Time Variant Identification

Course: Wb2301

Lecture 11

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Contents

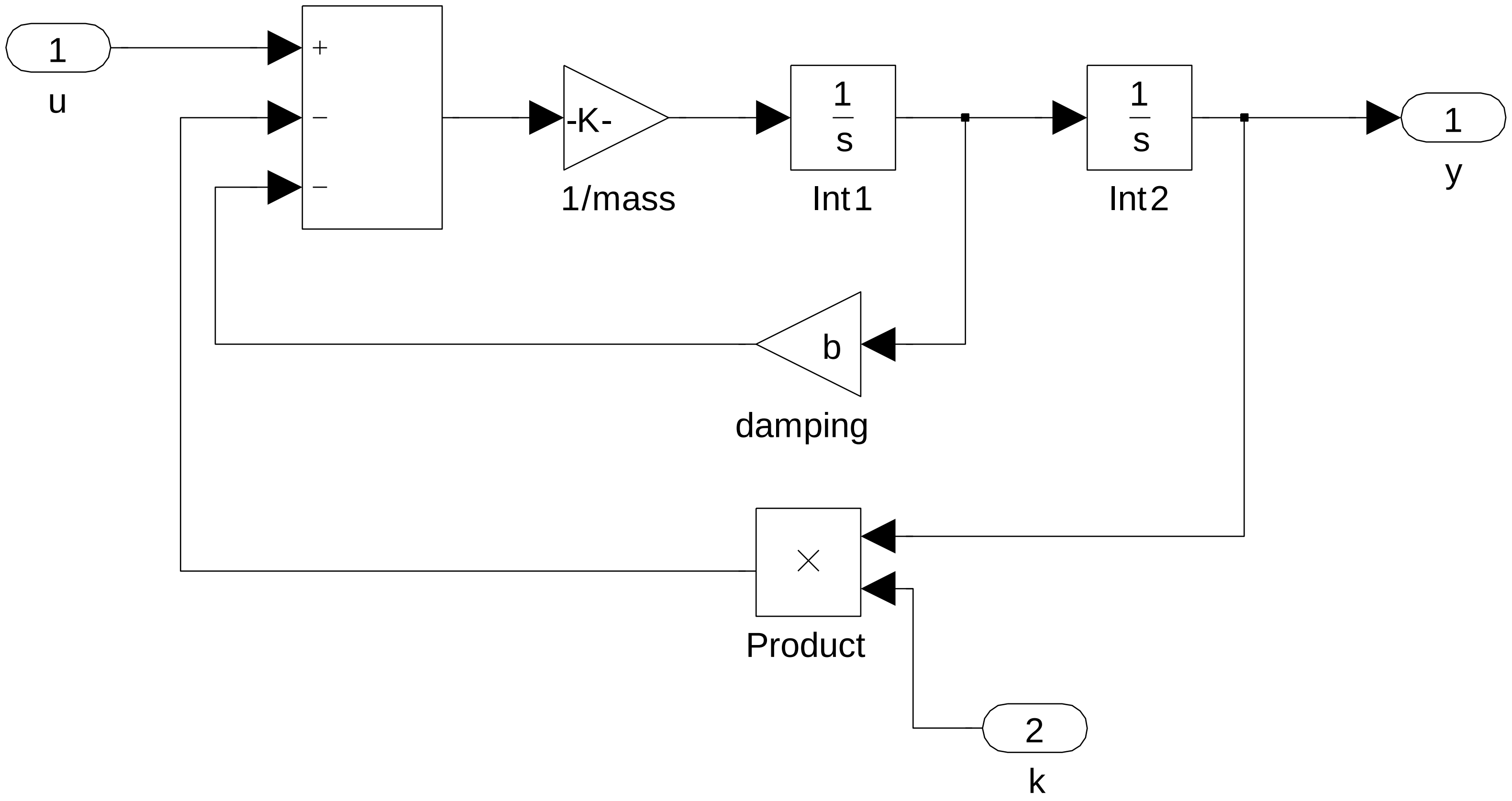
- Time variant (TV) systems
- TV identification using basis functions

TV Systems

- System properties change over time
- As a consequence, signal power at certain frequencies change over time
- Standard correlation or spectral estimators assume stationar systems, hence can not be applied

