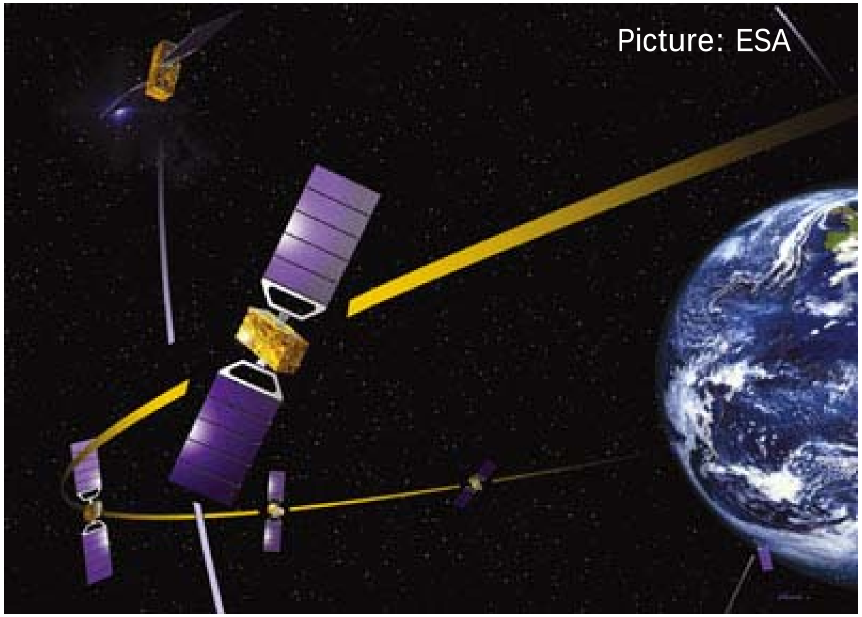
Satellite Navigation
Relative positioning



AE4E08

Sandra Verhagen

Course 2010 – 2011, lecture 10

TU Delft

Today's topics

- DGPS
- Relative positioning: observation differences
- Carrier phase integer ambiguities

- Study Sections 5.8, 7.1 – 7.3

Recap: Code and Carrier Phase measurements

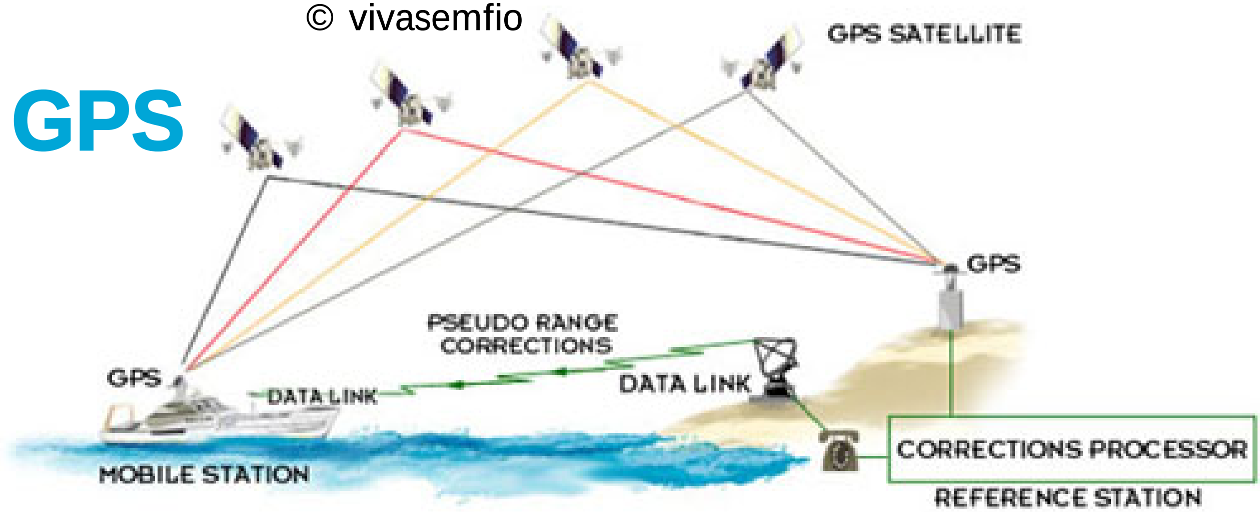
$$\rho_{Li} = r + \frac{f_1^2}{f_i^2} I_{L1} + T + c\left[\delta t_u - \delta t^{(k)}\right] + \varepsilon_{\rho_{Li}}$$

$$\Phi_{Li} = r - \frac{f_1^2}{f_i^2} I_{L1} + T + c\left[\delta t_u - \delta t^{(k)}\right] + \lambda_{Li} A_{Li} + \varepsilon_{\Phi_{Li}}$$

Lots of (big) errors…

use the fact that some errors are similar for two stations in each others vicinity

Differential GPS



- Reference station transmits corrections (radio, GSM, Internet) to users (rovers)
- DGPS corrections (range domain)
  - Correction and correction rate per satellite
  - IODE to ensure rover uses same satellite ephemeris data
- Standard format for corrections is RTCM SC-104
- Only corrections for L1
- Includes satellite ephemeris and ionosphere errors
- Accuracy decreases with the distance to the DGPS station

