# Out-of-plane behavior

- Based on standard plate theory, the strains are assumed to be at most linear through the thickness:

$$\varepsilon_x = \varepsilon_{xo} - \frac{\partial^2 w}{\partial x^2} z = \varepsilon_{xo} + z\kappa_x$$

$$\varepsilon_y = \varepsilon_{yo} - \frac{\partial^2 w}{\partial y^2} z = \varepsilon_{yo} + z\kappa_y \qquad (1.4)$$

$$\gamma_{xy} = \gamma_{xyo} - 2\frac{\partial^2 w}{\partial x \partial y} z = \gamma_{xyo} + z\kappa_{xy}$$

$\kappa_x$, $\kappa_y$, $\kappa_{xy}$ are laminate curvatures (1/radius of curvature)

Note the coordinate system for z has its origin at the laminate mid-plane

- For the case of pure bending, there are no in-plane strains so $\varepsilon_{xo} = \varepsilon_{yo} = \gamma_{xyo} = 0$

# Pure bending

- Take equations (1.1) again, multiply both sides by z and integrate through the thickness of the laminate. For example, the first equation gives:

$$\int_{-h/2}^{h/2} z\sigma_{11} dz = \int_{-h/2}^{h/2} Q_{11} z \left( -z \frac{\partial^2 w}{\partial x^2} \right) dz$$

- and using the definition for $M_x$

$$M_x = -\int_{-h/2}^{h/2} Q_{11} z^2 \frac{\partial^2 w}{\partial x^2} dz - \int_{-h/2}^{h/2} Q_{12} z^2 \frac{\partial^2 w}{\partial y^2} dz - \int_{-h/2}^{h/2} 2Q_{16} z^2 \frac{\partial^2 w}{\partial x \partial y} dz$$

- For pure bending, the curvatures $-\partial^2 w/\partial x^2$, etc., are constant and can come out of the integral sign:

$$M_x = \kappa_x \int_{-h/2}^{h/2} Q_{11} z^2 dz + \kappa_y \int_{-h/2}^{h/2} Q_{12} z^2 dz + \kappa_{xy} \int_{-h/2}^{h/2} Q_{16} z^2 dz$$

# Pure bending

$$M_x = \kappa_x \int_{-h/2}^{h/2} Q_{11} z^2 dz + \kappa_y \int_{-h/2}^{h/2} Q_{12} z^2 dz + \kappa_{xy} \int_{-h/2}^{h/2} Q_{16} z^2 dz$$

- or, since $Q_{ij}$ are constant for each ply:

$$M_x = \underbrace{\kappa_x \sum_{k=1}^{N} (Q_{11})^{(k)} \frac{z_k^{\,3} - z_{k-1}^{\,3}}{3}}_{D_{11}} + \underbrace{\kappa_y \sum_{k=1}^{N} (Q_{12})^{(k)} \frac{z_k^{\,3} - z_{k-1}^{\,3}}{3}}_{D_{12}} + \underbrace{\kappa_{xy} \sum_{k=1}^{N} (Q_{16})^{(k)} \frac{z_k^{\,3} - z_{k-1}^{\,3}}{3}}_{D_{16}}$$

- Repeating for the remaining two equations from (1.1) and using matrix notation:

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \qquad (1.5)$$

# Bending-Stretching coupling

- It is possible, for some laminates, to undergo bending or twisting under in-plane loads, or, undergo stretching under applied bending or torsional moments
- Returning to eq. (1.1) and using (1.4), $\sigma_{11}$ is written as:

$$\sigma_x = Q_{11}(\varepsilon_{xo} + z\kappa_x) + Q_{12}(\varepsilon_{yo} + z\kappa_y) + Q_{16}(\gamma_{xyo} + z\kappa_{xy})$$

- Integrating with respect to z and using eq. (1.2):

$$N_x = A_{11}\varepsilon_{xo} + \int_{-h/2}^{h/2} Q_{11} z dz \;\; \kappa_x + A_{12}\varepsilon_{yo} + \int_{-h/2}^{h/2} Q_{12} z dz \;\; \kappa_y + A_{16}\gamma_{xyo} + \int_{-h/2}^{h/2} Q_{16} z dz \;\; \kappa_{xy}$$

- The three integrals can be turned into summations:

$$N_x = A_{11}\varepsilon_{xo} + A_{12}\varepsilon_{yo} + A_{16}\gamma_{xyo} +$$

$$\underbrace{\kappa_x \sum_{k=1}^{N} Q_{11}{}^{(k)} \left[ \frac{z_k{}^2 - z_{k-1}{}^2}{2} \right]}_{B_{11}} + \underbrace{\kappa_y \sum_{k=1}^{N} Q_{12}{}^{(k)} \left[ \frac{z_k{}^2 - z_{k-1}{}^2}{2} \right]}_{B_{12}} + \underbrace{\kappa_{xy} \sum_{k=1}^{N} Q_{16}{}^{(k)} \left[ \frac{z_k{}^2 - z_{k-1}{}^2}{2} \right]}_{B_{16}}$$

# Putting it all together…

- The equivalent stress-strain relations for a laminate:

$$\text{forces}\left\{\begin{matrix} N_x \\ N_y \\ N_{xy} \end{matrix}\right. \quad \text{moments}\left\{\begin{matrix} M_x \\ M_y \\ M_{xy} \end{matrix}\right.$$

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \gamma_{xyo} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (1.6)$$

$\left.\begin{matrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \gamma_{xyo} \end{matrix}\right\}$ mid-plane strains

$\left.\begin{matrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{matrix}\right\}$ curva-tures

- Reminders:
  - –For symmetric laminates the B matrix is zero
  - –For balanced laminates (for each +θ there is a –θ somewhere) $A_{16}=A_{26}=0$

when is $D_{16}=D_{26}=0$ ??

# Inverted stress-strain relations

- Usually, we do not know the strains and curvatures but the forces and moments. It is more convenient then, to use the inverted relations:

$$\begin{Bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \gamma_{xyo} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{16} & \beta_{11} & \beta_{12} & \beta_{16} \\ \alpha_{12} & \alpha_{22} & \alpha_{26} & \beta_{21} & \beta_{22} & \beta_{26} \\ \alpha_{16} & \alpha_{26} & \alpha_{66} & \beta_{61} & \beta_{62} & \beta_{66} \\ \beta_{11} & \beta_{21} & \beta_{61} & \delta_{11} & \delta_{12} & \delta_{16} \\ \beta_{12} & \beta_{22} & \beta_{62} & \delta_{12} & \delta_{22} & \delta_{26} \\ \beta_{16} & \beta_{26} & \beta_{66} & \delta_{16} & \delta_{26} & \delta_{66} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} \quad (1.7)$$

$$[\alpha] = [A]^{-1} + [A]^{-1}[B]\big[[D] - [B][A]^{-1}[B]\big]^{-1}[B][A]^{-1}$$

$$[\beta] = -[A][B]\big[[D] - [B][A]^{-1}[B]\big]^{-1}$$

$$[\delta] = \big[[D] - [B][A]^{-1}[B]\big]^{-1}$$

# Inverted stress-strain relations

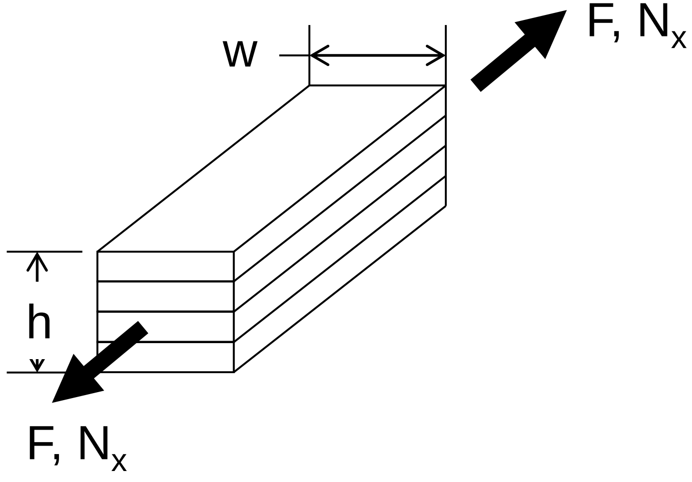
- For a symmetric laminate:

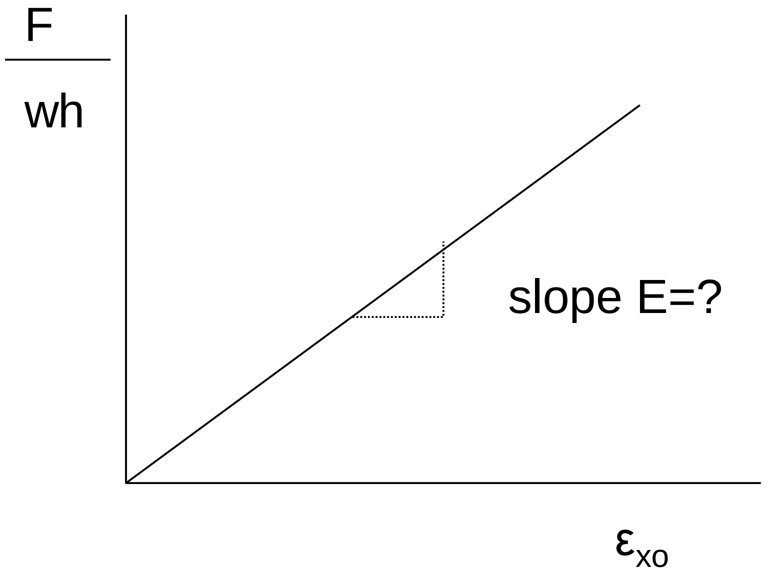
$$\begin{Bmatrix} \varepsilon_{xo} \\ \varepsilon_{yo} \\ \gamma_{xyo} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} & 0 & 0 & 0 \\ a_{12} & a_{22} & a_{26} & 0 & 0 & 0 \\ a_{16} & a_{26} & a_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & d_{11} & d_{12} & d_{16} \\ 0 & 0 & 0 & d_{12} & d_{22} & d_{26} \\ 0 & 0 & 0 & d_{16} & d_{26} & d_{66} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}$$

Note that, in general, the little $a_{ij}$ and $d_{ij}$ matrices are not the same as $\alpha_{ij}$ $\delta_{ij}$ (they are only for symmetric laminates)

# Elastic constants for a laminate

- What is the Young's modulus for a laminate?




Note: $\varepsilon_{xo}$ is not necessarily what one would measure with a strain gage at the top of the laminate. Why?

# Young's modulus for a laminate

- For a symmetric laminate under uniaxial tension: $N_y$=$N_{xv}$=0. Then from (1.6):

$$N_x = A_{11}\varepsilon_{xo} + A_{12}\varepsilon_{yo}$$
$$0 = A_{12}\varepsilon_{xo} + A_{22}\varepsilon_{yo}$$

- Solve for $\varepsilon_{yo}$ and substitute in the first equation to obtain:

$$N_x = \left(A_{11} - \frac{A_{12}^2}{A_{22}}\right)\varepsilon_{xo}$$

- and using the relationships between $N_x$ and applied stress $\sigma_o$:

$$\sigma_o = \frac{F}{wh} = \frac{1}{h}\left[\frac{A_{11}A_{22} - A_{12}{}^2}{A_{22}}\right]\varepsilon_{xo}$$

# Elastic constants for a laminate

$$\sigma_o = \frac{F}{wh} = \frac{1}{h}\left[\frac{A_{11}A_{22} - A_{12}{}^2}{A_{22}}\right]\varepsilon_{xo}$$

- The last relation can be rewritten if one uses the fact that:

$$a_{11} = \frac{A_{22}}{A_{11}A_{22} - A_{12}^2}$$

- to obtain:

$$\sigma_o = \frac{1}{ha_{11}}\varepsilon_{xo}$$

- from which, the membrane laminate modulus (in stretching) is:

$$E_{1m} = \frac{1}{ha_{11}}$$

# Elastic constants for a laminate

- Similarly, one can show that:

$$E_{1m} = \frac{1}{ha_{11}} \qquad E_{1b} = \frac{12}{h^3 d_{11}}$$

$$E_{2m} = \frac{1}{ha_{22}} \qquad E_{2b} = \frac{12}{h^3 d_{22}}$$

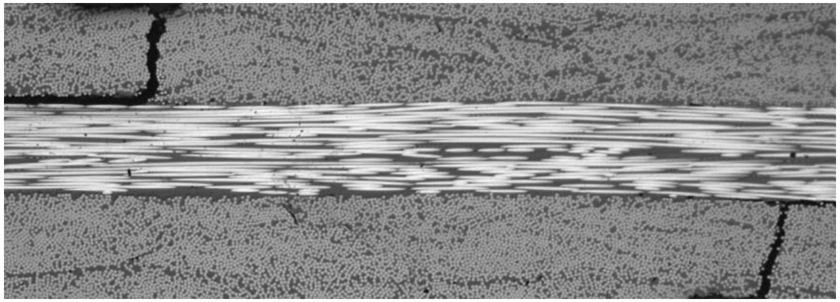
$$G_{12m} = \frac{1}{ha_{66}} \qquad G_{12b} = \frac{12}{h^3 d_{66}}$$

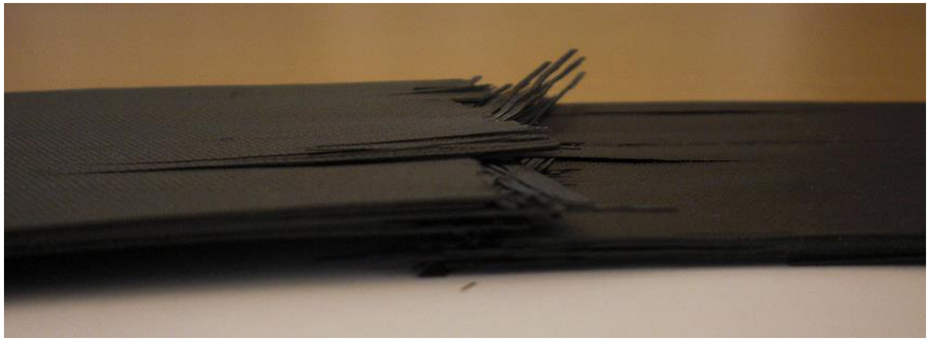
$$\nu_{12m} = -\frac{a_{12}}{a_{11}} \qquad \nu_{12b} = -\frac{d_{12}}{d_{11}}$$

$$\nu_{21m} = -\frac{a_{12}}{a_{22}} \qquad \nu_{21b} = -\frac{d_{12}}{d_{22}}$$

# Failure in composites

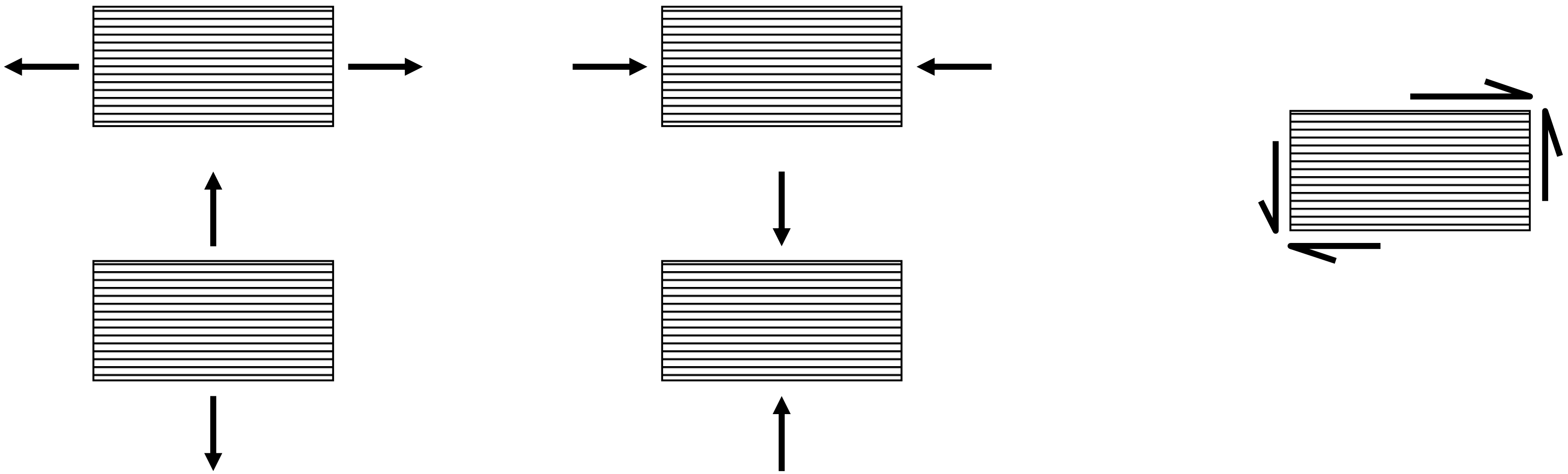
- In a structure, failure, as a rule starts at the weakest link
- Unlike stiffness, where the overall stiffness is a weighted average of the stiffnesses of the constituents, fiber and matrix, strength is characterized by failure of the weakest link, the matrix
- Usually, matrix failure does not mean final failure.  Load is transferred to fibers.
- For final failure fibers must fail





