.3) and (5.4.2.4)

Panel Buckling = Bay Buckling

- consider the continuous function

$$y = D_{11}x^2 + D_{22}\frac{(\overline{AR})^4}{x^2}$$



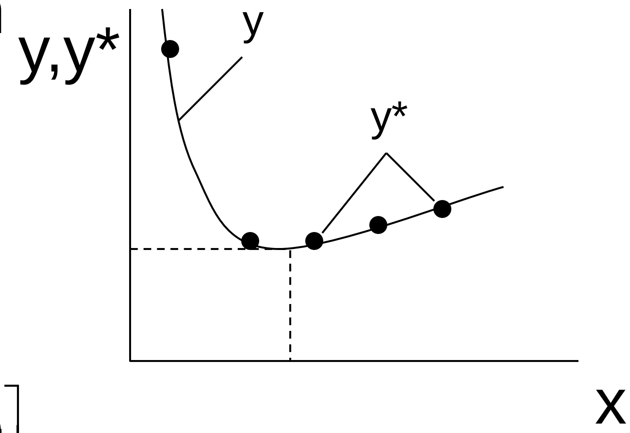
- this function has one and only one minimum determined by

$$\frac{dy}{dx} = 0 \Rightarrow x_{\min} = \left(\frac{D_{22}}{D_{11}}\right)^{1/4}\left(\overline{AR}\right)$$

Panel Buckling = Bay Buckling

- therefore, the discontinuous function

$$y^* = D_{11}k^2 + D_{22}\frac{(\overline{AR})^4}{k^2}$$



- is minimized when

$$k = \text{int}\left[\left(\frac{D_{22}}{D_{11}}\right)^{1/4}\left(\overline{AR}\right)\right] \quad \text{or} \quad k = \text{int}\left[\left(\frac{D_{22}}{D_{11}}\right)^{1/4}\left(\overline{AR}\right)\right]+1$$

whichever makes y* lowest, with one important note:

if $\text{int}\left[\left(\frac{D_{22}}{D_{11}}\right)^{1/4}\left(\overline{AR}\right)\right] = 0$ then k (or m) = 1

Panel Buckling = Bay Buckling

- for the case of m,

$$x_{\min} = \left(\frac{D_{22}}{D_{11} + \frac{EI}{d_s}} \right)^{1/4} \left(\frac{a}{b} \right)$$

- for typical applications, x_{min}<1 because D_{11}>D_{22} and b>a

- so setting m=1 covers most cases of interest

Panel Buckling = Bay Buckling

- now approximate k with its corresponding x_{min}

$$k \approx \left(\frac{D_{22}}{D_{11}}\right)^{1/4}\left(\overline{AR}\right)$$

- and substitute for m and k in eq. (5.4.2.8)

$$D_{11} + \frac{(EI)_{stif}}{d_s} + 2\left[D_{12} + 2D_{66}\right](AR)^2 + D_{22}(AR)^4 = \frac{\left(A_{11} + \frac{EA}{d_s}\right)}{A_{11}}\left[D_{11}\sqrt{\frac{D_{22}}{D_{11}}}\,\overline{AR}^{\,2} + 2\left[D_{12} + 2D_{66}\right](\overline{AR})^2 + D_{22}\frac{(\overline{AR})^4}{\sqrt{\frac{D_{22}}{D_{11}}}\,\overline{AR}^{\,2}}\right]$$

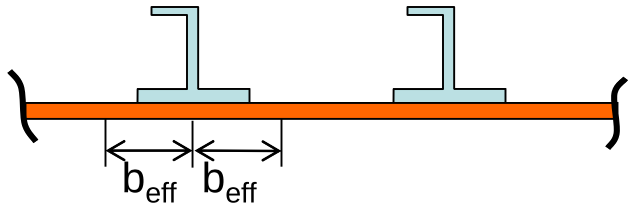
$=\lambda$, $\lambda>1$

- solving for $(EI)_{stiff}$ and dropping the subscript "stiff"

$$EI = D_{11}d_s\left[\sqrt{\frac{D_{22}}{D_{11}}}\left(2\lambda\,\overline{AR}^{\,2} - \sqrt{\frac{D_{22}}{D_{11}}}(AR)^4\right) + \frac{2(D_{12} + 2D_{66})}{D_{11}}\left(\lambda\,\overline{AR}^{\,2} - (AR)^2\right) - 1\right] \quad (5.4.2.9)$$