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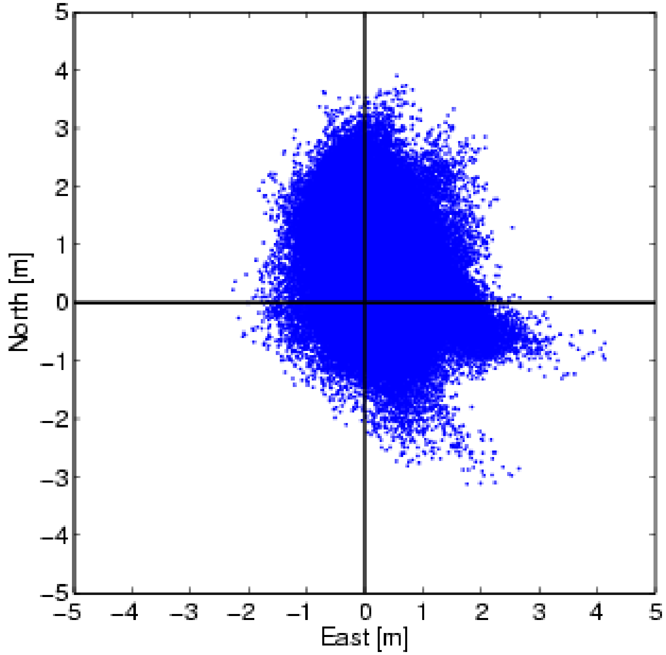
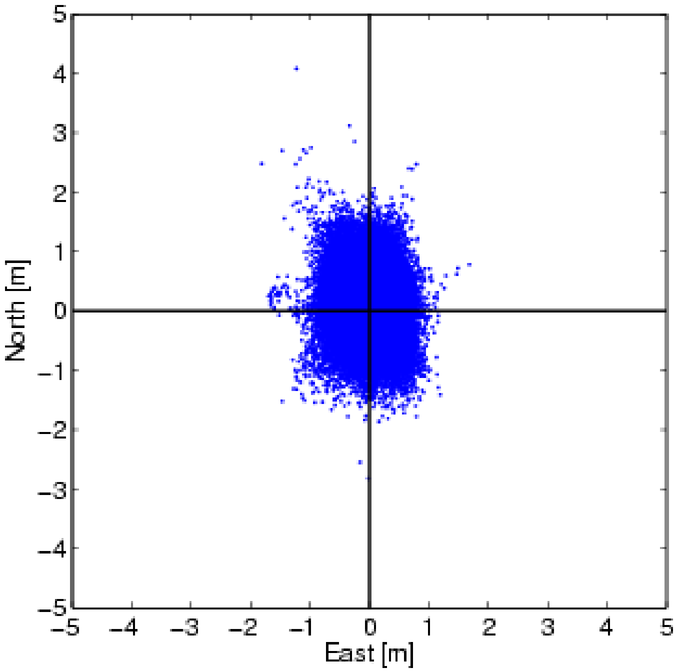
Differential GPS

GPS



DGPS



Impact much bigger in case S/A enabled

Recap: Code and Carrier Phase measurements

$$\rho_{Li} = r + \frac{f_1^2}{f_i^2} I_{L1} + T + c\left[\delta t_u - \delta t^{(k)}\right] + \varepsilon_{\rho_{Li}}$$

$$\Phi_{Li} = r - \frac{f_1^2}{f_i^2} I_{L1} + T + c\left[\delta t_u - \delta t^{(k)}\right] + \lambda_{Li} A_{Li} + \varepsilon_{\Phi_{Li}}$$

Recap: Code and Carrier Phase measurements

$$\rho_{Li} = r + \mu_i I + T + c\left[\delta t_u - \delta t^{(k)}\right] + \varepsilon_{\rho_{Li}}$$

$$\Phi_{Li} = r - \mu_i I + T + c\left[\delta t_u - \delta t^{(k)}\right] + \lambda_{Li} A_{Li} + \varepsilon_{\Phi_{Li}}$$

Note: parameters depend on *t* except for ambiguities

Carrier phase observation and ambiguity

Carrier phase observation from receiver u to satellite k:

$$\Phi_u^{(k)} = r_u^{(k)} - \mu I_u^{(k)} + T_u^{(k)} + c\left[\delta t_u - \delta t^{(k)}\right] + \lambda A_u^{(k)} + \varepsilon_\Phi$$

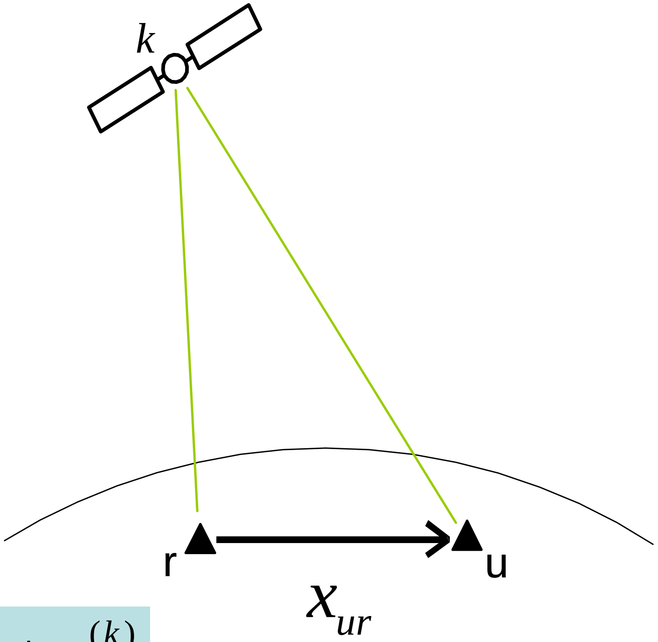
for notational convenience: frequency index omitted

Carrier phase ambiguity:

lecture 4

$$A_u^{(k)} = \underbrace{\phi_u(t_0) - \phi^{(k)}(t_0)}_{\text{initial phases}} - f \cdot \underbrace{\left(\delta t_u(t_0) - \delta t^{(k)}(t_0)\right)}_{\text{clock biases at } t_0} + \underbrace{N_u^{(k)}}_{\text{integer ambiguity}}$$

Single differencing (SD)



- Subtract observations at two receivers to same satellite

$$\Phi_u^{(k)} = r_u^{(k)} - \mu I_u^{(k)} + T_u^{(k)} + c\delta t_u - c\delta t^{(k)} + \lambda A_u^{(k)} + \varepsilon_{\Phi,u}^{(k)}$$

$$\Phi_r^{(k)} = r_r^{(k)} - \mu I_r^{(k)} + T_r^{(k)} + c\delta t_r - c\delta t^{(k)} + \lambda A_r^{(k)} + \varepsilon_{\Phi,r}^{(k)}$$

$$(\bullet)_{ur} = (\bullet)_u - (\bullet)_r$$

$$\begin{aligned} \Phi_{ur}^{(k)} &= \Phi_u^{(k)} - \Phi_r^{(k)} \\ &= r_{ur}^{(k)} - \mu I_{ur}^{(k)} + T_{ur}^{(k)} + c\delta t_{ur} + \lambda A_{ur}^{(k)} + \varepsilon_{\Phi,ur}^{(k)} \end{aligned}$$