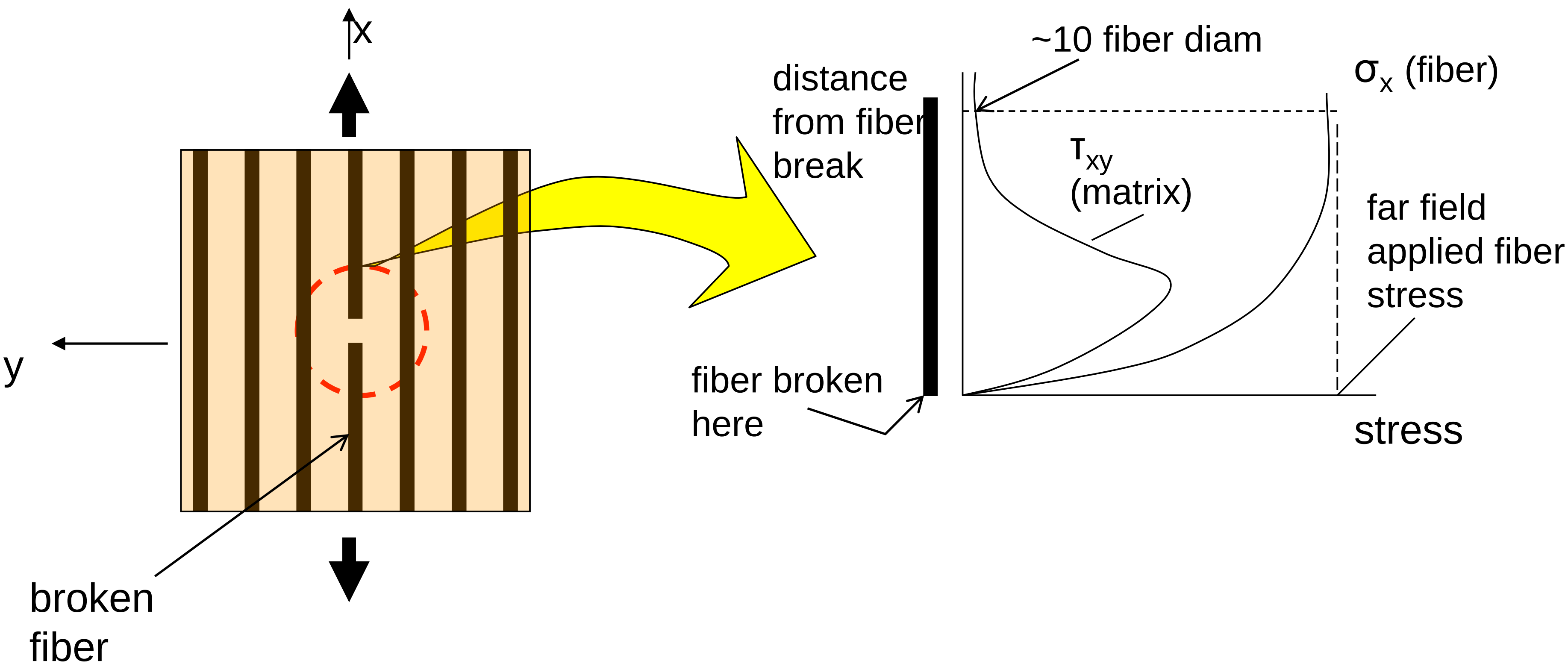
# Failure modes of a ply

- A single ply can fail in (at least) 5 different ways:
  - tension along fibers (fiber pull-out and fiber failure)
  - tension perpendicular to the fibers (matrix failure)
  - compression along fibers (local shearing of matrix and fibers)
  - compression perpendicular to fibers (matrix shear failure)
  - shear



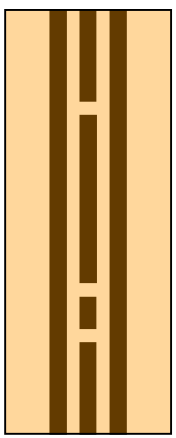
# Is this the right approach?

- Test for these five failure modes and, somehow, put them together in a prediction model (failure criterion)
- Alternatively, one could start at constituent level but it is hard to translate to ply level



# Role of matrix

- Transfers load around cut fibers through shear
- Distance over which this transfer occurs is very small (<20 fiber diameters)



which fiber breaks first?

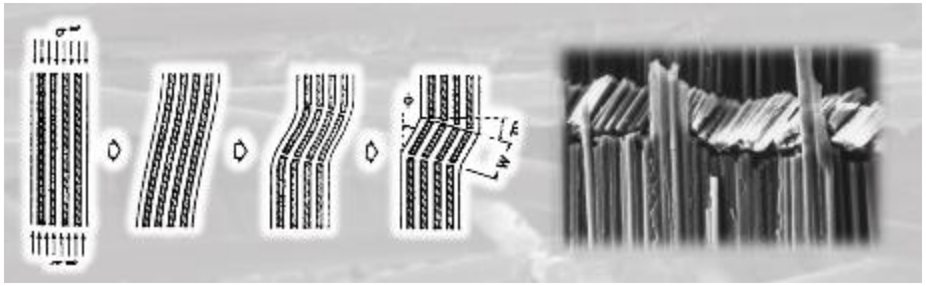
where along its length?

how is load of adjacent fibers affected?

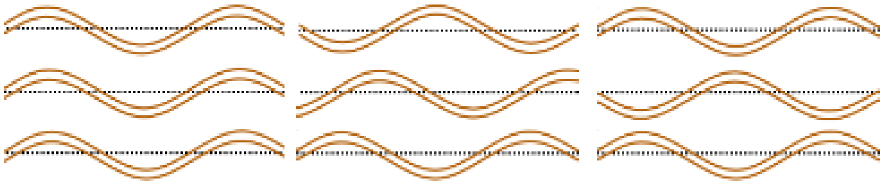
how many breaks before a fiber is ineffective?

# Situation is even more complex under compression

- At the fiber/matrix level, there are multiple failure modes:
  - fiber kinking
  - fiber failure due to compression and bending (wavy fibers)
  - matrix cracking
  - failure of fiber/matrix interface



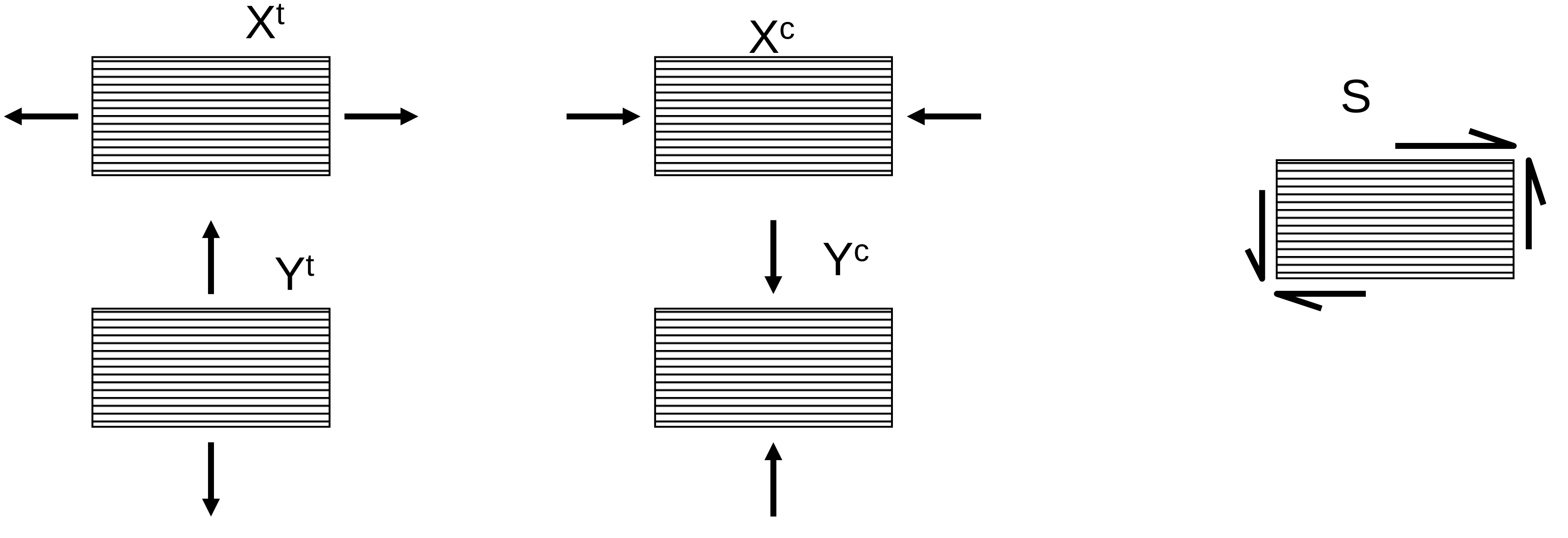
fiber kinking from W. De Backer MS thesis



wavy fibers under compression from W. De Backer MS thesis

# Retreat to the ply level measured strength properties

- Combine these in a failure criterion
- There are very many failure criteria
- To paraphrase president A. Lincoln, “you can have a criterion that works for some cases all the time or for all cases some of the time but you cannot have a criterion that works for all cases all the time”



