System Identification & Parameter Estimation (SIPE)

application to physiological systems

Wb2301

Lecture 1: Introduction and Background

Delft University of Technology

People

Lectures

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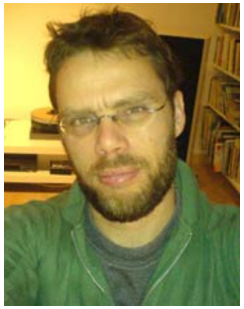

Assignments and 'SIPE helpdesk'

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General Course Info

- Blackboard:
  - Course schedule
  - Announcements
  - Lecture Notes
  - Assignments + Final Assignment
  - Chapters Reader
  - Demonstration programs (Matlab)
  - Matlab Command History (if applicable)

- 7 ECTS => 7 * 28 = 196 hours, for 14 lectures
- Work-load => 14 hours/week !!!!!!

- Theory
  - Book chapters (on Blackboard)
    - Westwick & Kearney: Identification of Nonlinear Physiological Systems
    - Pintelon & Schoukens: System identification
  - Scientific Articles (on Blackboard)

- Regular Assignments
  - Do it yourself. Basic Matlab experience helps !

- Final assignment
  - Identification of a physical system from real experimental data (to be announced)
  - Written report

- Written exam: register via TAS.tudelft.nl

Grading

- Final grade
  - 20% average of class assignments
  - 20% final assignment
  - 60% written examination

Related Courses

Previous

- Wb 2207: Systeem- en Regeltechniek 2 (SR 2)
  - Bode, Nyquist, Matlab
- Wb 2310: Systeem- en Regeltechniek 3 (SR 3)
  - Fourier

=>Overview: A Student's Guide to Classical Control, Bernstein 1997 (available on Blackboard)

Related

- SC4110: System identification (Bombois & van den Hof)
  - (Linear) control theory

Aim of the Course

- Parameterization input-output behavior of unknown systems
  - Non-parametric system representation
  - Parametric representation: model structure, linear, nonlinear
  - Estimation of model parameters
  - Model validation

- Course end terms
  - Intuition and understanding: Lectures
  - Theoretical background: Reader
  - Practical skills: Assignments

Course Contents

- System Identification
  - signals, systems, models, estimators
  - discrete and continuous domain, impulse response function
  - frequency domain, multivariable systems
  - time domain models, time-varying identification
- Parameter Estimation
  - parameter search routines
  - direct, indirect parameterization
  - parameter accuracy, model validation
  - parameterization of nonlinear models

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Enjoy!

Signals

Systems are expressed by their observed signals

Signals **domain**: time, space, frequency

Signal **range**: meter, Newton, Voltage, etc…

E.g. $s(t)$ represents a mapping from the *time domain set* to a certain *range set*

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Deterministic and Stochastic Signals

If future signal values are obtained from known equations, the signal is called **deterministic**.

E.g.:

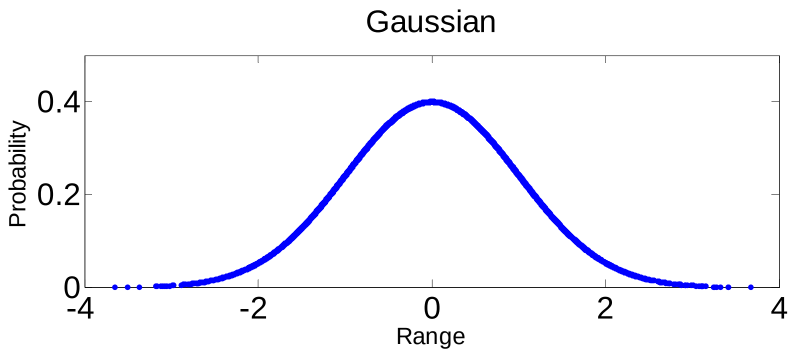
$$y(t) = \sin(\omega t)$$

If future signal values are random, the signal is called **stochastic**, or one realization from a stochastic process.

In reality, most signals fall between these two extremes.