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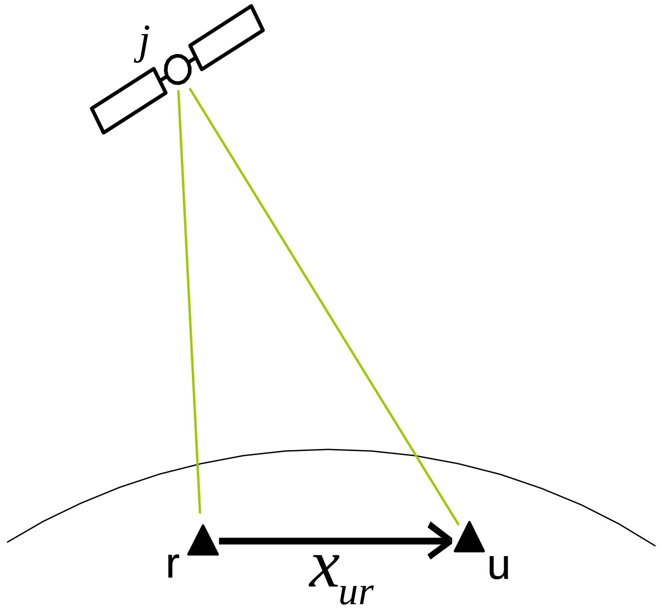
Single differencing (SD)

- Subtract observations at two receivers to same satellite



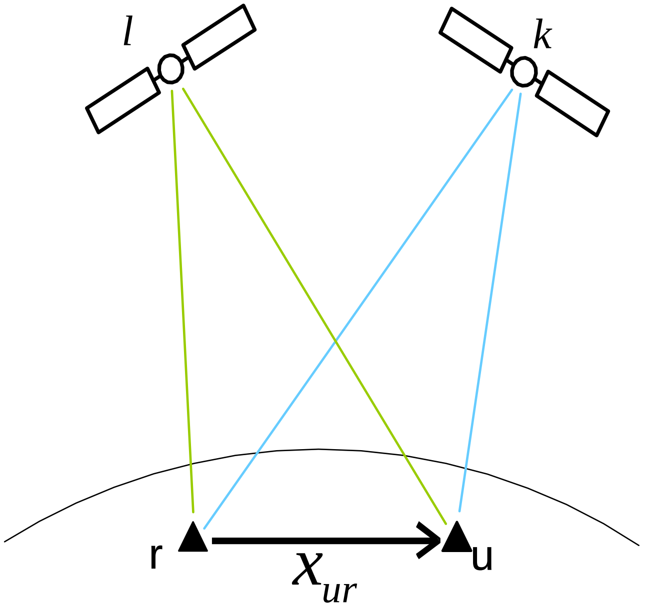
$$\Phi_{ur}^{(k)} = r_{ur}^{(k)} - \mu I_{ur}^{(k)} + T_{ur}^{(k)} + c\delta t_{ur} + \lambda A_{ur}^{(k)} + \varepsilon_{\Phi,ur}^{(k)}$$

$$A_u^{(k)} = \phi_u(t_0) - \cancel{\phi^{(k)}(t_0)} - f\,\delta t_u(t_0) - \cancel{f\,\delta t^{(k)}(t_0)} + N_u^{(k)}$$

$$A_r^{(k)} = \phi_r(t_0) - \cancel{\phi^{(k)}(t_0)} - f\,\delta t_r(t_0) - \cancel{f\,\delta t^{(k)}(t_0)} + N_r^{(k)}$$

$$A_{ur}^{(k)} = \phi_{ur}(t_0) - f\,\delta t_{ur}(t_0) + N_{ur}^{(k)}$$

Double differencing (DD)



- Subtract SD observations from two satellites

$$\Phi_{ur}^{(k)} = r_{ur}^{(k)} - \mu I_{ur}^{(k)} + T_{ur}^{(k)} + c\delta t_{ur} + \lambda A_{ur}^{(k)} + \varepsilon_{\Phi,ur}^{(k)}$$

$$\Phi_{ur}^{(l)} = r_{ur}^{(l)} - \mu I_{ur}^{(l)} + T_{ur}^{(l)} + c\delta t_{ur} + \lambda A_{ur}^{(l)} + \varepsilon_{\Phi,ur}^{(l)}$$

$$\begin{aligned}\Phi_{ur}^{(kl)} &= \Phi_{ur}^{(k)} - \Phi_{ur}^{(l)} \\ &= r_{ur}^{(kl)} - \mu I_{ur}^{(kl)} + T_{ur}^{(kl)} + \lambda A_{ur}^{(kl)} + \varepsilon_{\Phi,ur}^{(kl)}\end{aligned}$$

Both receivers must track same satellites at same time (i.e. must be synchronized)

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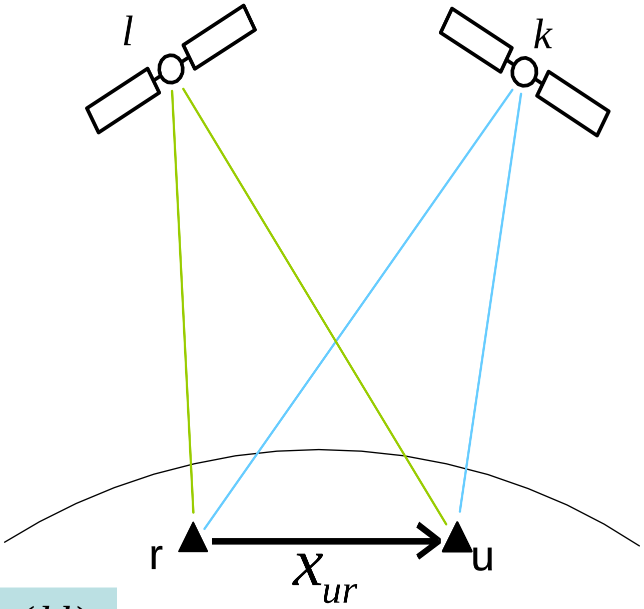
Double differencing (DD)



