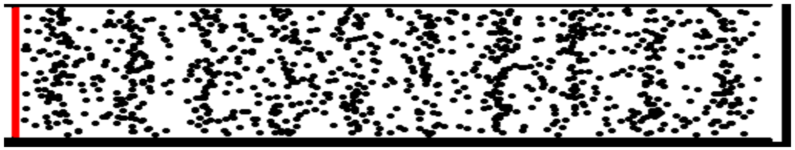
Fundamentals of Acoustics

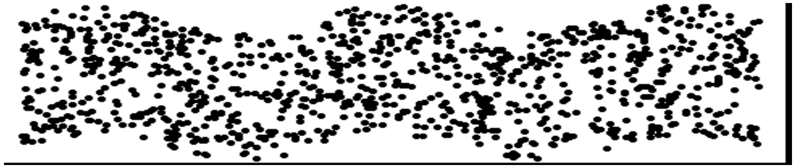
Sound is a wave phenomenon

- Propagating disturbance

- Longitudinal waves



Transverse waves

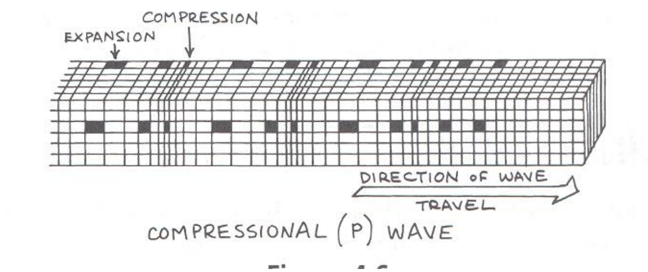


Surface waves

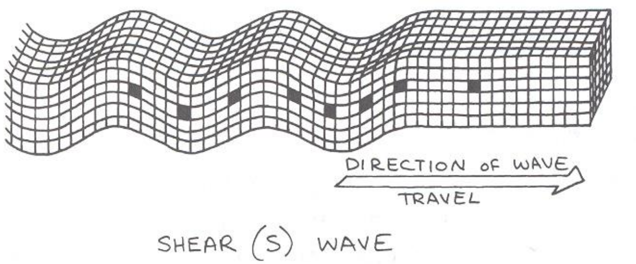
Recall: Slide from previous lecture on "elements marine geology"

Geo-acoustic modelling of the seafloor

P- and S- waves



(sound)



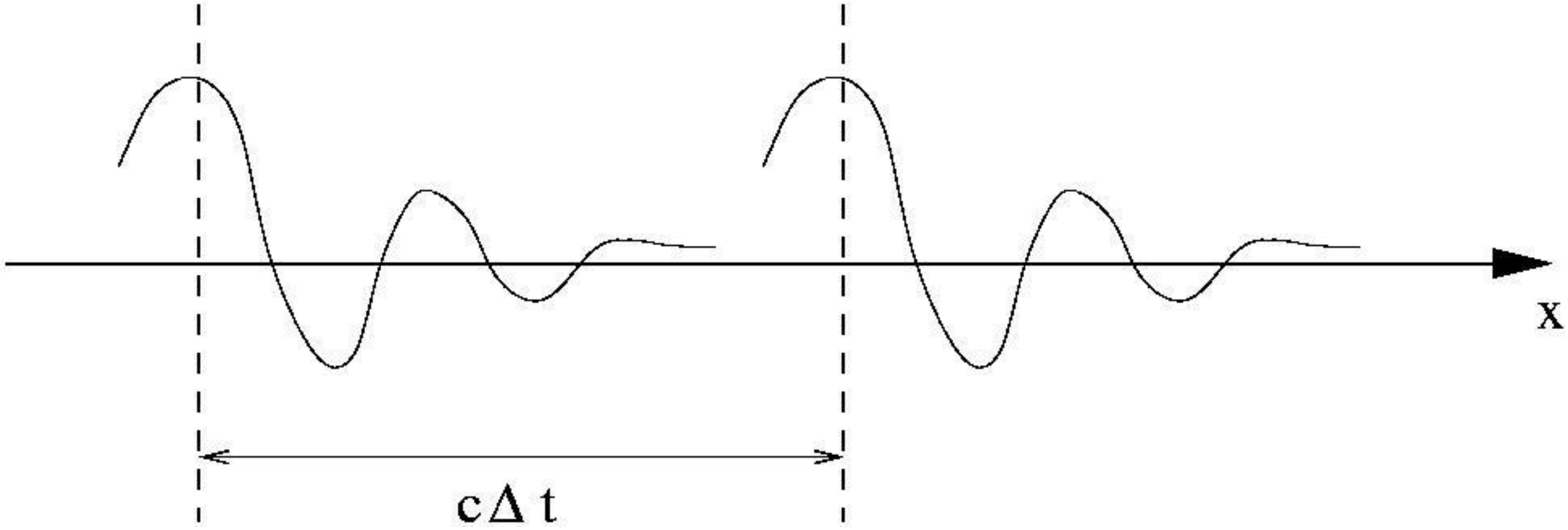
(only solids)