Probability Density Function

A stochastic process is described by its **probability density function** (PDF)



Statistical properties (e.g. mean, variance) of a random variable x are derived from the PDF, $f(x)$, by **ensemble integrals**:

$$\mu_x = \int_{-\infty}^{\infty} x f(x) dx$$

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f(x) dx$$

Stationary and Ergodic Signals

- for a random time signal, $x(t)$, the PDF is also time dependent: $f(x, t)$
- if the PDF is independent of time, i.e. $f(x, t) = f(x)$, then the process is called **stationary**
- practical cases => PDF unknown => only finite time realizations => properties by **time integrals**:

$$\hat{\mu}_x = \int_0^T x(t)dt$$

- if ensemble and finite time integrals are equal, then process is called **ergodic**

Systems and Models

- system **N** transforms input $u(t)$ to output $y(t)$:

$$y(t) = N(u(t))$$

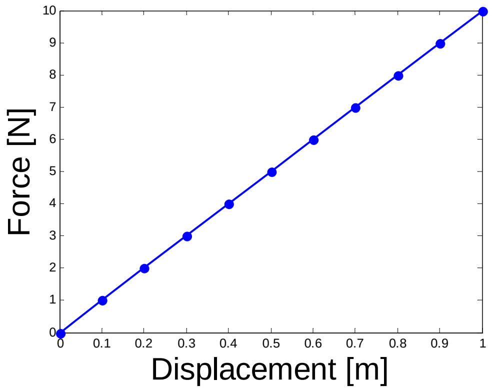
- model **M** estimates system's output (hat) from the input $u(t)$ and the model parameters θ:

$$\hat{y}(\theta,t) = M(\theta,u(t))$$

Parametric and Nonparametric Models

- **parametric** models: few parameters, in many cases with a physical meaning. E.g. a spring (*k*) model:

$$\hat{y}(t) = ku(t)$$



- **nonparametric** models: many parameters with no physical meaning. Mappings from domain variable (time, frequency) to output values. E.g. the dots in the above figure.

Static and Dynamic Systems

- **static** system: output at one time instant depends only on the input at the same time instant. E.g. a full-wave rectifier:

$$y(t) = |u(t)|$$

- **dynamic** system: output depends on some or all of the input history. E.g. a time delay:

$$y(t) = u(t - \tau)$$

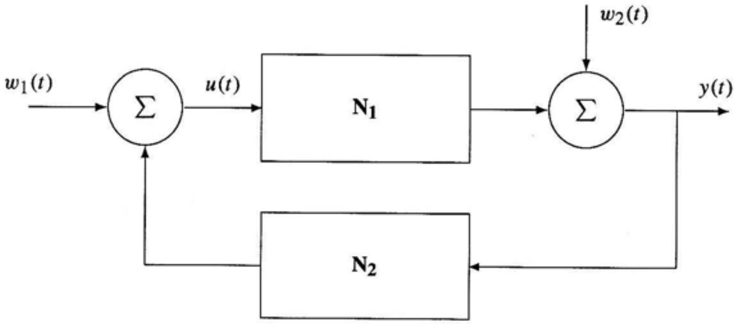
Causality

- **causal** systems: output depends only on previous inputs (all physical systems)

- **anti-causal** systems: output depends only on future inputs

- **noncausal** systems: output depends both on previous and future inputs

Feedback mixes-up causality



- only *u* and *y* are available
- systems N1, N2, include time delays
- w_1 arrives first at *u*, then at *y* (seems causal)
- w_2 arrives first at *y*, then at *u* (seems anti-causal)
- while the total system is causal (from w1 to y), feedback let the relationship between u and y appear noncausal

Linear and Nonlinear Systems

- System: $y(t) = N(u(t))$
- scaling property: $cy(t) = N(cu(t))$
- superposition property: $y_1(t) = N(u_1(t))$

$$y_2(t) = N(u_2(t))$$

$$y_1(t) + y_1(t) = N(u_1(t) + u_2(t))$$

- Systems that obey both scaling and superposition property are called **linear**. Otherwise, the system is **nonlinear**.

! In practice: linearity is approximate, depending on the range of the input => systems operate in their '**linear range**'.