- Subtract SD observations from two satellites

$$\Phi_{ur}^{(kl)} = r_{ur}^{(kl)} - \mu I_{ur}^{(kl)} + T_{ur}^{(kl)} + \lambda N_{ur}^{(kl)} + \varepsilon_{\Phi,ur}^{(kl)}$$

$$A_{ur}^{(k)} = \phi_{ur}(t_0) - f\,\delta t_{ur}(t_0) + N_{ur}^{(k)}$$

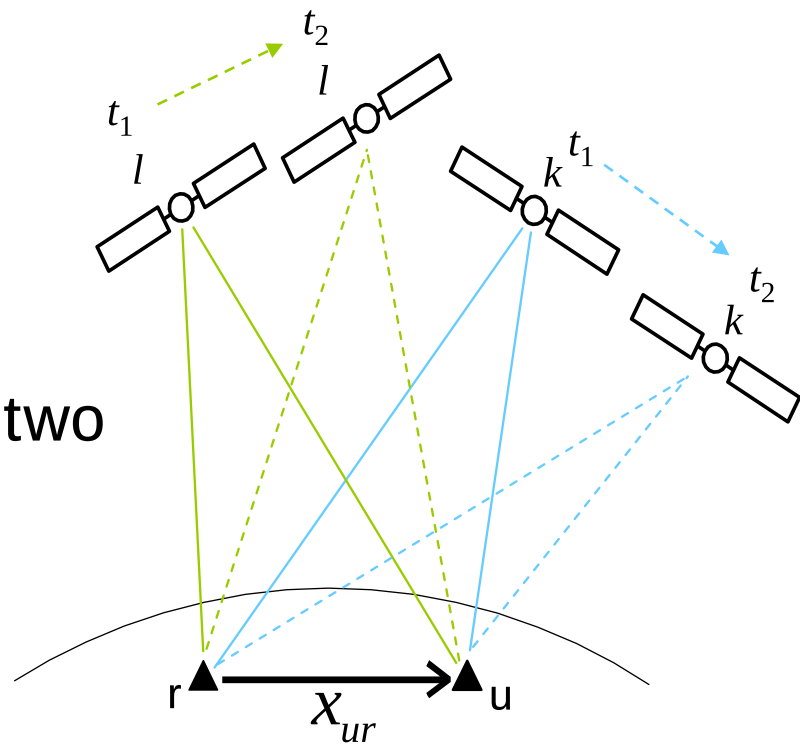
$$A_{ur}^{(l)} = \phi_{ur}(t_0) - f\,\delta t_{ur}(t_0) + N_{ur}^{(l)}$$

$\Rightarrow$ $A_{ur}^{(kl)} = N_{ur}^{(kl)}$ = integer !!

Triple differencing

- Subtract DD observations from two epochs

$$\delta\Phi_{ur}^{(kl)}(i) = \Phi_{ur}^{(kl)}(t_{i+1}) - \Phi_{ur}^{(kl)}(t_i)$$



only position change can be determined

SD variance matrix

$$\Phi_{ur}^{(k)} = r_{ur}^{(k)} - \mu I_{ur}^{(k)} + T_{ur}^{(k)} + c\delta t_{ur} + \lambda A_{ur}^{(k)} + \varepsilon_{\Phi,ur}^{(k)}$$

$$\begin{bmatrix} \Phi_{ur}^{(j)} \\ \Phi_{ur}^{(k)} \\ \Phi_{ur}^{(l)} \end{bmatrix} = \begin{bmatrix} \Phi_{u}^{(j)} - \Phi_{r}^{(j)} \\ \Phi_{u}^{(k)} - \Phi_{r}^{(k)} \\ \Phi_{u}^{(l)} - \Phi_{r}^{(l)} \end{bmatrix} = \underbrace{\begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix}}_{=M} \begin{bmatrix} \Phi_{r}^{(j)} \\ \Phi_{u}^{(j)} \\ \Phi_{r}^{(k)} \\ \Phi_{u}^{(k)} \\ \Phi_{r}^{(l)} \\ \Phi_{u}^{(l)} \end{bmatrix}$$

$$Q^{\mathrm{SD}} = M \cdot \sigma_{\Phi}^{2} I_6 \cdot M^T$$

DD variance matrix

$$\Phi_{ur}^{(kl)} = r_{ur}^{(kl)} - \mu I_{ur}^{(kl)} + T_{ur}^{(kl)} + \lambda N_{ur}^{(kl)} + \varepsilon_{\Phi,ur}^{(kl)}$$

$$\begin{bmatrix} \Phi_{ur}^{(kj)} \\ \Phi_{ur}^{(lj)} \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Phi_{ur}^{(j)} \\ \Phi_{ur}^{(k)} \\ \Phi_{ur}^{(l)} \end{bmatrix}$$

$$Q^{\mathrm{DD}} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \cdot 2\sigma_{\Phi}^{2} I_3 \cdot \begin{bmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \sigma_{\Phi}^{2} \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

UD, SD, DD

UD: $$\Phi_u^{(k)} = r_u^{(k)} - \mu I_u^{(k)} + T_u^{(k)} + c\delta t_u - c\delta t^{(k)} + \lambda A_u^{(k)} + \varepsilon_{\Phi,u}^{(k)}$$

Recall, linearization: $$\delta r_u^{(k)} = \left(-\mathbf{1}_u^{(k)}\right)^T \delta \mathbf{x}_u$$

Then “observed-minus-computed” observations!

but...