A *geo-acoustic model* of the seafloor comprises the following parameters

- compressional (P-) wave speed
- shear (S-) speed
- compressional wave attenuation
- shear wave attenuation
- density

as a function of depth.

Propagating disturbance



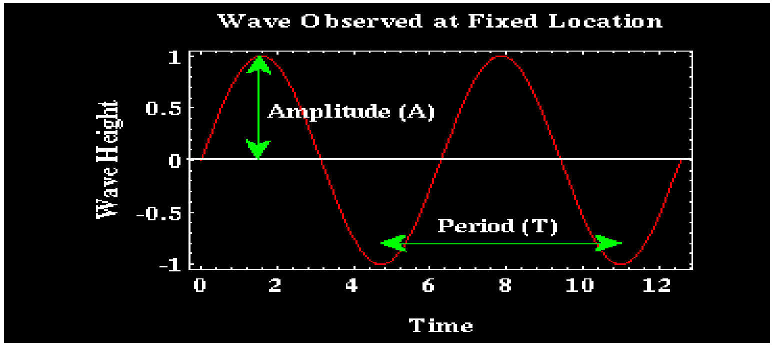
$$\xi(x,t) = f(x - ct)$$

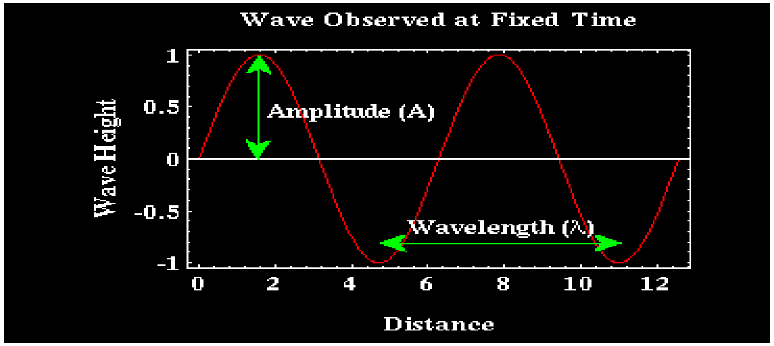
Plane harmonic waves

$$p(x,t) = p_0 \cos(kx - \omega t + \varphi)$$

p_0:	amplitude
k:	wavenumber
ω:	radial frequency
$kx - \omega t + \varphi$:	phase

Period and wavelength of the signal





$$c = \frac{\Delta x}{\Delta t} = \frac{\omega}{k} = \frac{\lambda}{T}$$

Typical values

- Audible sound: 20 Hz-20000Hz
- Sound speed in water: 1500 m/s
- Sound speed in air: 343 m/s
-

Frequency-range	Sonar
'0' - 1000 Hz	passive
5 kHz - 10 kHz	hull-mounted
≈ 40 kHz	active torpedo
≈ 100 kHz	echo-sounder, mine detection
≈ 400 kHz	mine classification
≈ 10 MHz	medical imaging