# Some of the most commonly used failure criteria

- Maximum stress criterion

$$\sigma_x < X^t \; or \; X^c \text{ depending on whether } \sigma_x \text{ is tensile or compressive}$$

$$\sigma_y < Y^t \; or \; Y^c \text{ depending on whether } \sigma_y \text{ is tensile or compressive}$$

$$|\tau_{xy}| < S$$

- Stresses are examined separately and their interaction is not accounted for
- Maximum strain criterion (analogous to max stress)

$$\varepsilon_x < \varepsilon_{xu}^t \; or \; \varepsilon_{xu}^c \text{ depending on whether } \varepsilon_x \text{ is tensile or compressive}$$

$$\varepsilon_y < \varepsilon_{yu}^t \; or \; \varepsilon_{yu}^c \text{ depending on whether } \varepsilon_y \text{ is tensile or compressive}$$

$$|\gamma_{xy}| < \gamma_{xyu}$$

# Some of the most commonly used failure criteria

- Tsai-Hill failure criterion:

$$\frac{\sigma_x^2}{X^2} - \frac{\sigma_x \sigma_y}{X^2} + \frac{\sigma_y^2}{Y^2} + \frac{\tau_{xy}^2}{S^2} = 1$$

(X and Y are tensile or compressive stresses accordingly)

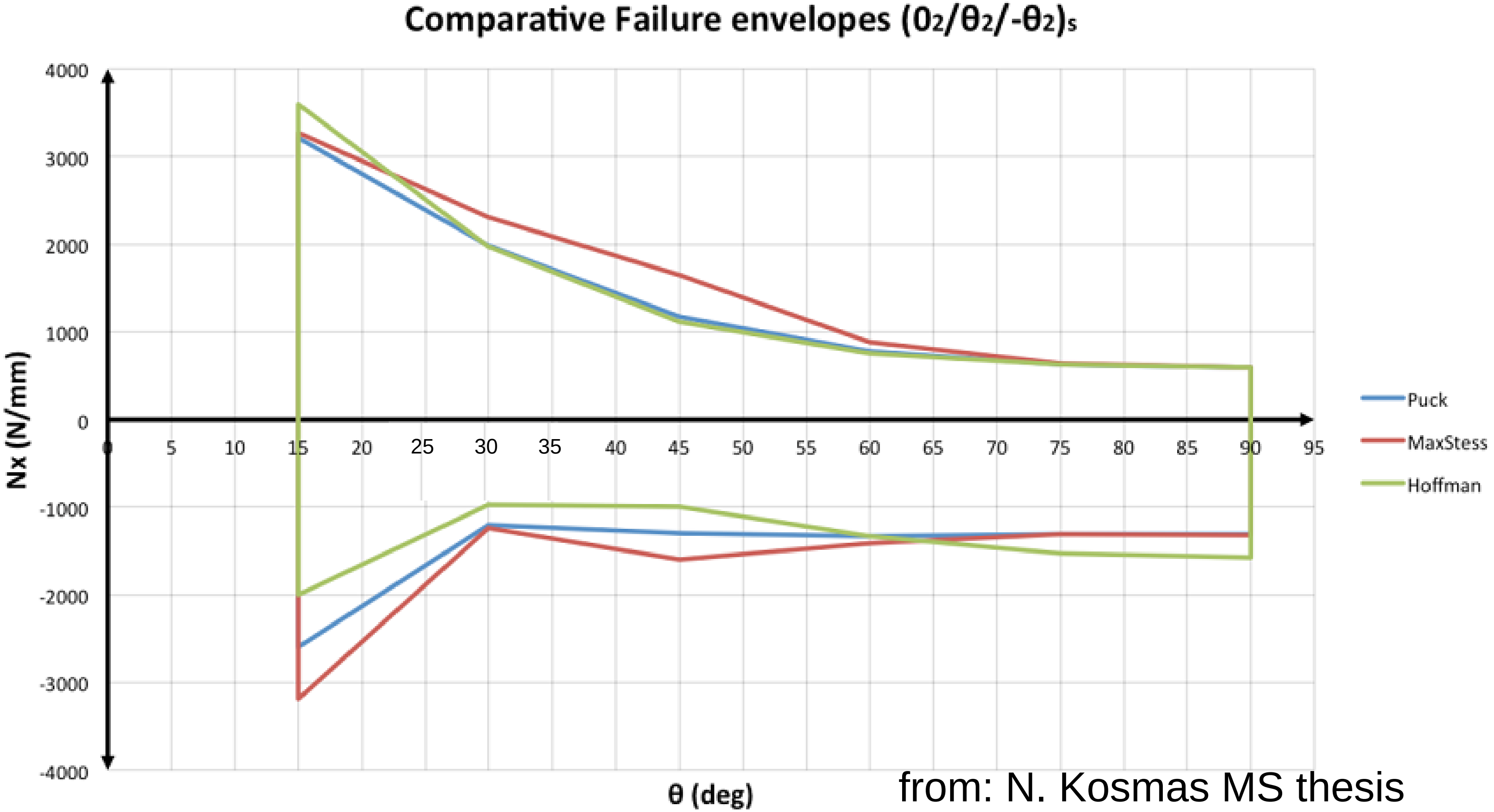
- Tsai-Wu failure criterion:

$$\frac{\sigma_x^2}{X^{\mathrm{t}} X^{\mathrm{c}}} + \frac{\sigma_y^2}{Y^{\mathrm{t}} Y^{\mathrm{c}}} - \sqrt{\frac{1}{X^{\mathrm{t}} X^{\mathrm{c}}} \frac{1}{Y^{\mathrm{t}} Y^{\mathrm{c}}}} \sigma_x \sigma_y + \left(\frac{1}{X^{\mathrm{t}}} - \frac{1}{X^{\mathrm{c}}}\right) \sigma_x + \left(\frac{1}{Y^{\mathrm{t}}} - \frac{1}{Y^{\mathrm{c}}}\right) \sigma_y + \frac{\tau_{xy}^2}{S^2} = 1$$

- Note that these two criteria recover the von-Mises yield criterion in metals if the material is isotropic; this does not mean they are any better than other failure criteria

# Comparison of criteria with each other

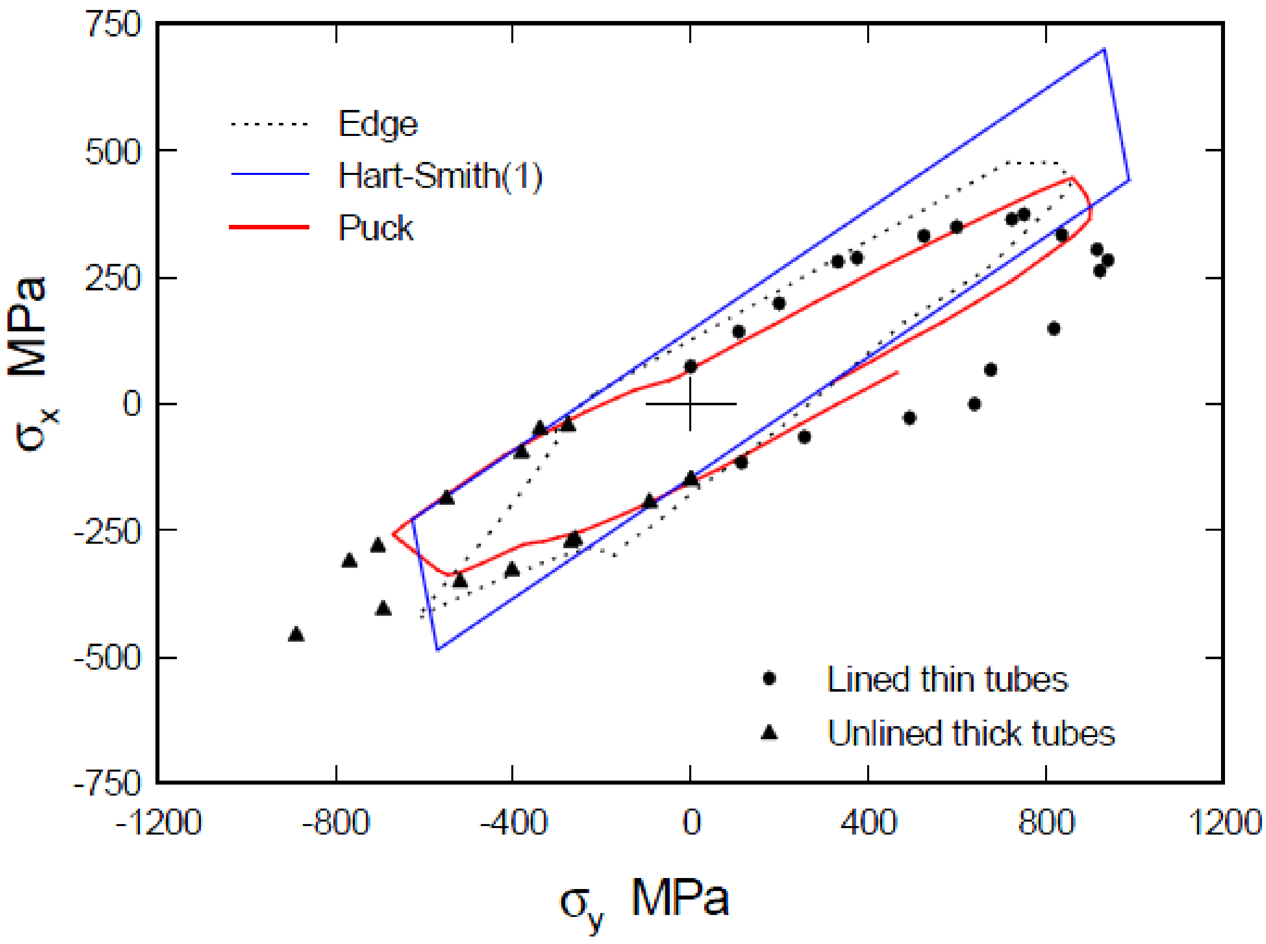
- Under tension, most criteria tend to be reasonably close to each other




from: N. Kosmas MS thesis

# Some comparisons with test results

- Under more generalized loading, there can be serious disagreement between different criteria and tests