Linearization (lecture 6)

$$\mathbf{y} = \rho^{(k)} = \left\| \mathbf{x}^{(k)} - \mathbf{x} \right\| + b + \tilde{\varepsilon}_{\rho}^{(k)}$$

$$\mathbf{y} = \mathbf{H}(\mathbf{v}_0) + \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{v}} (\mathbf{v} - \mathbf{v}_0) + \ldots + \boldsymbol{\varepsilon}$$

$$H(\mathbf{v}_0) = \rho_0^{(k)} = \left\| \mathbf{x}^{(k)} - \mathbf{x}_0 \right\| + b_0$$

satellite position assumed known

$$\frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{v}} = \begin{bmatrix} \dfrac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial x} \\ \dfrac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial y} \\ \dfrac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial z} \\ \dfrac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial b} \end{bmatrix}^T = \begin{bmatrix} -\dfrac{x^{(k)} - x_0}{\left\| \mathbf{x}^{(k)} - \mathbf{x}_0 \right\|} \\ -\dfrac{y^{(k)} - y_0}{\left\| \mathbf{x}^{(k)} - \mathbf{x}_0 \right\|} \\ -\dfrac{z^{(k)} - z_0}{\left\| \mathbf{x}^{(k)} - \mathbf{x}_0 \right\|} \\ 1 \end{bmatrix}^T = \left[(-\mathbf{1}^{(k)})^T \quad 1 \right]$$

Linearization

$$\mathbf{y} = \rho^{(k)} = \left\| \mathbf{x}^{(k)} - \mathbf{x} \right\| + b + \tilde{\varepsilon}_{\rho}^{(k)}$$

$$\mathbf{y} = \mathbf{H}(\mathbf{v}_0) + \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{v}} \left(\mathbf{v} - \mathbf{v}_0 \right) + \ldots + \boldsymbol{\varepsilon}$$

$$H(\mathbf{v}_0) = \rho_0^{(k)} = \left\| \mathbf{x}^{(k)}{}_0 - \mathbf{x}_0 \right\| + b_0$$

$$\frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{v}} = \begin{bmatrix} \dfrac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{x}} \\ \dfrac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{x}^{(k)}} \\ \dfrac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial b} \end{bmatrix}^T = \begin{bmatrix} (-\mathbf{1}^{(k)})^T & \left(\mathbf{1}^{(k)} \right)^T & 1 \end{bmatrix}$$

$$\mathbf{v} - \mathbf{v}_0 = \begin{bmatrix} \delta \mathbf{x} \\ \delta \mathbf{x}^{(k)} \\ \delta b \end{bmatrix}$$

orbit error

UD, SD, DD

UD: $$\Phi_u^{(k)} = r_u^{(k)} - \mu I_u^{(k)} + T_u^{(k)} + c\delta t_u - c\delta t^{(k)} + \lambda A_u^{(k)} + \varepsilon_{\Phi,u}^{(k)}$$

$$\delta r_u^{(k)} = \left(-\mathbf{1}_u^{(k)}\right)^T \delta \mathbf{x}_u + \left(\mathbf{1}_u^{(k)}\right)^T \delta \mathbf{x}^{(k)}$$

SD: $$\Phi_{ur}^{(k)} = r_{ur}^{(k)} - \mu I_{ur}^{(k)} + T_{ur}^{(k)} + c\delta t_{ur} + \lambda A_{ur}^{(k)} + \varepsilon_{\Phi,ur}^{(k)}$$

$$\delta r_{ur}^{(k)} = \left(-\mathbf{1}_r^{(k)}\right)^T \delta \mathbf{x}_{ur}$$

DD: $$\Phi_{ur}^{(kl)} = r_{ur}^{(kl)} - \mu I_{ur}^{(kl)} + T_{ur}^{(kl)} + \lambda N_{ur}^{(kl)} + \varepsilon_{\Phi,ur}^{(kl)}$$

$$\delta r_{ur}^{(kl)} = \left(-\mathbf{1}_r^{(kl)}\right)^T \delta \mathbf{x}_{ur}$$

Short baselines:

iono and tropo terms can be ignored

Linearized DD observation equations (short baseline)

$$
\begin{bmatrix} \Phi_{ur}^{(kj)} \\ \Phi_{ur}^{(lj)} \\ \Phi_{ur}^{(mj)} \end{bmatrix} = \underbrace{\begin{bmatrix} \left(-\mathbf{1}_r^{kj}\right)^T & \lambda & 0 & 0 \\ \left(-\mathbf{1}_r^{lj}\right)^T & 0 & \lambda & 0 \\ \left(-\mathbf{1}_r^{mj}\right)^T & 0 & 0 & \lambda \end{bmatrix}}_{3\times 6} \underbrace{\begin{bmatrix} \mathbf{x}_{ur} \\ N_{ur}^{kj} \\ N_{ur}^{lj} \\ N_{ur}^{mj} \end{bmatrix}}_{6\times 1}
$$

→ Ambiguities are constant in time!

\# unknowns > # DD phase observations:
need more epochs or use code observations as well!

Linearized DD observation equations (short baseline)

$$\begin{bmatrix} \Phi_{ur}^{(21)} \\ \vdots \\ \Phi_{ur}^{(K1)} \end{bmatrix} = \begin{bmatrix} \left(-\mathbf{1}_r^{21}\right)^T & \lambda & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ \left(-\mathbf{1}_r^{K1}\right)^T & 0 & 0 & \lambda \end{bmatrix} \begin{bmatrix} \mathbf{x}_{ur} \\ N_{ur}^{21} \\ \vdots \\ N_{ur}^{K1} \end{bmatrix} = \begin{bmatrix} \mathbf{U} & \mathbf{\Lambda} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{ur} \\ N_{ur}^{21} \\ \vdots \\ N_{ur}^{K1} \end{bmatrix}$$

Linearized DD observation equations (short baseline)

$$\begin{bmatrix} P_{ur}^{(21)} \\ \vdots \\ P_{ur}^{(K1)} \\ \Phi_{ur}^{(21)} \\ \vdots \\ \Phi_{ur}^{(K1)} \end{bmatrix} = \begin{bmatrix} \mathbf{U} & \mathbf{O} \\ \mathbf{U} & \mathbf{\Lambda} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{ur} \\ N_{ur}^{21} \\ \vdots \\ N_{ur}^{K1} \end{bmatrix}$$

Try yourself:

- model with observations from 2 or more frequencies
- model for SD observations
- redundancy?