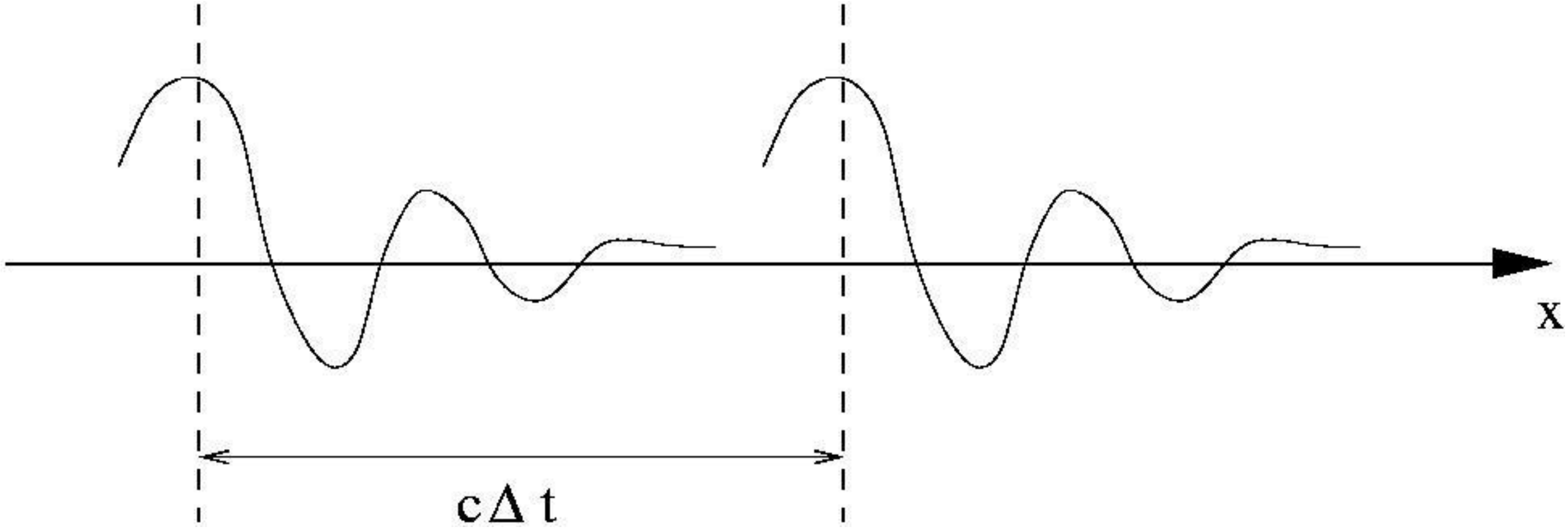
Sound level

- 2×10^{-5} Pa (hearing threshold) – 100 Pa (pain threshold)
  (1 atm = 101325 Pascals= 1.01325 bar)

- $$L = 20\,{}^{10}\log \frac{p}{p_{\text{ref}}}$$ dB

with p the effective pressure p_{rms}

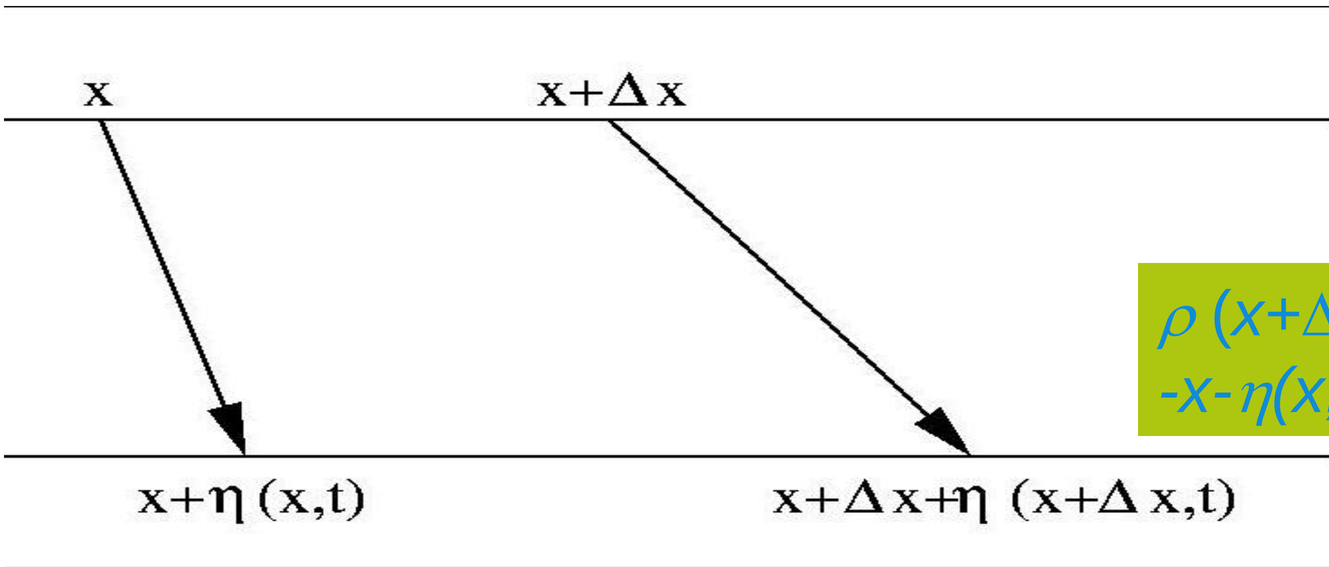
Propagating disturbance



$$\xi(x,t) = f(x - ct)$$

Wave equation for plane waves

- *The particle displacement* η
- *The density* ρ
- *The pressure* P
- *The particle speed* $v = \dfrac{\partial \eta}{\partial t}$
- *The particle acceleration* $a = \dfrac{\partial^2 \eta}{\partial t^2}$