TV System Example

- Increasing stiffness of a 2^{nd}-order system

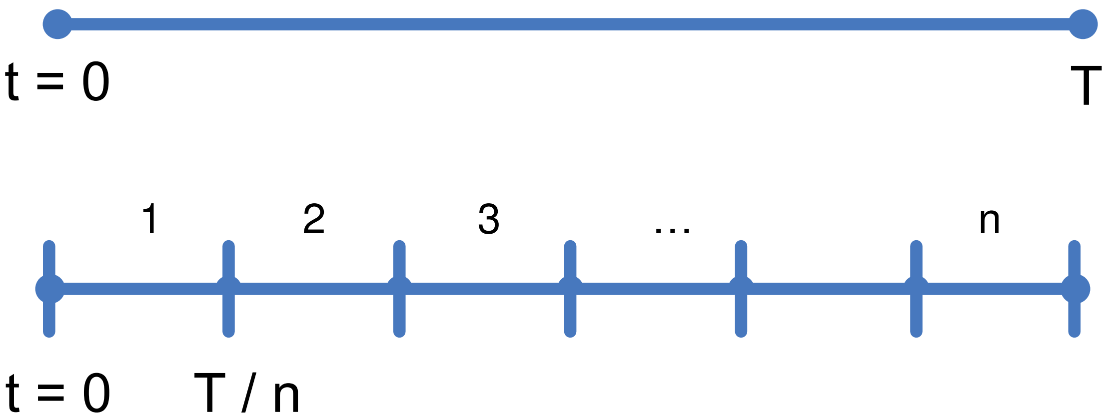


TV System Example

Matlab demo

Windowed Fourier Transform (WFT)

Divide T into adjacent segments (windows) and perform an fft over each segment



$\Delta f = 1 / T$

$\Delta f = n / T$

TU Delft

TV System ID using WFT

- Normal Fourier transform (spectral densities) applies to the whole observation time T (smears out TV behavior)
- WFT can identify TV behavior but still has poor temporal and spectral resolution
- Accurate time-frequency (t-f) estimators are required
- Signal decomposition using a-periodic basisfunctions provide for accurate t-f estimates

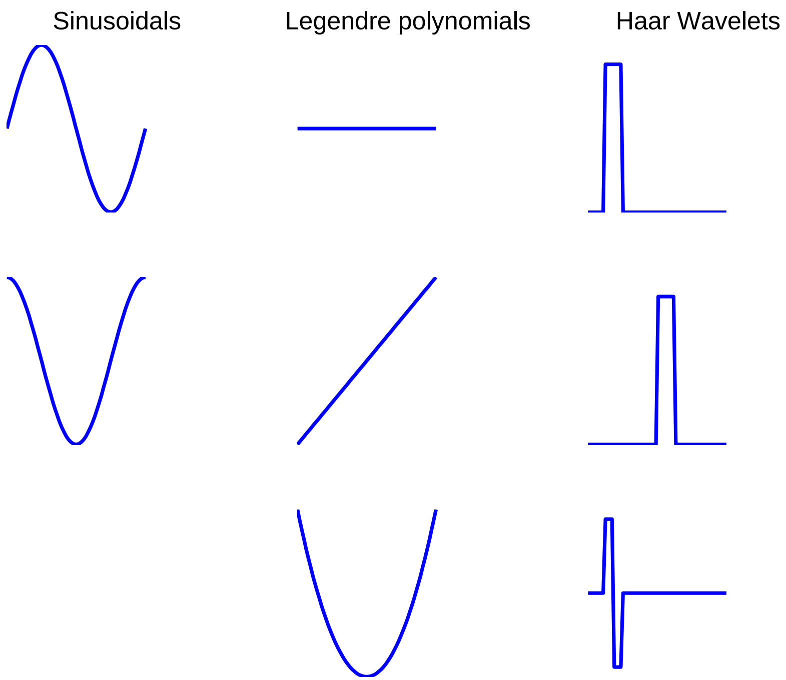
Basisfunctions

- Basisfunctions are known analytical functions of time
- Basisfunctions can be scaled (stretched) and translated (delayed) in time
- Basisfunctions can be superimposed to describe temporal changes of signals

Types of Basis Functions

- Trigoniometric functions: sines and cosines (periodic)
- Polynomial functions: e.g. Legendre polynomials (aperiodic, unbounded in time)
- Wavelet functions: e.g. Haar, Morlet (aperiodic, bounded in time)

Basis Functions: examples



Signal Decomposition

$$x(t) = l_1 f_1 + l_2 f_2 + ...$$

with f_n the n^{th} basisfunction and l_n the n^{th} expansion coefficient.

Note: a basisfunction can be selected for its own specific spectral (frequency) and temporal (timing) properties!