Integer ambiguity resolution

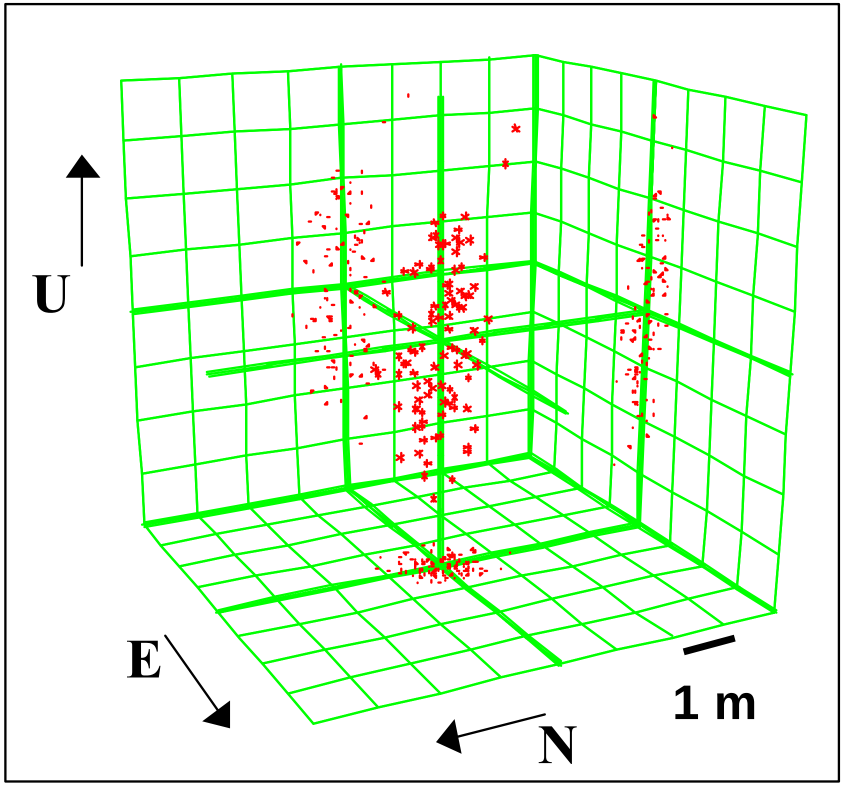
- code observation: **dm** precision

- phase observation: **mm** precision,

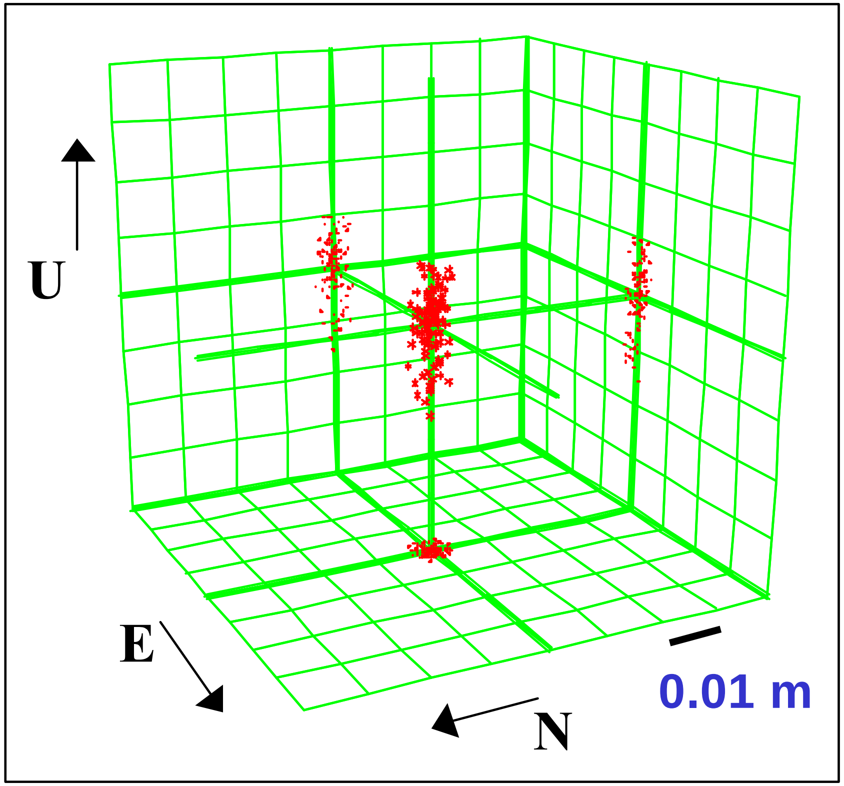
  if DD ambiguities are resolved to **integers within a short time (or instantaneously)**, positions (and other parameters) can be solved with mm-cm accuracy

Precision code vs. phase observations

code observations



phase observations



- both RELATIVE positioning
- phase: provided that the integer ambiguity is KNOWN

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Integer ambiguity resolution