# Reminders and Discussion

- The stresses or strains in these criteria are parallel and perpendicular to the fibers in each ply
- These criteria are for “onset of failure” only (first-ply-failure)
- They cannot predict final failure; Modifications for progressive failure analysis are possible with mixed results
- These criteria do not account for out-of-plane failure (delaminations); Special criteria for out-of-plane failure have been developed (mainly stress based)
- In 1982, prof. P.A. Lagacé wrote: “There are as many failure criteria as there are researchers in the field and a consensus has yet to be reached” (Lagacé known for the Brewer-Lagacé Quadratic Delamination Criterion)

# Discussion on failure criteria

- More than 30 years later, in 2014, the situation is much the same if not worse
- There have been two Worldwide Failure Exercises which concluded that no criterion is good enough
- Some interesting quotes:
- Z. Hashin (known, among other things, for the Hashin failure criterion): “My only work on this subject relates to failure criteria of uni-directional fiber composites, not to laminates…I must say to you that I personally do not know how to predict the failure of a laminate (and **furthermore that I do not believe that anybody does**)”

# Discussion on failure criteria

- J. Hart-Smith (known, among other things, for the Hart-Smith failure criterion):
- “The **irrelevance** of most composite failure criteria to conventional fiber-polymer composites is claimed to have remained undetected…”
- Nothing in Hill's work addresses more than one mode of failure and he should therefore **be spared the ignominy** of association with the many abstract mathematical failure theories for composite materials. Yet, in the UK and Europe, **Tsai's misinterpretation of Hill's theory** of anisotropic plasticity is referred to as the 'modified Hill theory
- It should now be evident that the innumerable abstract mathematical 'failure theories' for fibrous composites… **are beyond redemption as useful structural design tools**…”
- “It is clear, then, that the unstated simplifying assumptions of traditional composite failure theories **are so contradictory to basic laws of physics that the theories should be discarded**…”

# Discussion on failure criteria

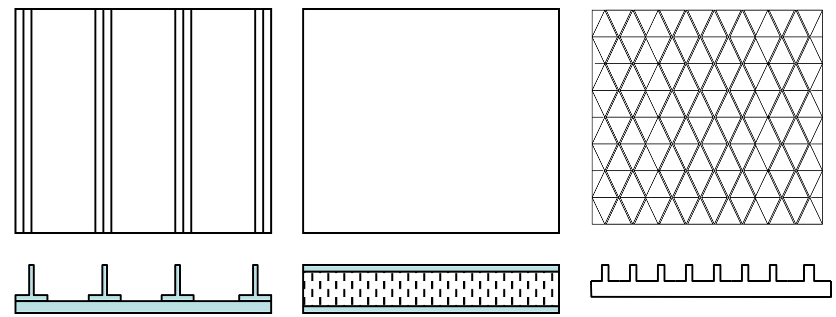
- The Puck and Larc3 failure criteria seem to be the best
- It is recommended to use whichever (legitimate) criterion one wishes provided it is supported by tests
- For this course, any (legitimate) criterion can be used

# Typical Scenario

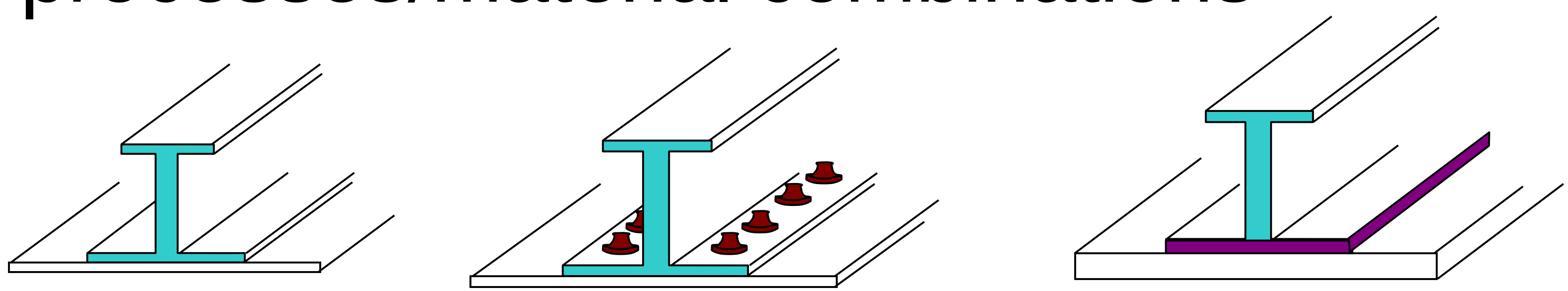
- Aircraft is designed for
  - flight maneuvers (take-off, climb, cruise, turn, approach, land, dive, etc.)
  - taxi
  - crash
  - static AND fatigue
- =>~2000 maneuvers

# Typical Scenario (cont'd)

- At each location, there are at least 3 design concepts
  - e.g. skins: stiffened panel
    sandwich panel
    isogrid



- For each concept you may want to consider, on the average, 3 fabrication processes/material combinations



# Number of trade studies

- For each location at the aircraft there are, therefore, 2000 x 3 x 3 = 18000 combinations **for a single layup**
- For a decent GA optimization run you need to consider at least 1000 generations with at least 15 design layups per generation or 15000 designs

# Analysis requirements

- Total number of analyses to be done (e.g. FE):

  = 18000 x 15000= **270 million analyses!!!**
- and this without including convergence checks, load redistribution runs, load changes during design, etc.
- Prohibitive to do with FE; need faster, reasonably accurate analysis methods

**more than 500 analyses/minute if you are working 24 hrs/day 365 days/year to finish them all in one year!**

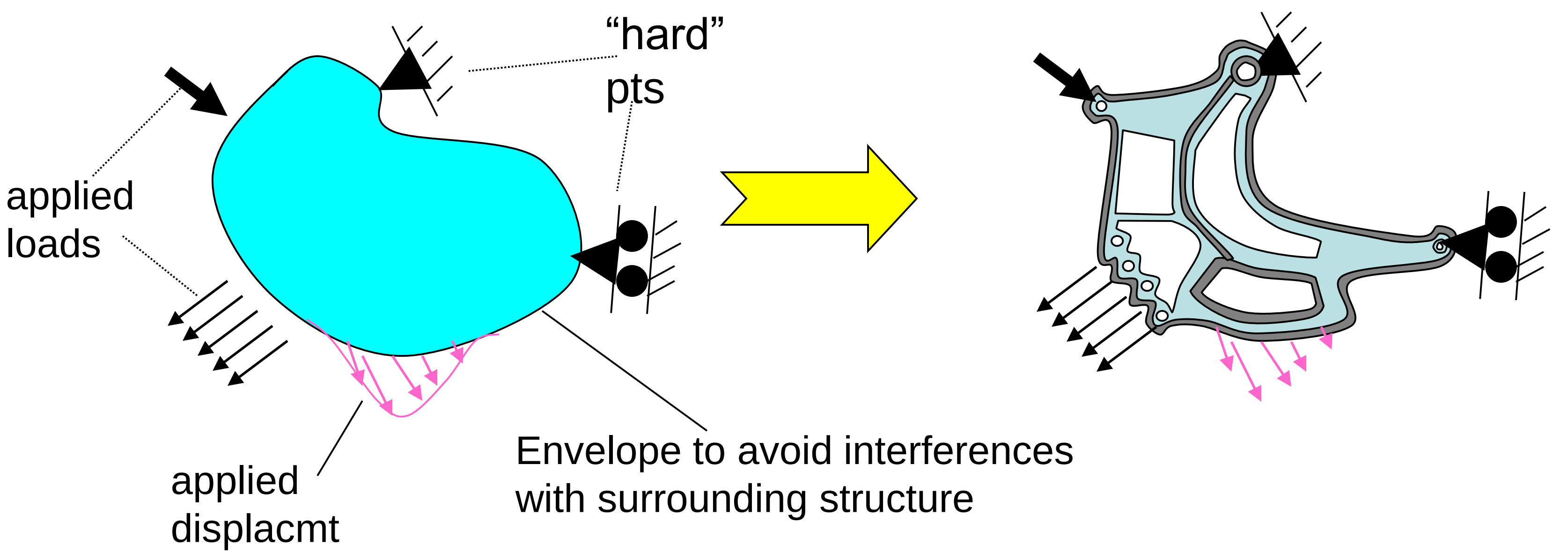
# Computer simulation limitations

The A30X has undergone several rounds of low-speed wind-tunnel tests using a subscale model. The trials have focused on airflow at landing, with speeds not exceeding Mach 0.2. Despite much work on computer-aided design, the tunnel tests are still seen as a cheaper way to gain needed airflow and loads data, the Airbus official says. Running computer models at different attitudes is costly; whereas in a wind tunnel, the model can simply be rotated and the impact measured, he adds.