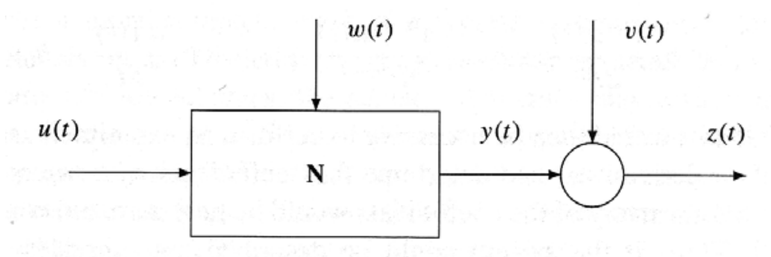
Time-Invariant and Time-Variant

- A system is **time-invariant** if its behavior does not depend on the passage of time:

$$y(t) = N(u(t)) \Rightarrow y(t-\tau) = N(u(t-\tau)) \qquad \forall \tau \in R$$

- Systems that for which the above equation does not hold are called **time-variant**

Deterministic and Stochastic Systems



- $v(t)$: additive noise, $w(t)$: process disturbance

$$z(t) = y(t) + v(t) = N(u(t), w(t)) + v(t)$$

- **Deterministic** system: $w(t) = 0$

- **Stochastic** system: $w(t) \neq 0$

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System Modeling from First Principles

- In many cases a mathematical model of a system can be constructed based on known physics (first principles). E.g. Hooke's law for modeling of a spring:

$$y = -ku$$

y : spring reaction force, u : imposed displacement, k : spring constant

- In many cases, the spring constant is an unknown parameter and needs to be estimated experimentally.

- If the spring model does not predict well, additional terms are needed, such as damping and inertia. The system may even be nonlinear!

System Identification

- In many cases, however, first principles do not give the right initial lead.
- General mathematical models are required. E.g. ordinary differential equations (linear case):

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \ldots + a_1\frac{dy(t)}{dt} + a_0 y(t)$$
$$= b_m\frac{d^m u(t)}{dt^m} + b_{m-1}\frac{d^{m-1}u(t)}{dt^{m-1}} + \ldots + b_1\frac{du(t)}{dt} + b_0 u(t)$$

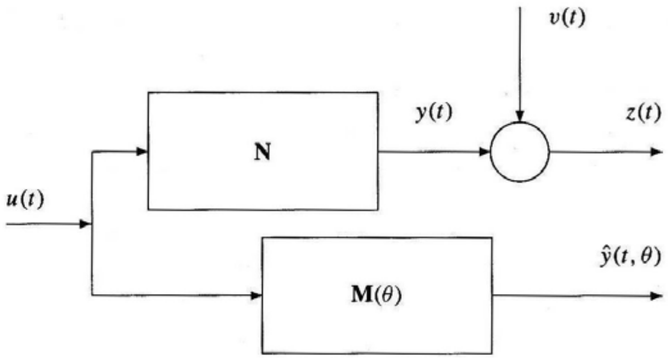
- Problem: many parameters in a general model structure (linear, nonlinear, parametric, nonparametric) are irrelevant and may lead to undefined (bad convergence) parameter values.
- Identification goal: find the minimum order of the model.

System Identification Problems

- Deterministic SYSID: find the relationship between $u(t)$ and $y(t)$, assuming no process noise $w(t)$
- Stochastic SYSID: find the relationship between $w(t)$ and $y(t)$, based on assumed statistics of $w(t)$
- Complete SYSID: both deterministic and stochastic components, e.g. for precise output predictions.
- This course is dedicated to **deterministic SYSID**

The Art of System Identification

Find a model (M) having the least amount of parameters (θ) providing an accurate description $\hat{y}(\theta, t)$ of the system's output $y(t)$