‘Float’ solution

Estimate position and carrier ambiguities

Least squares

LAMBDA method

Estimate integer ambiguities

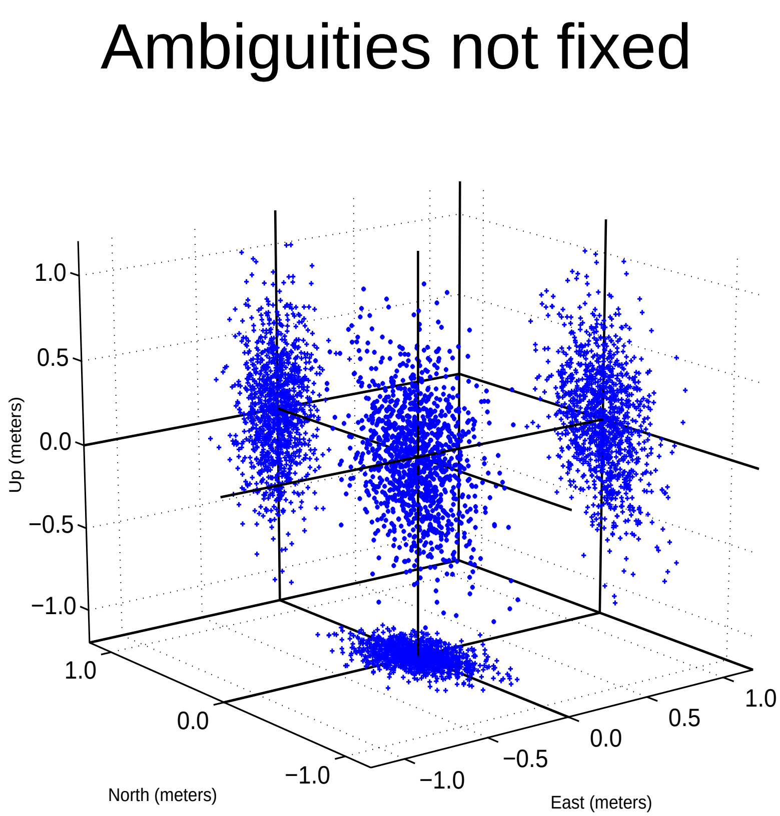
Integer least squares

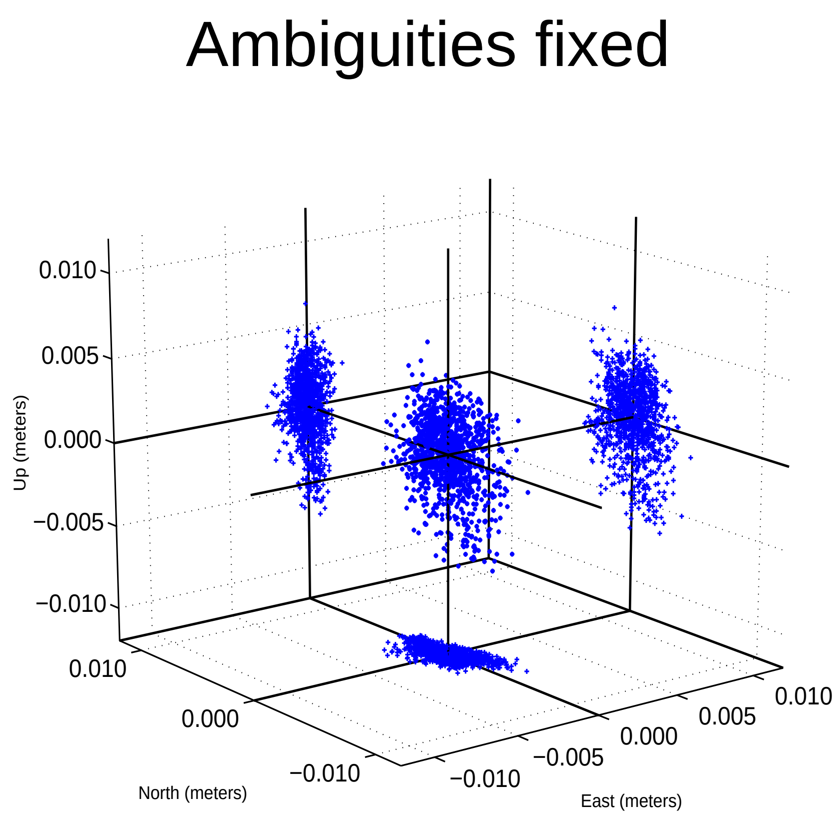
‘Fixed’ solution

Estimate position (ambiguities fixed)

Least squares

Float and fixed solution





Integer ambiguity resolution

Successful ambiguity resolution depends on:

- baseline length (tropo + iono delays)
- satellite geometry
- precision of code and phase observations
- # frequencies

→ Change in satellite geometry helps (long duration)

Real-time Kinematic (RTK)

- Permanent GPS station broadcasting L1 and L2 data over radio/GSM/Internet using the RTCM SC-104 / NTRIP format
- User receiver computes short base-line solution using L1 and L2 carrier phase data (code data is secondary), no ionosphere free l.c. (ionosphere fixed or weighted), L1 and L2 as separate data types!
- Relies on finding the correct integer ambiguities. Two stage process:
  - Initialization: float solution
  - Ambiguity fixed solution: centimeter accuracy

  Initialization takes a few seconds to minutes, depending on the distance to the reference station and atmospheric conditions
- Only within 20-30 km from reference station fast initialization times

Real-time Kinematic (RTK)