R. Wall reporting in Air Transport, Feb 2009

# Structural Design Process –
## The analyst's perspective

- **Objective**: Given specific requirements, "create" structure that meets requirements and at the same time has certain "desirable" attributes



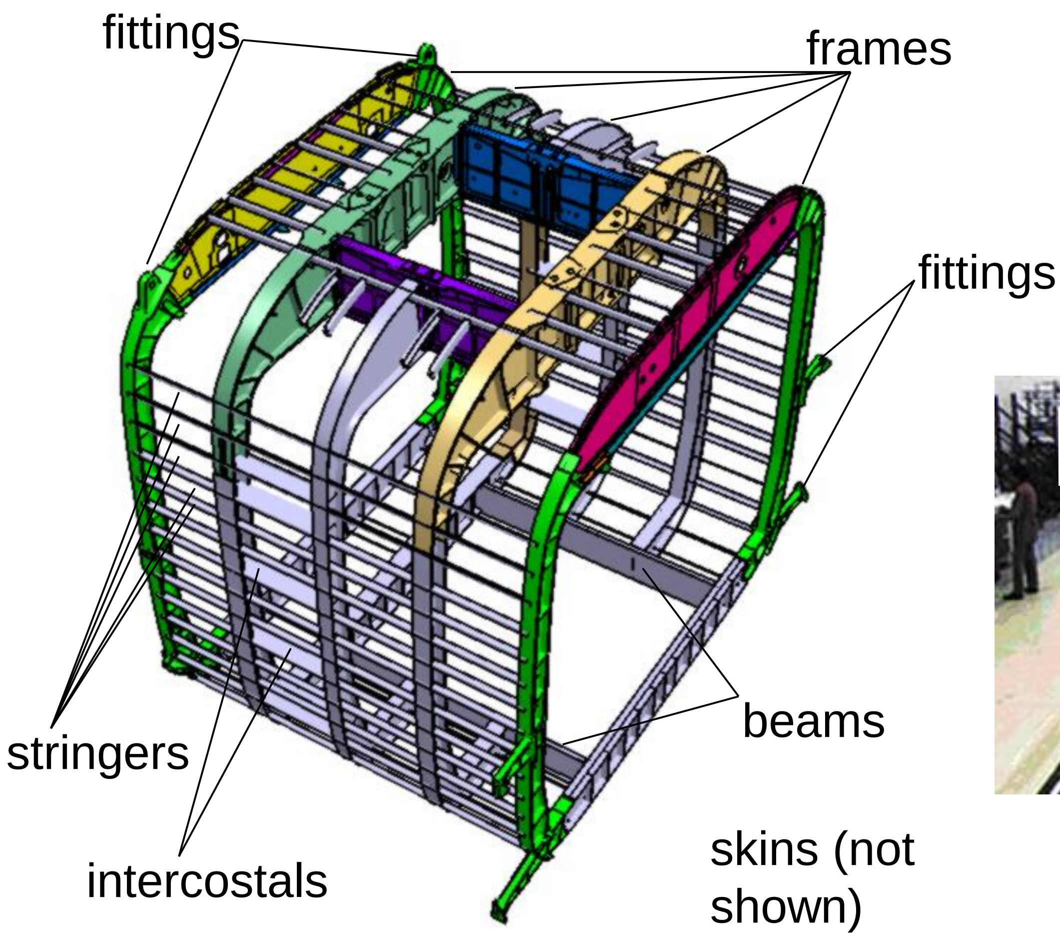
# Design Requirements

- Fit, form, function (fit within allowable envelope, have the appropriate matl/generic shape, perform the assigned function)
- Applied loads (static and fatigue)
- Corrosion resistance
- Natural frequency placement
- Thermal expansion coefficient
- Provide attachments for other structure (e.g. clips for electrical harnesses)
- Provide paths for other structure (e.g. ducts)
- Other

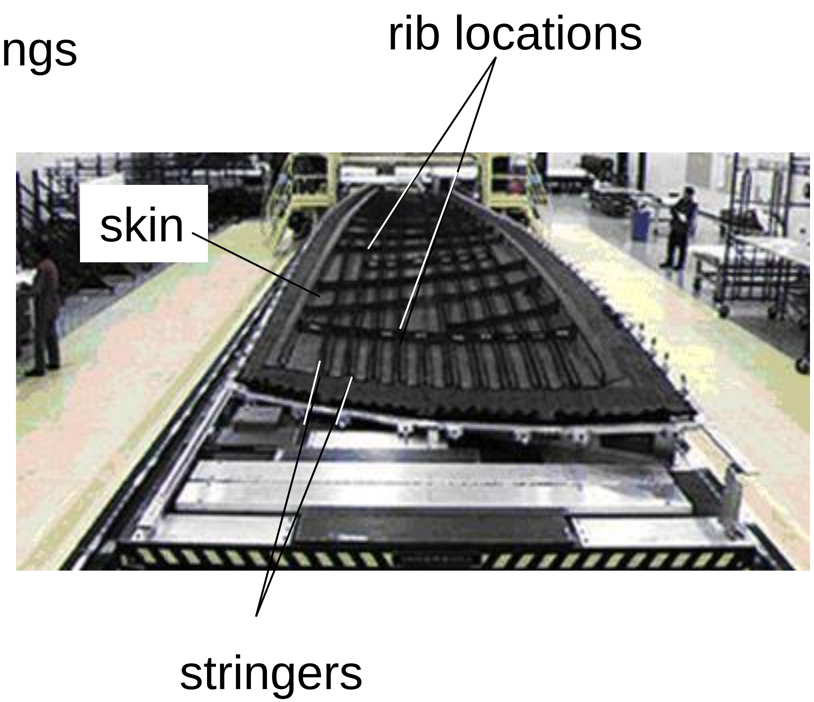
# Desirable Attributes

- Minimum weight
- Minimum cost (recurring, non-recurring, assembly,…)
- Low maintenance
- Replaceability across assemblies
- Specific natural frequency placement (e.g. helicopter fuselage vs main and tail rotor harmonics)
- Zero CTE (Space applications)
- Other
- Any combination of the above

# Airframe Structures




Fuselage

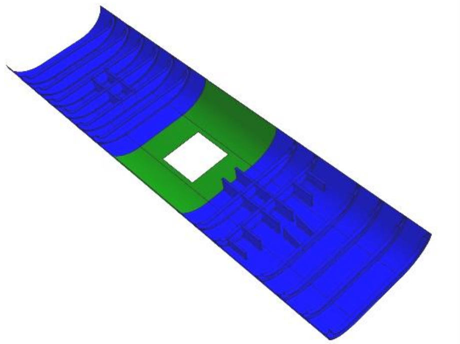



Wing

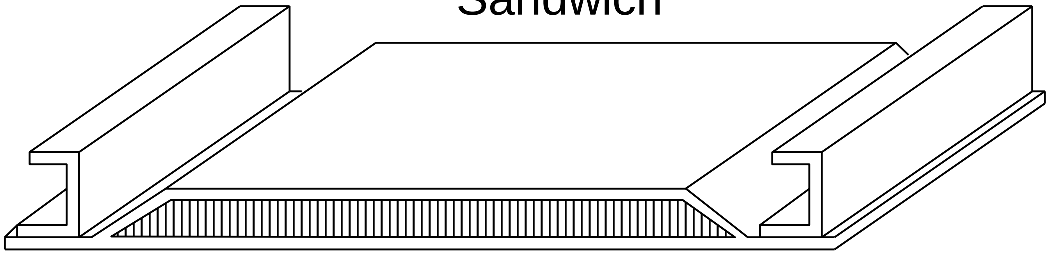
oea.larc.nasa.gov (ACT program)

# Skins

Monolithic



Sandwich



- **Failure Modes:**
  - Material strength
  - Notched strength
    - – OHT, OHC
    - – CAI, SAI, TAI
  - Buckling
  - Skin/Stringer separation
  - Delamination
  - Bearing, Bearing/Bypass