$$\boxed{\hat{y}(\theta,t) = M(\theta, u(t))}$$

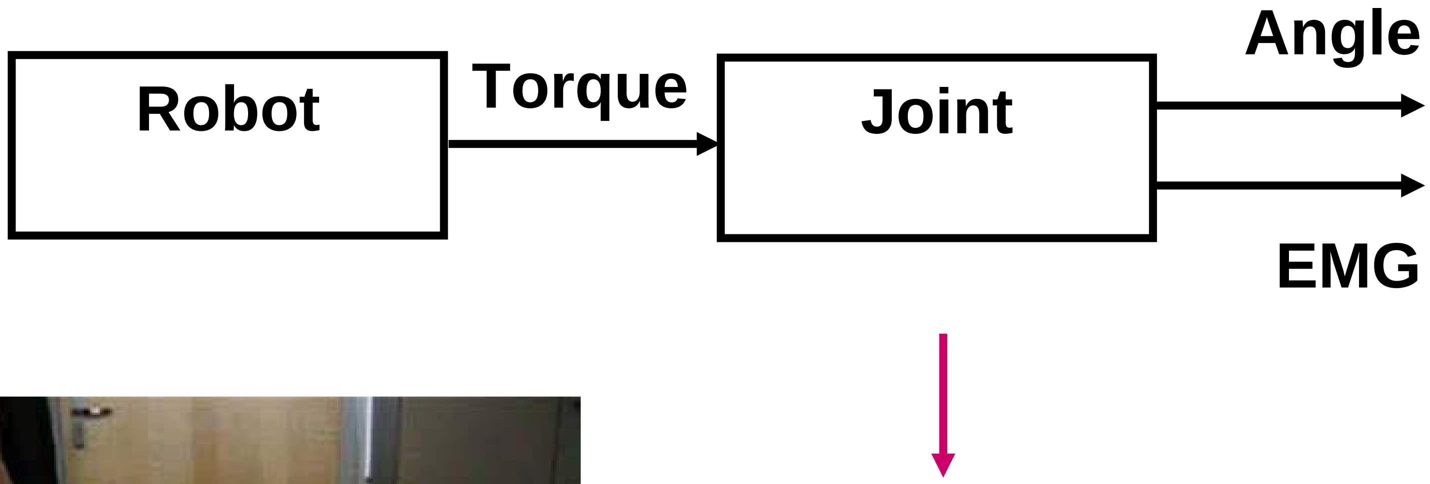
SYSID Applications

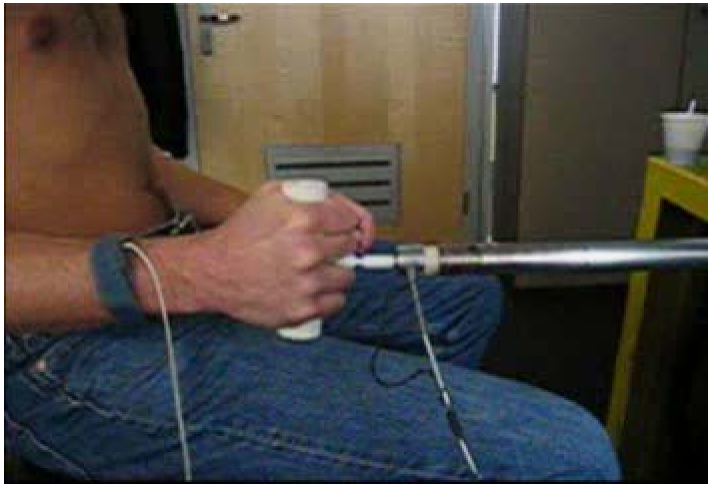
- Control: aim is to improve system performance
- Analysis: aim is to understand system functioning
- This course is dedicated to system **analysis**. Focus is on human physiological systems, in particular the neuromuscular system.

Nonlinearities

- Nonlinearities play a crucial role in SYSID
- Nonlinear functioning in the neuromuscular system:
  - **sensors**: e.g. joint rotation is encoded and used by the nervous system to control muscle force. Sensors are directional and amplitude sensitive
  - **actuators** (muscles): muscles only produce pulling forces and exhibit nonlinear force-length and force-velocity behavior
  - **tissues**: visco-elasticity of binding tissue increases exponentially with stretch amplitude
- Linear models are valid in one point of operation but generally do not provide much insight in the functional organization of the system
- This course mainly discusses linear SYSID techniques and some nonlinear issues.

Measurement Setup





Parameters:
sensor gains
muscle visco-elasticity
tissue visco-elasticity

Gaussian Random Variables

- Noise is always present and approximately Gaussian:

$$PDF = f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

- The PDF is fully determined by its mean μ and variance σ^2:

$$\mu_x = E[x]$$

$$\sigma_x^2 = E\left[(x-\mu)^2\right]$$

- with E[…] indicates the expectation.