1st equation

Stiffener buckling = PB x skin buckling



- in post-buckling regime, skin is replaced by the effective skin portion; strain compatibility then gives

$$F_{skin} = \frac{A_{11}\frac{b}{d_s}2b_{eff}}{2A_{11}\frac{b}{d_s}b_{eff} + EA\frac{b}{d_s}}F_{TOT} \Rightarrow F_{skin} = \frac{A_{11}2b_{eff}}{2A_{11}b_{eff} + EA}F_{TOT}$$

$$F_{stiffeners} = \frac{EA}{2A_{11}b_{eff} + EA}F_{TOT} \qquad (5.4.2.10)$$

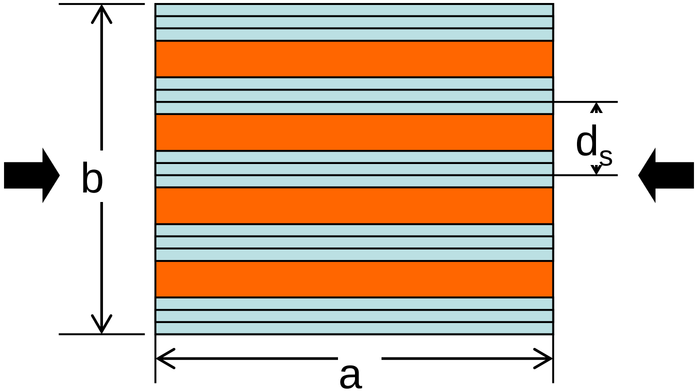
- and for a single stiffener, dividing by $n_s = b/d_s$

$$F_{stif} = \frac{d_s}{b}\frac{EA}{2A_{11}b_{eff} + EA}F_{TOT} \qquad (5.4.2.11)$$

Stiffener buckling = PB x skin buckling

- single stiffener buckles when

$$F_{stif} = \frac{\pi^2 EI}{a^2} \qquad (5.4.2.12)$$



- combining (5.4.2.11) and (5.4.2.12):

$$\frac{d_s}{b}\frac{EA}{2A_{11}b_{eff} + EA}F_{TOT} = \frac{\pi^2 EI}{a^2} \Rightarrow F_{TOT} = \frac{\pi^2 EI}{a^2}\frac{b}{d_s}\frac{2A_{11}b_{eff} + EA}{EA} \qquad (5.4.2.13)$$

Stiffener buckling = PB x skin buckling

- final failure occurs when the post-buckling factor (PB) is reached; the force in the skin at that load is given by

$$F_{skin} = F_{skin\,buckling}(PB) \qquad (5.4.2.14)$$

- with the skin buckling load given by

$$F_{skin\,buckling} = b\frac{\pi^2}{a^2}\left[(D_{11})k^2 + 2[(D_{12}) + 2(D_{66})](\overline{AR})^2 + (D_{22})\frac{(\overline{AR})^4}{k^2}\right] \qquad (5.4.2.15)$$

- combining (5.4.2.10), (5.4.2.14), and (5.4.2.15)

$$F_{TOT} = \frac{2A_{11}b_{eff} + EA}{2A_{11}b_{eff}}b\frac{\pi^2}{a^2}\left[(D_{11})k^2 + 2[(D_{12}) + 2(D_{66})](\overline{AR})^2 + (D_{22})\frac{(\overline{AR})^4}{k^2}\right](PB) \qquad (5.4.2.16)$$

Stiffener buckling = PB x skin buckling

- Equations (5.4.2.13) and (5.4.2.16) are combined to give

$$\frac{EI}{d_s EA} = \frac{(PB)}{2A_{11}b_{eff}}\left[D_{11}k^2 + 2\left[D_{12} + 2D_{66}\right](\overline{AR})^2 + D_{22}\frac{(\overline{AR})^4}{k^2}\right]$$

- use the definition of λ to simplify things

$$\lambda = \frac{A_{11} + \dfrac{EA}{d_s}}{A_{11}} \Rightarrow \lambda A_{11} - A_{11} = \frac{EA}{d_s} \Rightarrow A_{11}(\lambda - 1) = \frac{EA}{d_s} \Rightarrow \frac{EA}{A_{11}} = (\lambda - 1)d_s$$