Decomposition

- Find a minimum number of expansion coefficients l_n
- Number of expansion coefficients determined by ’degree of orthogonality’ of f_n
- Choose type of basisfunctions to comply with the expected TV-dynamics of the signal

Wavelet Transform

Convolution of a time series x_n with a wavelet $\Psi(\eta)$

$$W_n(s) = \sum_{n'=0}^{N-1} x_{n'} \Psi^* \left[\frac{(n' - n)\Delta t}{s} \right]$$

* denots the complex conjugate

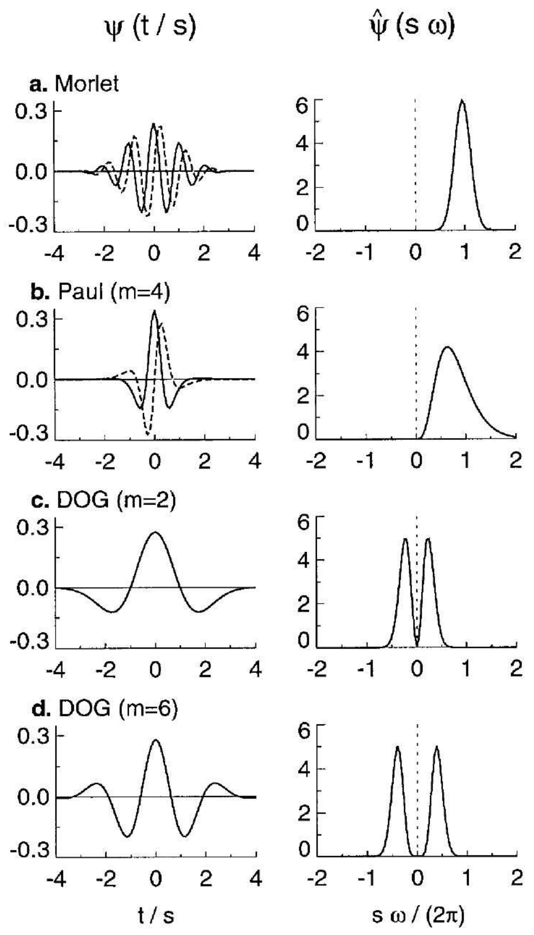
- Convolution with a wavelet equals band-pass filtering
- One filter for each scale s at each time instanst n (filter bank)

Note: s is here the scaling factor, not the Laplace operator!

Wavelet transform faster via the inverse fft of the product:

$$W_n(s) = \sum_{k=0}^{N-1} X(k\Delta\omega)\Psi(sk\Delta\omega)^* e^{jk\Delta\omega n\Delta t}$$