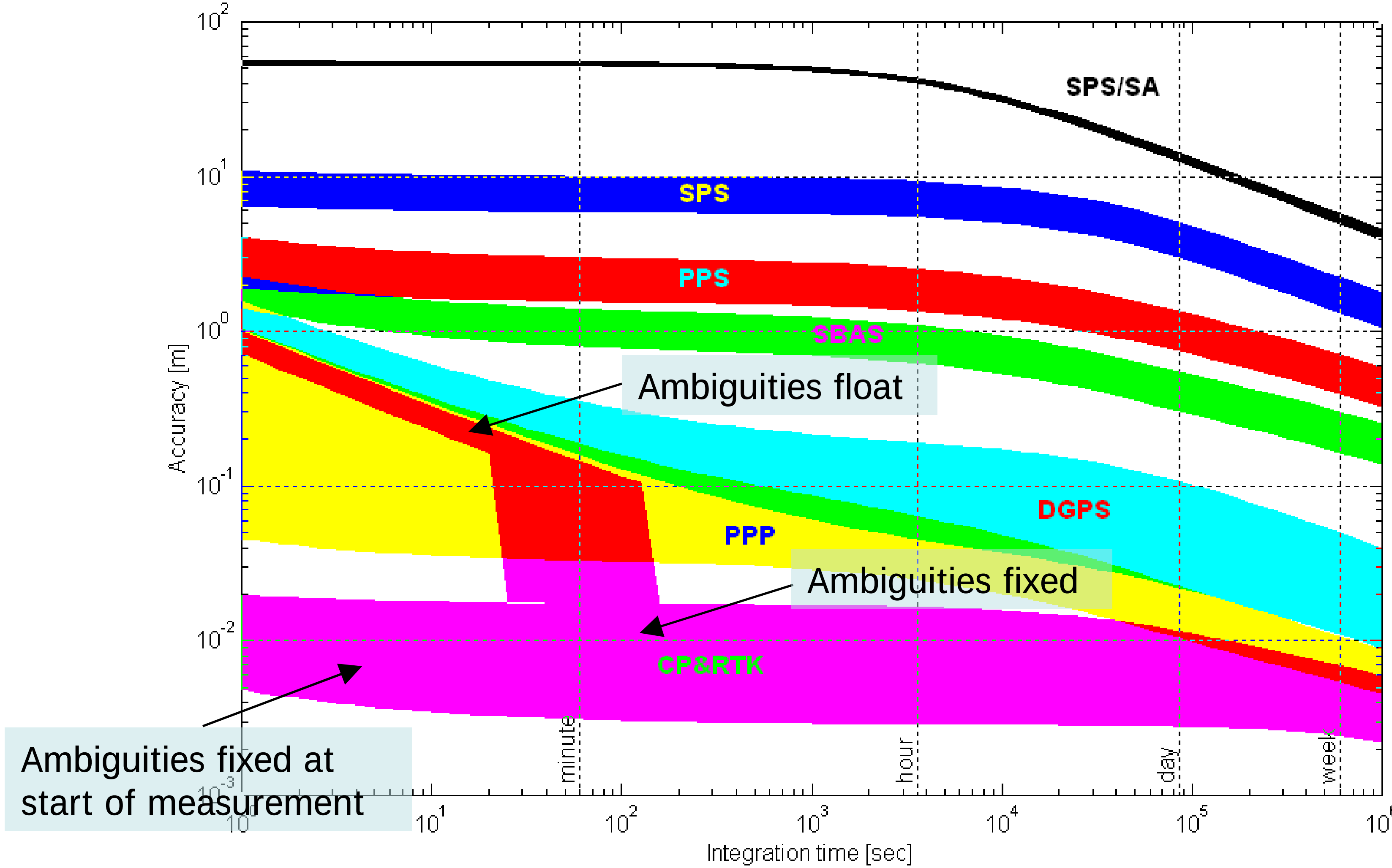
Examples:

- LNR GlobalCom network (NL), ‘network’ of individual RTK stations
- your ‘own’ temporary or permanent GPS-RTK basestation (e.g. on the roof of the town-hall, office, ...)

Overview GPS Services

SPS / SA	SPS	PPS	SBAS	DGPS	PPP	CP&RTK
Standard positioning service before May 2000	Standard positioning service after May 2000	Precise positioning service	Satellite Based Augmentation System	Differential-GPS	Precise Point Positioning	Carrier Phase processing and RTK
1-freq mass market receiver		2-freq receiver	1-freq SBAS enabled receiver	1-freq DGPS enabled receiver	2-freq receiver	2-freq geodetic receiver
Uses free to air signals from GPS only			GEO satellite or Internet (via GPRS)	Radio link or Internet (via GPRS)	Internet (via GPRS) or GEO satellite	Radio link; GSM; or Internet (via GPRS)
Pseudo-range (code) measurements only; optional carrier-phase smoothing					Code & Carrier phase	Carrier phase mainly
Global				< 500 km	Global	Local - Global

Accuracy (1σ) of GPS Services



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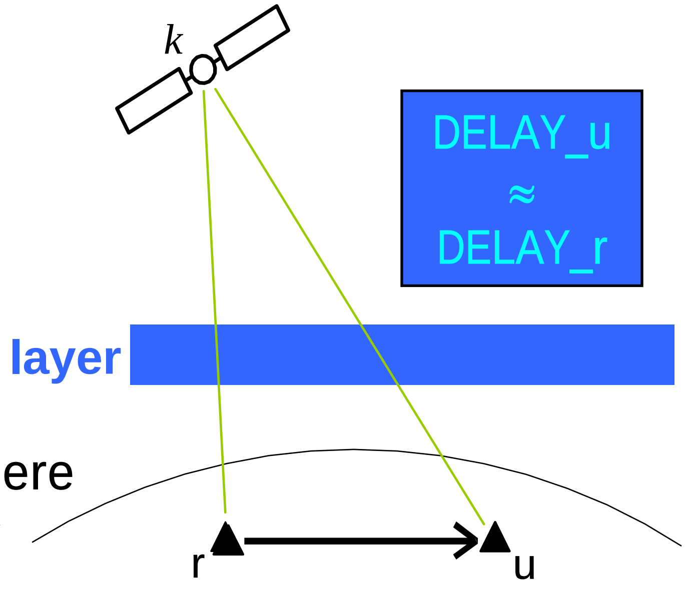
Short vs. long baselines



- **short baselines (few km):**
  signals travel through same part of atmosphere
  $\Rightarrow$ *differential atmospheric delays neglected*

$$\boxed{I_{ur}^{(k)}=0,\quad T_{ur}^{(k)}=0}$$

  $\Rightarrow$ beneficial for ambiguity resolution (less parameters)

- **long baselines (> few km):**
  differential ionospheric delays and Zenith Troposphere Delays (ZTD) need to be modeled (otherwise: wrong ambiguities and biased position)

Summary and outlook

- Signals, observations, errors, PVT estimation
- Today: DGPS and RTK (short baselines)

Next:

11. Network RTK (long baselines), PPP, SBAS

12. Quality control & ambiguity resolution

13,14. Applications