$\rho_0 \Delta x$

$\rho\,(x+\Delta x+\eta(x+\Delta x,t)$
$-x-\eta(x,t))$

$$\rho_1 = -\rho_0 \frac{\partial \eta}{\partial x}$$

Variation of displacement with *x*: change in density

$$P = P(\rho)$$

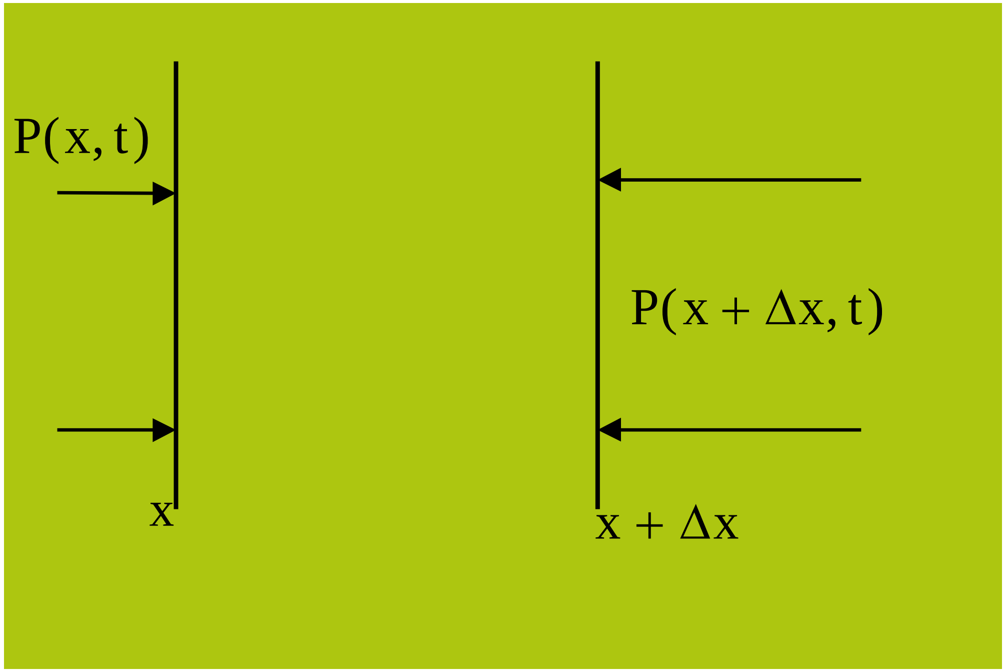
$$P_0 + p = f(\rho_0 + \rho_1) = f(\rho_0) + \rho_1 f^{'}(\rho_0) = P_0 + \rho_1 \left(\frac{dP}{d\rho} \right)_0$$

$$p = \rho_1 c^2$$

with

$$c^2 = \left(\frac{dP}{d\rho} \right)_0$$

Variation of density: change in pressure



$$\rho_0 \Delta x \frac{\partial^2 \eta(x,t)}{\partial t^2} = P(x,t) - P(x+\Delta x,t) = -\Delta x \frac{\partial P(x,t)}{\partial x}$$

Inequalities in pressure: movement of the medium

Wave equation

$$\rho_1 = -\rho_0 \frac{\partial \eta}{\partial x}$$

$$p = \left(\frac{dP}{d\rho}\right)_0 \rho_1$$

$$\rho_0 \frac{\partial^2 \eta(x,t)}{\partial t^2} = -\frac{\partial P(x,t)}{\partial x}$$

$$\frac{\partial^2 \eta}{\partial t^2} = c^2 \frac{\partial^2 \eta}{\partial x^2}$$