- substitute in the above equation to obtain

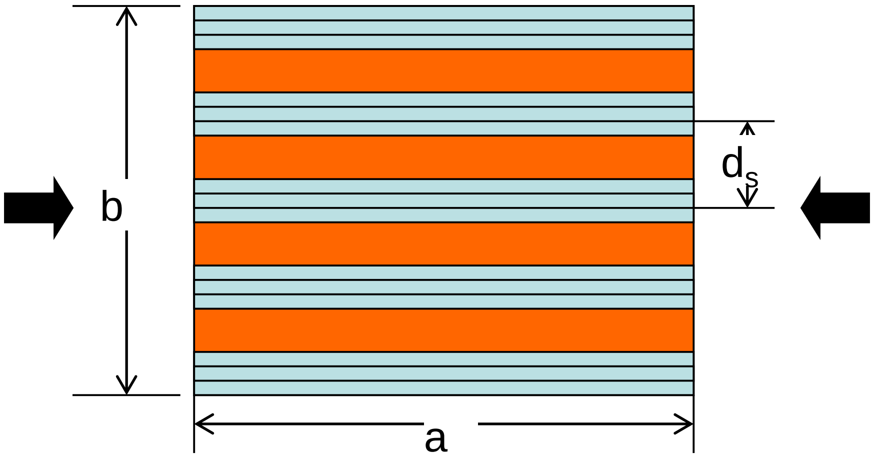
$$EI = (\lambda - 1)(PB)d_s\frac{d_s}{2b_{eff}}\left[D_{11}k^2 + 2\left[D_{12} + 2D_{66}\right](\overline{AR})^2 + D_{22}\frac{(\overline{AR})^4}{k^2}\right] \qquad (5.4.2.17)$$

Stiffener buckling = PB x skin buckling

- from the expression for b_{eff} derived earlier,

$$\frac{d_s}{b_{eff}} = 2\left[1 + 2\left(1 + \frac{A_{12}}{A_{11}}\right)\left(1 - \frac{1}{(PB)}\right)\frac{A_{11}}{A_{11} + 3A_{22}}\right]$$

(note that instead of a in the original b_{eff} expression we have d_s because this is the width of the plate that buckles)



Results: Min EI required for stiffeners

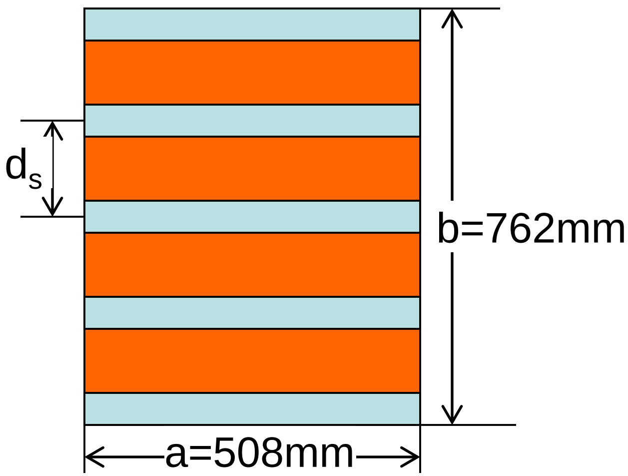
- eqns (5.4.2.9) and (5.4.2.17) are used to determine the minimum required bending stiffness for stiffeners
- EI is evaluated using both equations and the highest value is used
- Note that this does not guarantee that there is no crippling or inter-rivet buckling failure of the stiffener; separate checks must be made for these and they (usually) supersede the EI requirement (i.e. meeting the crippling and/or inter-rivet buckling requirements usually leads to higher EI than eqs (5.4.2.9) and (5.4.2.17))