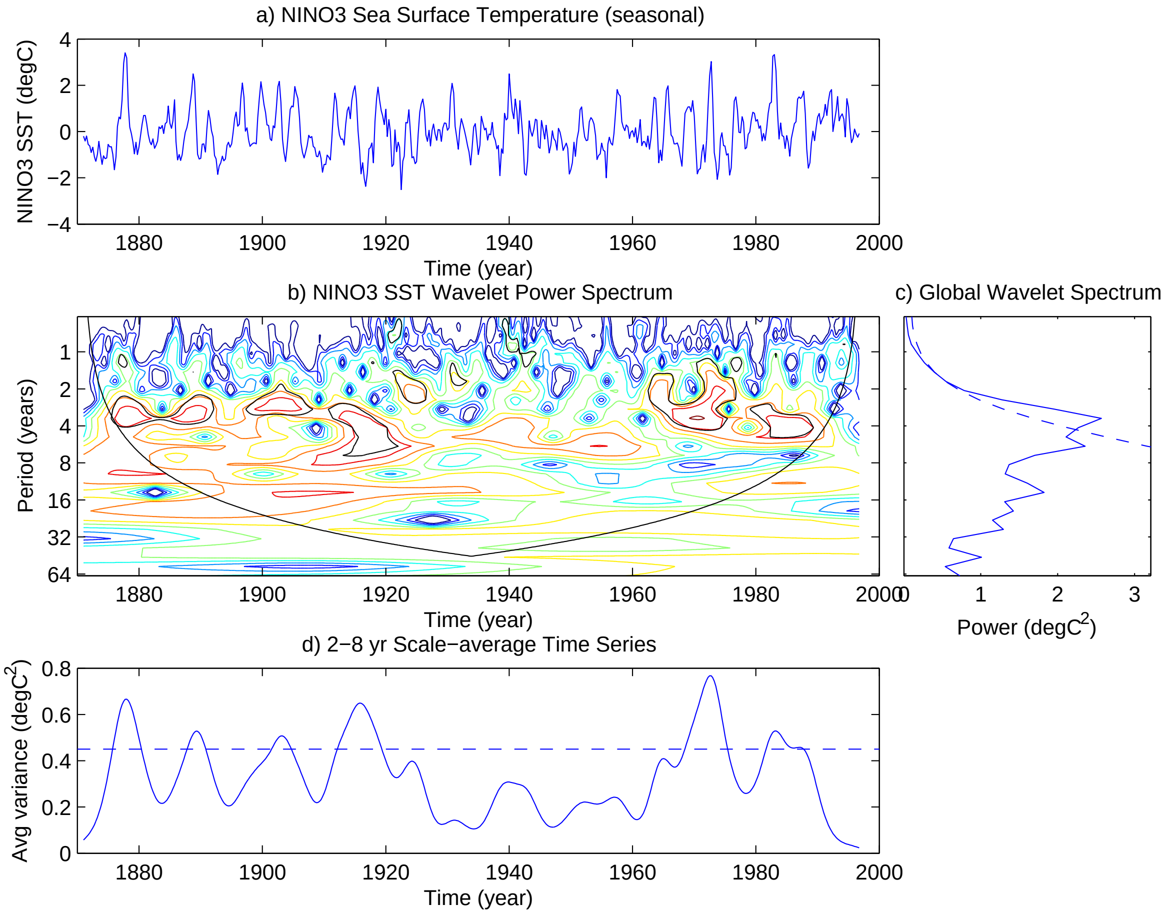
Wavelet Types



Result: increasing stiffness

Matlab

Example: Niño3 Sea Temperature



Polynomial Basisfunctions

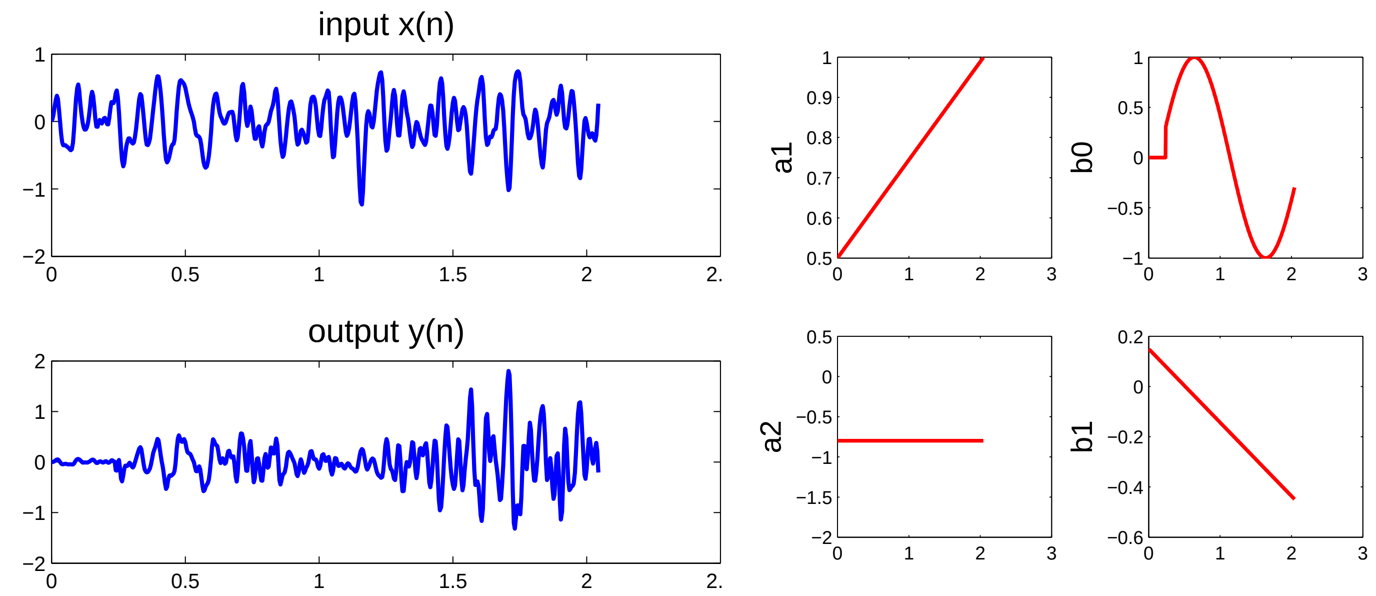
Application using an ARX model.
Process (ARX):

$$y(n) = \sum_{i=1}^{P} a(i,n)y(n-i) + \sum_{j=0}^{Q} b(j,n)x(n-j) + e(n)$$

where $a(i,n)$ and $b(i,n)$ are the TV ARX coefficients to be determined and are a function of time (i.e. n)!

Example: Estimating an LTV model

Time courses $a(i,n)$, $b(i,n)$ and signals:



Choose Legendre polynomials

Example: Estimating an LTV model

Expand $a(i,n)$ and $b(i,n)$ onto a set of basisfunctions $\pi_k(n)$:

$$
\begin{aligned}
a(i,n) &= \sum_{k=0}^{V} \alpha(i,k)\pi_k(n) \\
b(j,n) &= \sum_{k=0}^{V} \beta(j,k)\pi_k(n)
\end{aligned}
$$

with $\alpha(i,k)$ and $\beta(j,k)$ the expansion coefficients and V the maximum number of basisfunctions.

Example: Estimating an LTV model

Substituting the expansion into the ARX model:

$$y(n) = \sum_{i=1}^{P}\sum_{k=0}^{V}\alpha(i,k)\pi_k(n)y(n-i) + \sum_{j=0}^{Q}\sum_{k=0}^{V}\beta(j,k)\pi_k(n)x(n-j) + e(n)$$

Now, create new variables:

$$\begin{aligned} y_k(n-i) &= \pi_k(n)y(n-i) \\ x_k(n-j) &= \pi_k(n)x(n-j) \end{aligned}$$

The model is now TIV because the regression parameters $\alpha(i,k)$ and $\beta(j,k)$ are no longer functions of time!