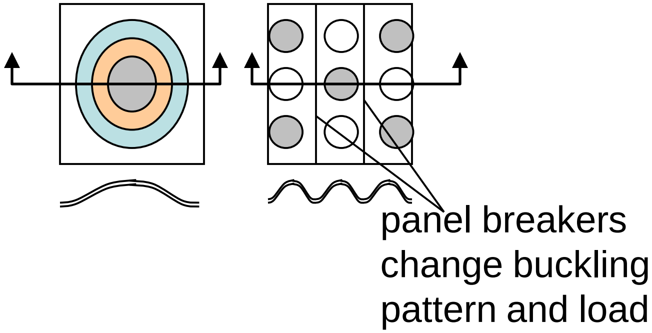
- **Failure Modes:**
  - Material strength (facesheet, core, adhesive)
  - Notched strength
    - – OHT, OHC
    - – CAI, SAI, TAI
  - Buckling
  - Wrinkling
  - Crimping
  - Intra-cell buckling
  - Delamination/Disbond
  - Rampdown

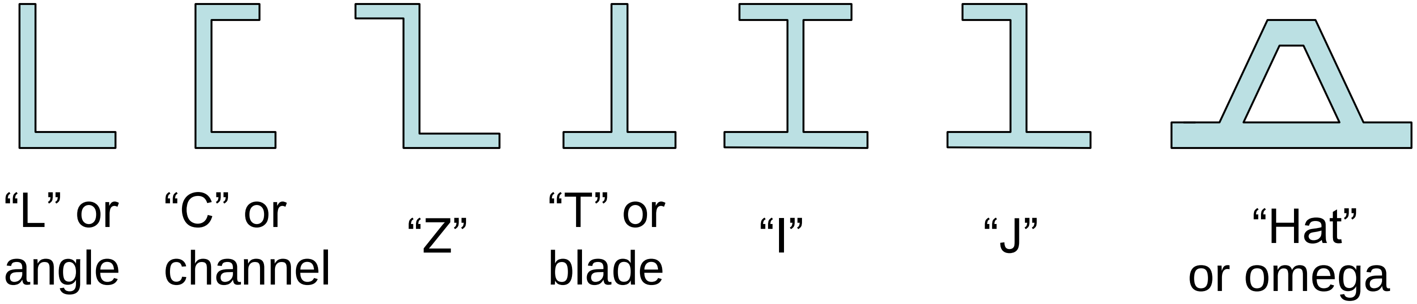
# Skins – Design/Manuf. Issues

- Stringer attachment to skin
  - Co-cured
  - Bolted
  - Secondarily bonded
- Frame attachment to skin and stringers
  - Co-cured
  - Bolted
  - Secondarily bonded
- Use of shear ties between frames/stringers/skins
- Cut-outs
  - cut after curing or molded in?
  - doubler or flange design (co-cured, bolted, bonded?)

# Stringers, Stiffeners, Panel Breakers

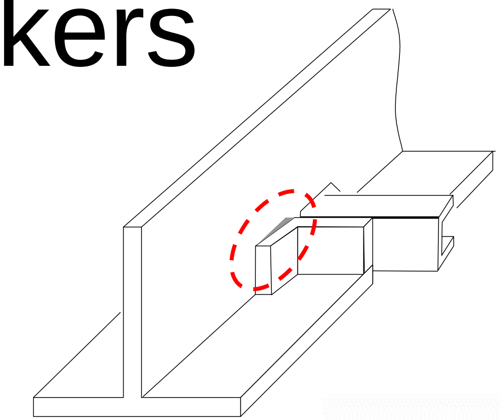
- carry longitudinal loads
- break-up skin in smaller panels to increase buckling load
- various cross-sections