Example of min stiffener EI required

- [(±45)/(0/90)/(±45)] skin

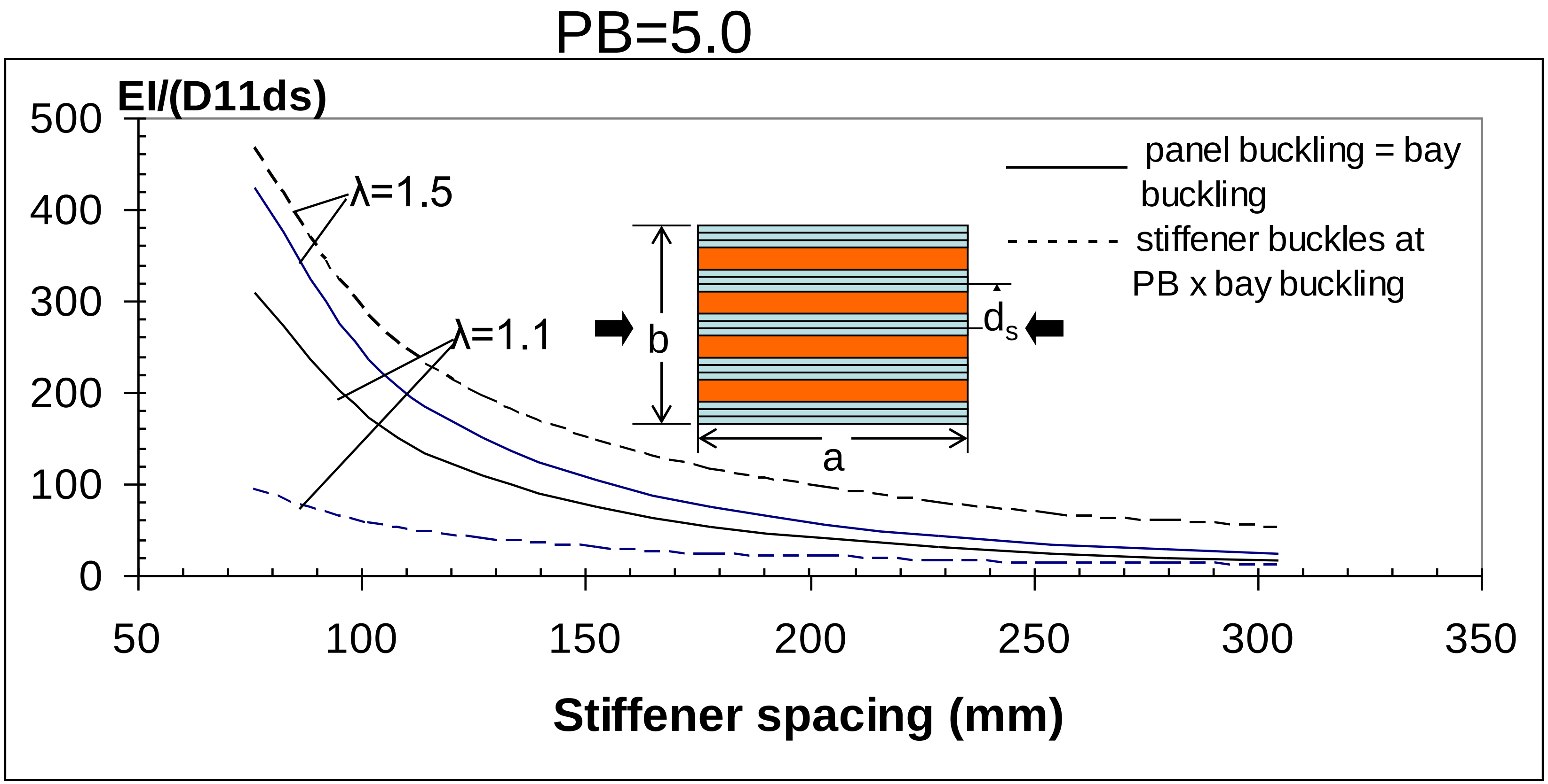
D_{11}	659.7	Nmm
D_{12}	466.9	Nmm
D_{22}	659.7	Nmm
D_{66}	494.0	Nmm

A_{11}	28912.44	N/mm
A_{12}	12491.43	N/mm
A_{22}	28912.44	N/mm
A_{66}	13468.58	N/mm