Linear Systems and Noise

- Filtering Gaussian noise by a linear system produces again Gaussian noise.
- Consequently, only the first two PDF moments (mean, variance) are required for linear SYSID

Correlation Functions

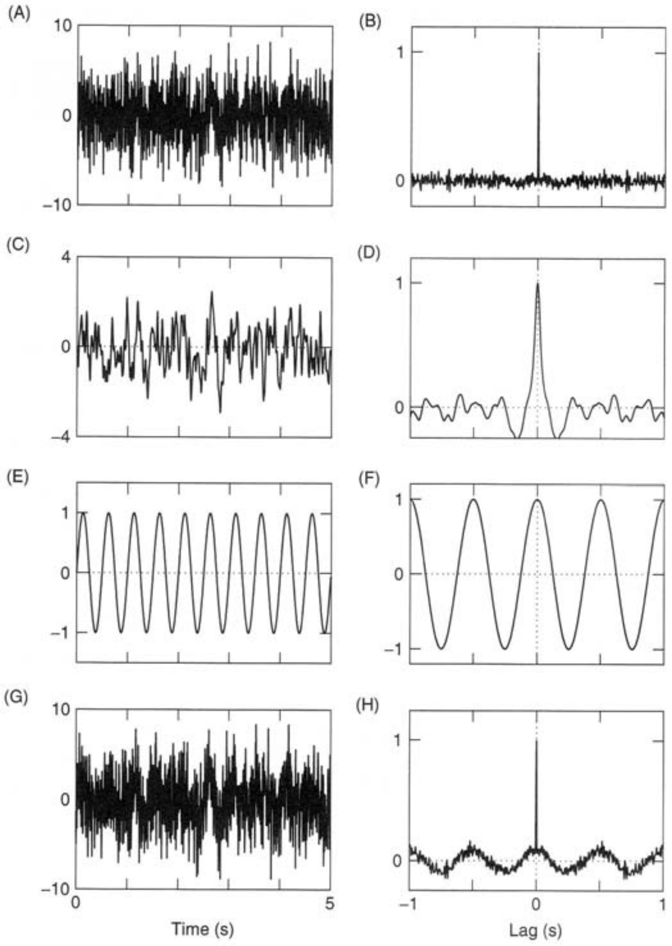
- Correlation functions reveal structures of signals that are not apparently detectable in the time series.
- Used to analyze relationships between signals, usually between inputs and outputs.
- Three types:
  - Autocorrelation function
  - Autocovariance function
  - Autocorrelation coefficient

Autocorrelation Function

- Autocorrelation function:

$$\Phi_{xx}(\tau) = E\left[x(t-\tau)x(t)\right]$$

- Maximal for $\tau = 0$.



Autocovariance Function

- Autocovariance function:

$$
\begin{aligned}
C_{xx}(\tau) &= E\left[\left(x(t-\tau)-\mu_x\right)\left(x(t)-\mu_x\right)\right] \\
&= E\left[x(t-\tau)x(t)\right]-E\left[x(t-\tau)\mu_x\right]-E\left[\mu_x x(t)\right]+E\left[\mu_x^2\right] \\
&= \Phi_{xx}(\tau)-\mu_x^2-\mu_x^2+\mu_x^2 \\
&= \Phi_{xx}(\tau)-\mu_x^2
\end{aligned}
$$

- If $\mu = 0$, then autocovariance and autocorrelation functions are identical
- At zero lag,

$$
C_{xx}(0) = E\left[\left(x(t)-\mu_x\right)^2\right] = \sigma_x^2
$$

Autocorrelation coefficient

- Dividing the autocovariance by the variance gives the autocorrelation coefficient:

$$r_{xx}(\tau) = \frac{C_{xx}(\tau)}{C_{xx}(0)}$$

- The autocorrelation coefficient ranges from +1 (full positive correlation), to 0 (no correlation), to -1 full negative correlation).

Book: Westwick & Kearney

- Chapter 1, all
- Chapter 2, sec. 2.1 – 2.3