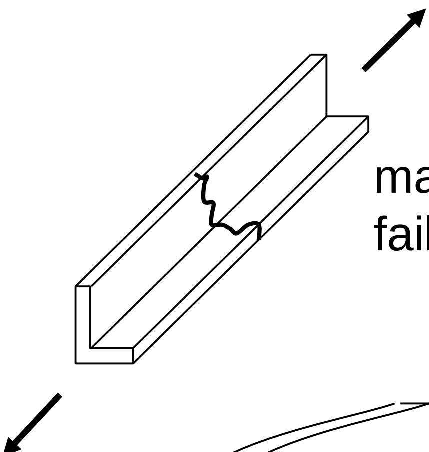



# Stringers, Stiffeners, Panel Breakers

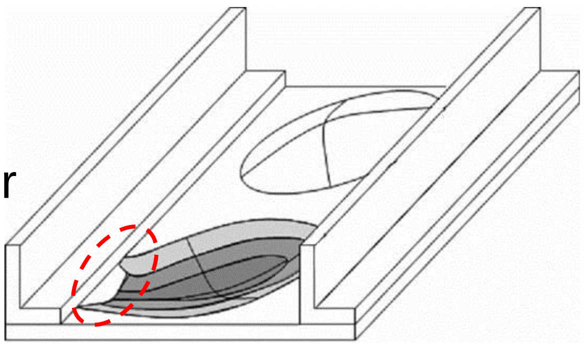
- Failure modes



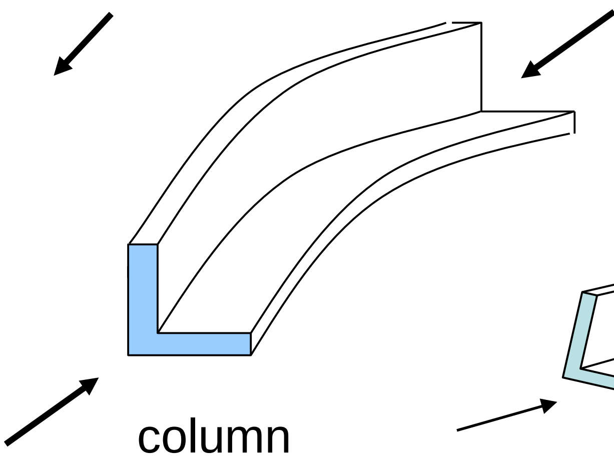
shear-tie failure



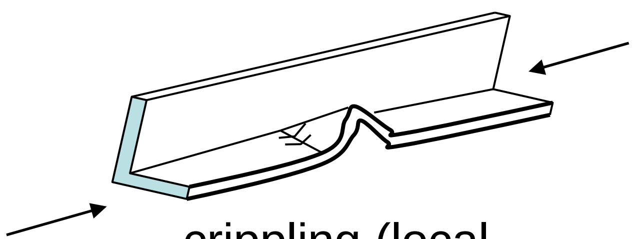
material failure



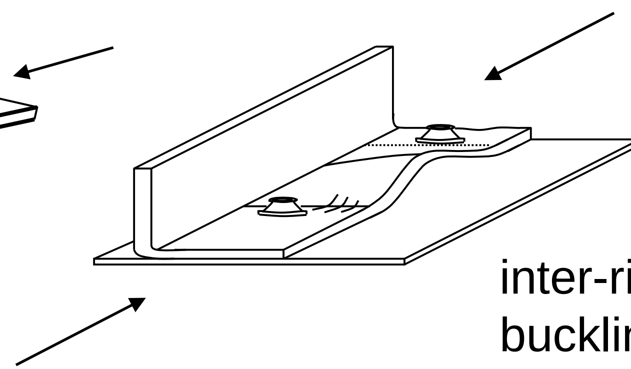
skin/stiffener separation



column buckling

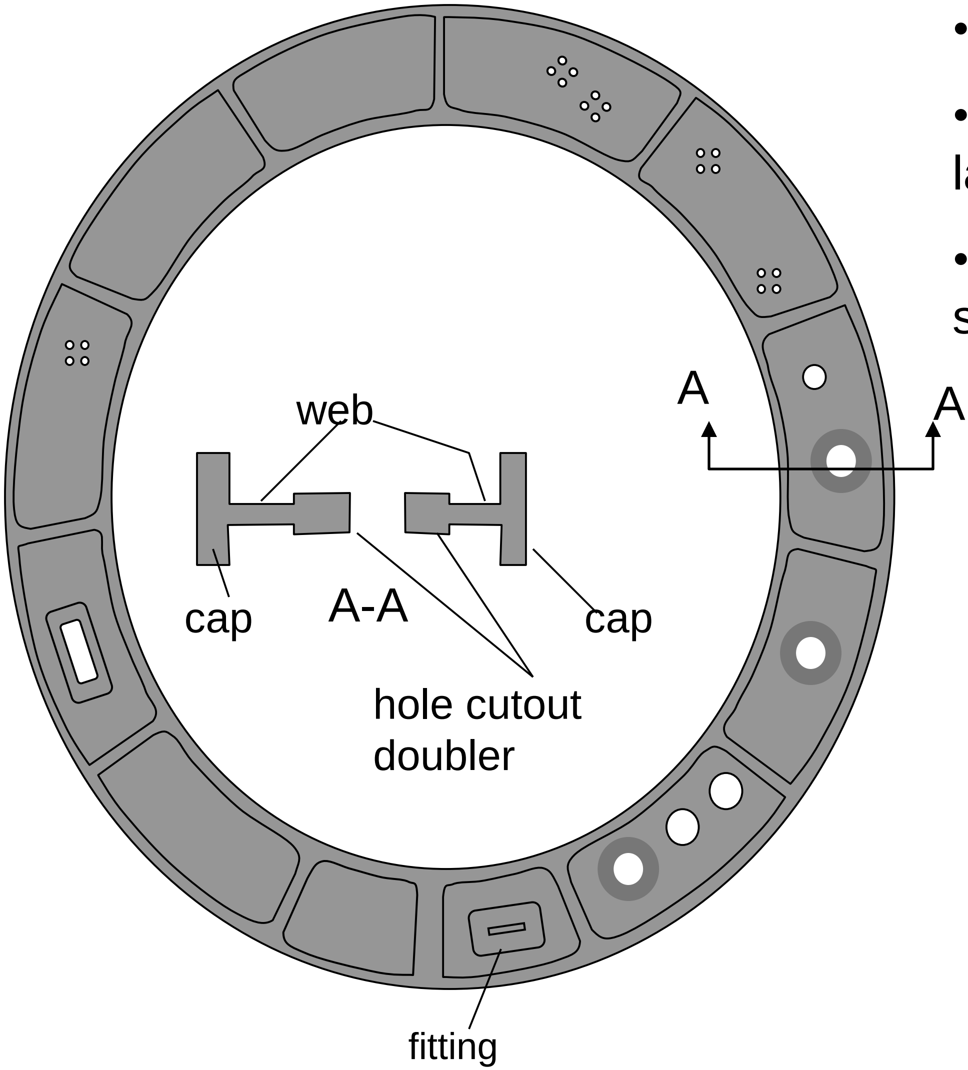


crippling (local buckling)



inter-rivet buckling

# Frames, Bulkheads

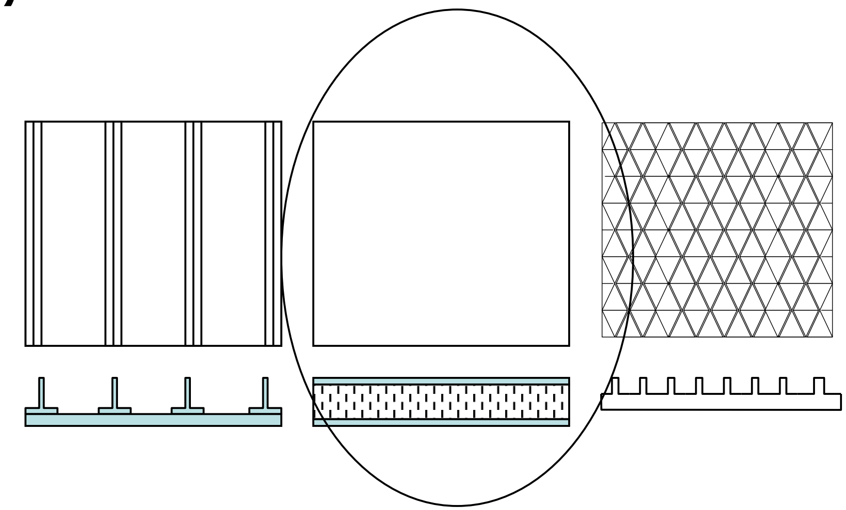



- maintain fuselage shape
- carry concentrated loads (e.g. landing gear)
- provide back-up support for other structure or equipment attachments

- Failure modes:
  - Material strength
  - Buckling of individual webs
  - Crippling of stiffeners or doublers
  - Crippling of caps

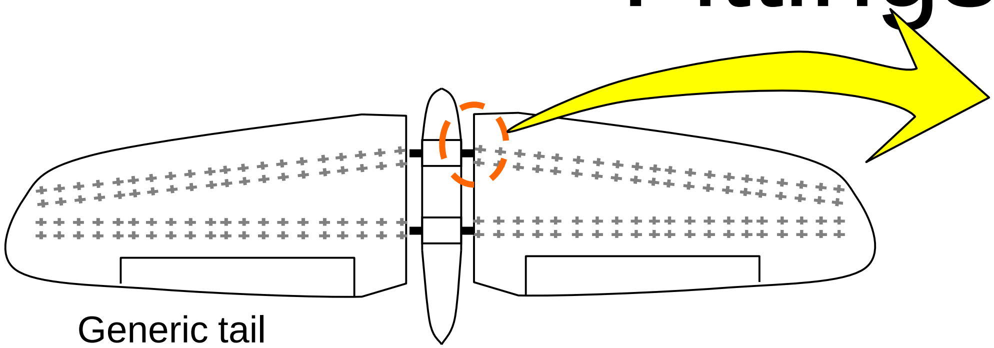
# Decks, Floors

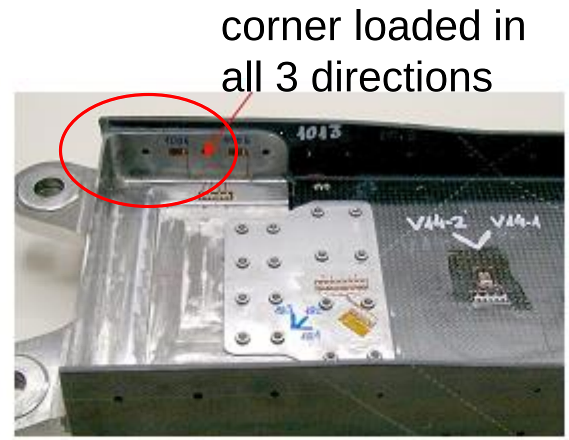
- Decks/floors:
  - need out-of-plane attachment capability



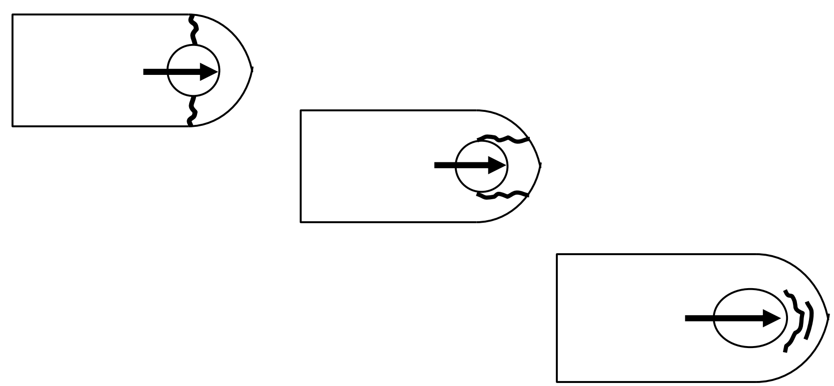

- bending and out-of-plane shear loads
  - seat loads
  - impact loads (dropped tools during maintenance etc.)

# Fittings



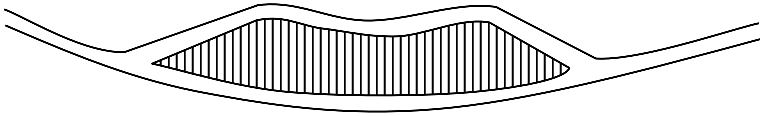




- Load transfer in all three directions
  - through-the-thickness reinforcement
- Lug failure modes
  - Net tension
  - Shear-out
  - Bearing

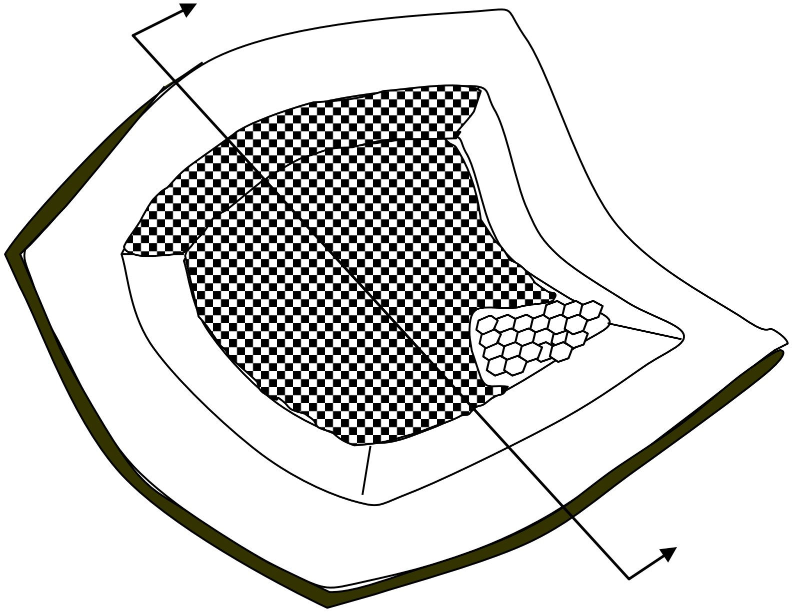


# Doors, Covers

- Aerodynamic pressure loads
- Skin shear loads
- Geometry complexity (compound curvature)
- Presence of cut-outs



cross-section



lightly loaded doors=> min gage!