Stiffener geometry and spacing unknown. Determine minimum EI for the stiffeners

Results: Min EI required for stiffeners



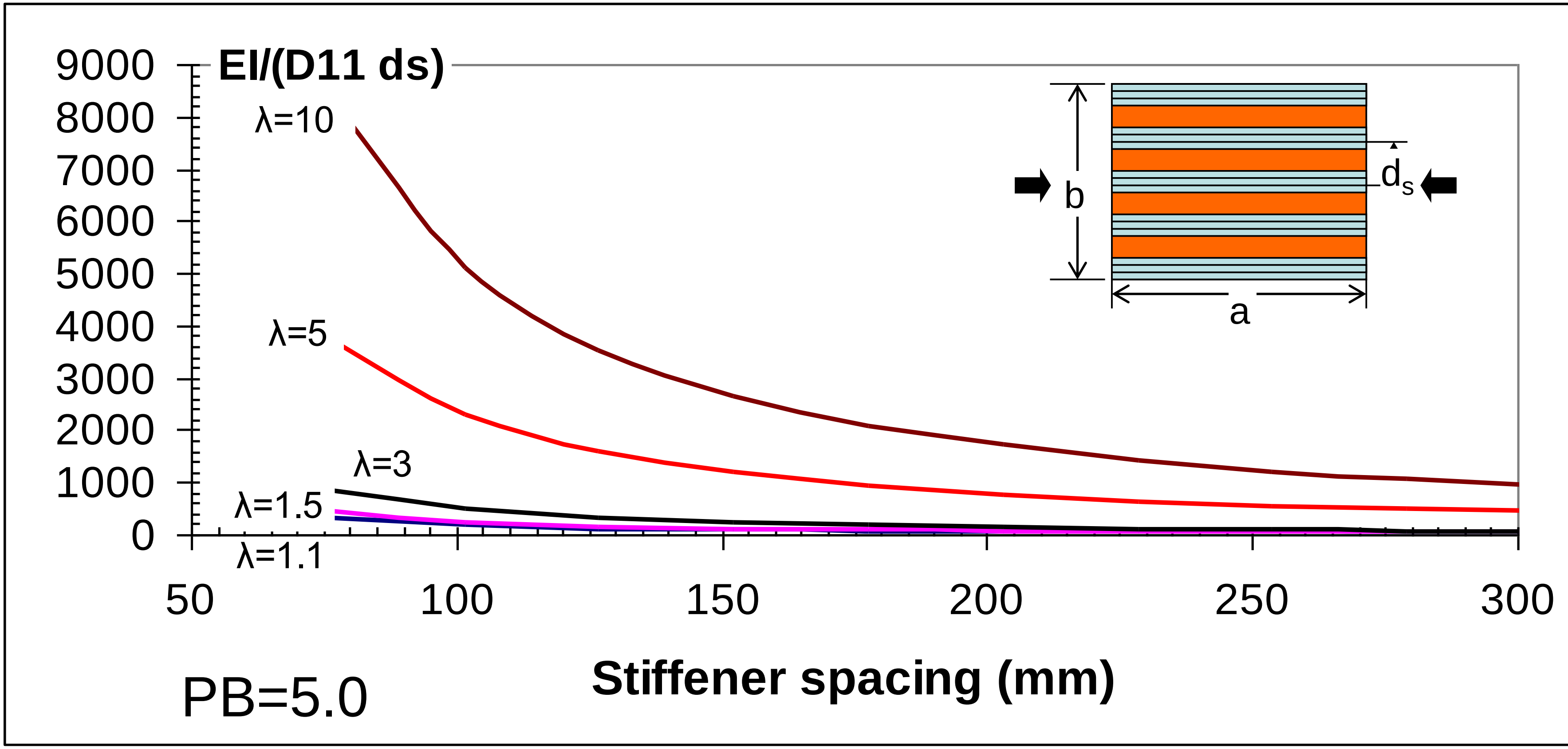
- between λ=1.1 and λ=1.5, stiffener buckling condition becomes more critical (design driver changes)

why?

- note that as d_s increases, the min req'd stiffness decreases

Results: Min EI required for stiffeners

(using the more critical of the two conditions)



Min stiffener EI: some notes

- comparing the min EI for PB=5 and PB=1.5 when λ=1.1 we see that they are identical; the reason is that the "active" constraint is that bay buckling=panel buckling which is independent of post-buckling factor PB
- when the axial stiffness EA of the stiffeners is low (λ=1.1-1.2) the min required bending stiffness for the stiffeners is independent of the ratio of the final failure load to the skin buckling load (PB)
- for higher axial stiffnesses (λ>1.2) the higher the PB the higher the bending stiffness EI required for the stiffener