Example: Estimating an LTV model

$$
\begin{aligned}
\mathbf{y_k}(n-i) &= \pi_\mathbf{k}\mathbf{y}(n-i) \\
&= [\pi_k(1)y(1-i),\ \pi_k(2)y(2-i), \ldots, \pi_k(N)y(N-i)]^T
\end{aligned}
$$

and similar for $\mathbf{x_k}(n-j)$. Now make a new matrix M of regressors:

$$M = [\mathbf{y_0}(n-1), \ldots, \mathbf{y_V}(n-1), \mathbf{x_0}(n), \ldots, \mathbf{x_V}(n), \mathbf{y_0}(n-2), \ldots, \mathbf{y_V}(n-2), \ldots]$$

and reject the linear dependent regressors resulting in a regression matrix W:

$$W = [\mathbf{w_1}, \mathbf{w_2}, \ldots, \mathbf{w_S}]$$

Example: Estimating an LTV model

The model in basisframe:

$$y(n) = \Theta^T W + e(n)$$

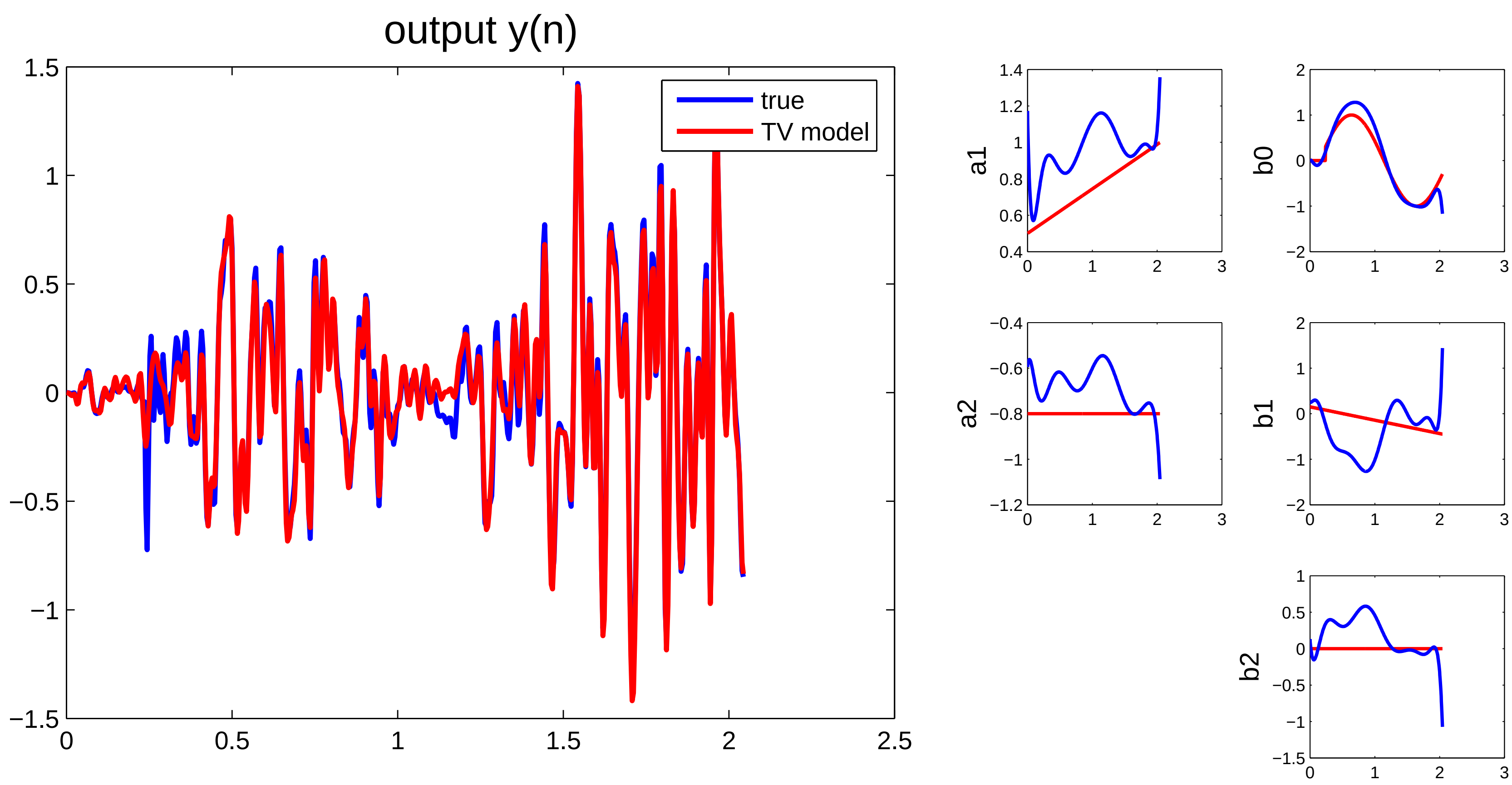
where $\Theta = [\theta_1, \theta_2, \ldots, \theta_S]$ the vector of regression coefficients.
Solution to LS analysis:

$$\hat{\Theta} = [WW^T]^{-1} W y(n)$$

Convert to original TV ARX modelparmeters $a(i, n)$ and $b(j, n)$ from Θ

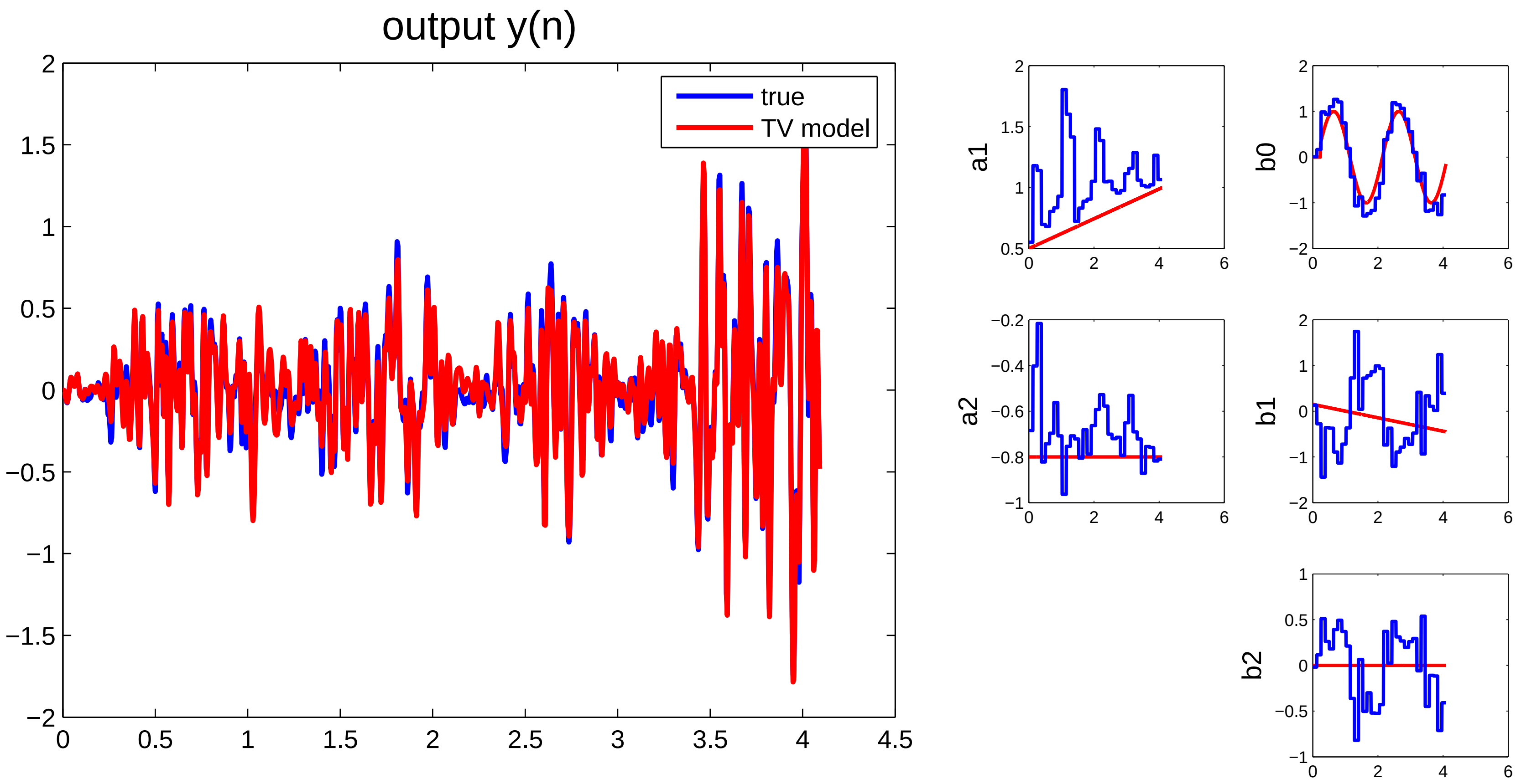
Example: Results

Estimated TV modelparameters:



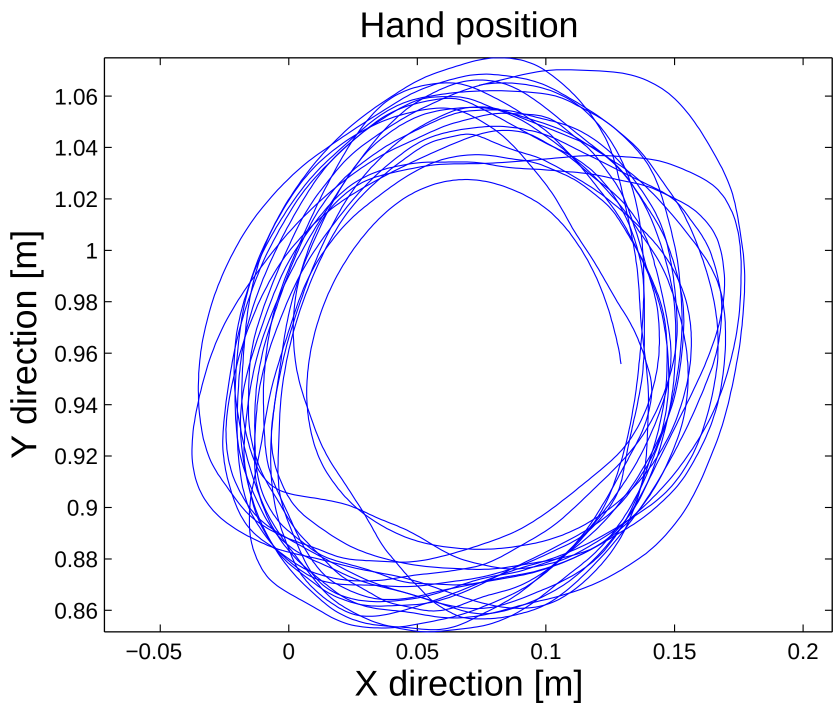
Example: Results

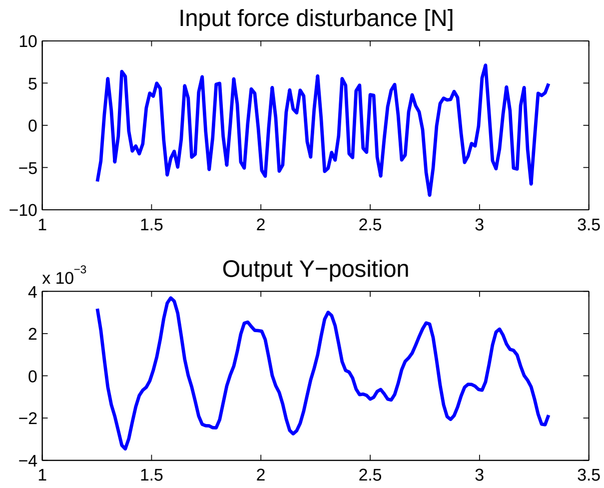
Estimated TV modelparameters using Haar wavelets:



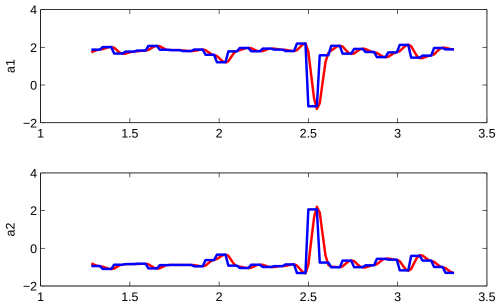
Example: 2D movement experiment

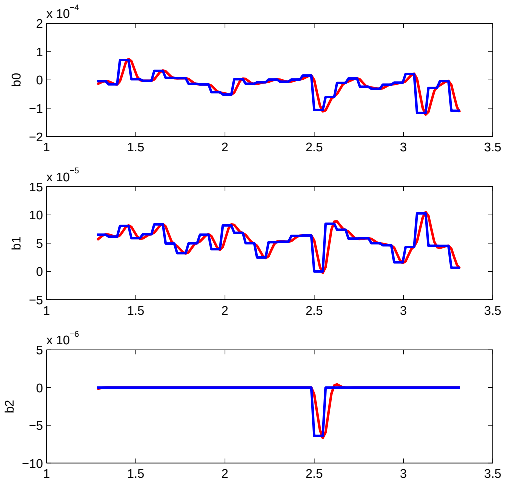
Goal: Estimate an TV MBK model

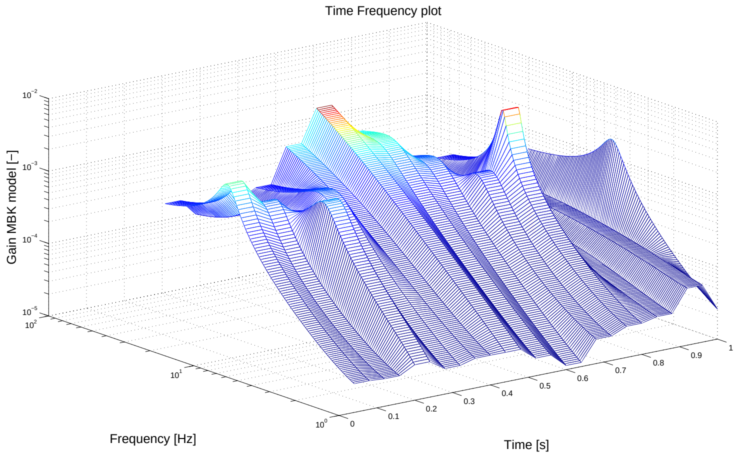




Example: 2D movement experiment



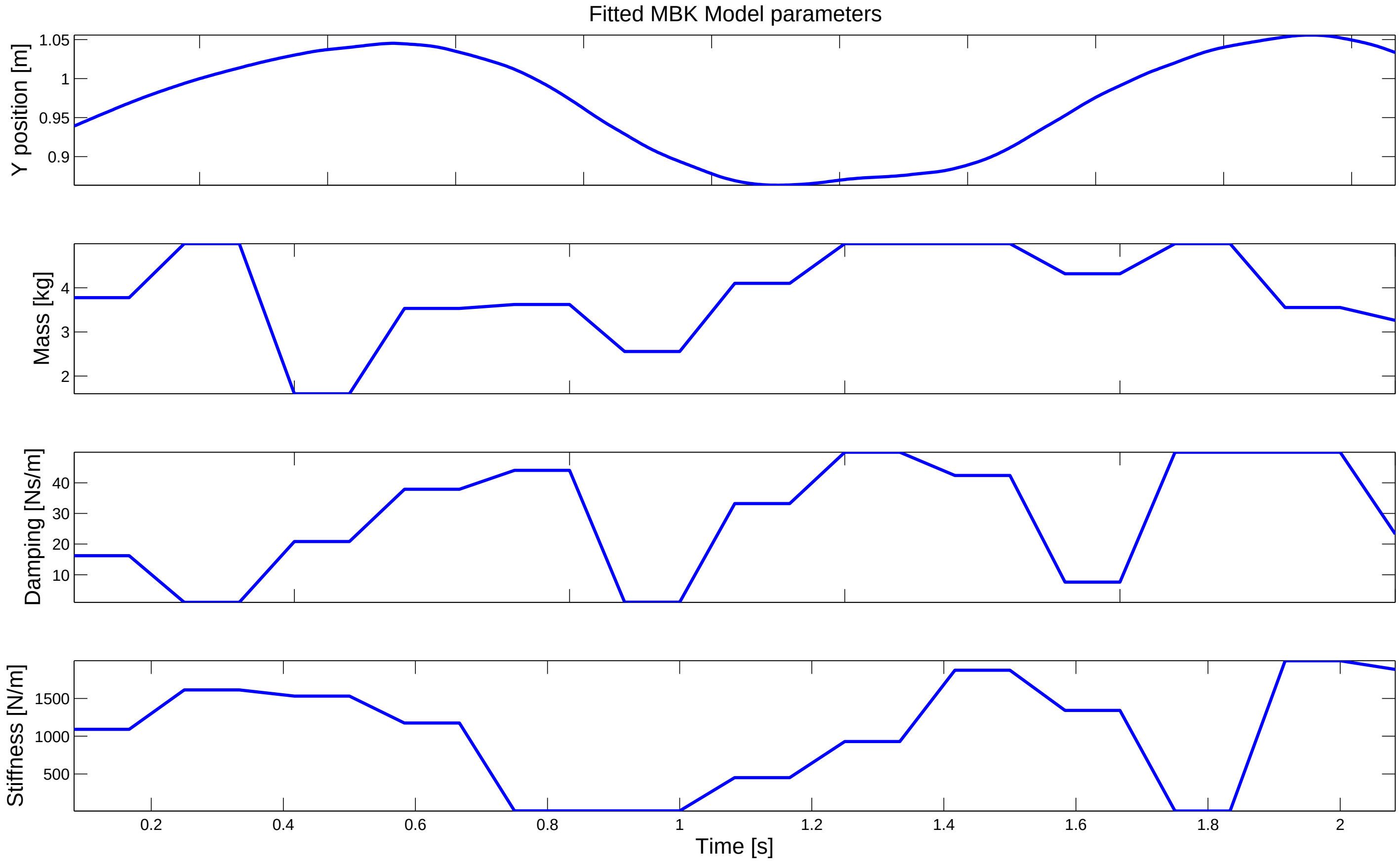




Inverse Z-transform:

$$H(q^{-1}, t) \rightarrow H(f, t)$$

Example: 2D movement experiment



Summary and Conclusions

- Basisfunctions are useful to describe TV system dynamics
- Time and frequency are decoupled
- Basisfunctions have to be chosen on apriori knowledge
- Trading-off accuracy vs number of coefficients