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2}$$

$$c^2 = \left(\frac{dP}{d\rho}\right)_0$$

Example

- Harmonic plane wave
- f = 1000 Hz
- Sound pressure level = 100 dB
- Undisturbed density = 1.21 kg/m^3, c = 343 m/s

$$p_{rms} = 10^{\frac{100}{20}} \cdot p_{ref}$$

$$= 10^5 \cdot 2 \cdot 10^{-5} = 2\ \mathrm{Pa}$$

$\eta \approx 10^{-6}$ m

$$\lambda = \frac{c}{f} = \frac{343}{1000} = 0.34\ \mathrm{m}$$

$$v = \omega \eta_{rms} = 5 \cdot 10^{-3}\ \frac{\mathrm{m}}{\mathrm{s}}$$

Sound speed for ideal gas

$$c^2 = \left(\frac{dP}{d\rho}\right)_0$$

$$P = constant\ \rho^\gamma$$

For adiabatic processes

$$c^2 = \left(\frac{dP}{d\rho}\right)_{(0)} = \text{constant}\ \gamma\rho^{\gamma-1} = \frac{\gamma}{\rho}P$$

use

$$PV = RT$$

$$c = \sqrt{\frac{\gamma RT}{\rho V}} = \sqrt{\frac{\gamma RT}{M}}$$

air

$$c = \sqrt{\frac{1.4\cdot 8.31\cdot 273}{28.8\cdot 10^{-3}}} = 332\ \frac{\text{m}}{\text{s}}$$

Sound speed for liquids and solids

$$c = \sqrt{\frac{K}{\rho_0}}$$

K is the bulk modulus=reciprocal of the compressibility

Pure water : K = 2x109 N/m^2, ρ_0 = 1000 kg/m^3 : c = 1414 m/s

$$c = \sqrt{\frac{E}{\rho_0}}$$

E is Young's elasticity modulus

Steel E = 2x10^{11} N/m^2 and ρ_0 = 7800 kg/m^3 : c = 5100 m/s.

Empirical relation:

$$c = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + (1.34 - 0.01T)(S - 35) + 0.017z$$

with
T the temperature in °C
z the depth in m
S the salinity (salt content) in ppt

Acoustic intensity

Definition:
mean rate of flow of energy through a unit area normal to the direction of the propagation

$$I = \left\langle \frac{F\partial\eta}{A\partial t} \right\rangle = < pv >$$

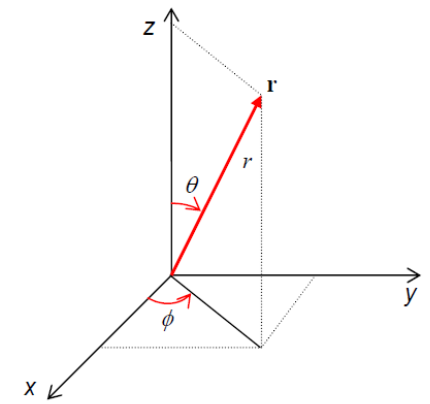
Harmonic wave: $\frac{p}{v} = \rho_0 c$ ← ACOUSTIC IMPEDANCE

$$I = \left\langle \frac{p^2}{\rho_0 c} \right\rangle = \frac{{p_{rms}}^2}{\rho_0 c}$$

Spherical waves

The wave equation

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$



$$r = \sqrt{x^2 + y^2 + z^2},$$

$$\theta = \arccos\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right),$$

$$\phi = \arctan\left(\frac{y}{x}\right).$$

The wave equation in spherical coordinates

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rp) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial p}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 p}{\partial\varphi^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

For a spherical wave, the pressure is independent of φ and θ:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rp) = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

or:

$$\frac{\partial^2}{\partial r^2}(rp) = \frac{1}{c^2}\frac{\partial^2 (rp)}{\partial t^2}$$

Spherical waves, continued

$$r\,p(r,t) = f(r-ct) + g(r+ct)$$

$$p(r,t) = \frac{A}{r} f(r-ct)$$

Harmonic solution

$$p(r,t) = \frac{A}{r} e^{i(kr-\omega t)}$$

Acoustic impedance, spherical harmonic waves

$$\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial r}$$

$$\frac{\partial v}{\partial t} = -i\omega v$$

$$v = \frac{1}{i\omega\rho_0}\frac{\partial p}{\partial r}$$

with

$$p(r,t) = \frac{A}{r}e^{i(kr-\omega t)}$$

$$v(r,t) = \frac{p(r,t)}{\omega\rho_0}\left[\frac{i}{r}+k\right]$$

$$Z = \frac{p(r,t)}{v(r,t)} = \rho_0 c\left[\frac{k^2r^2}{1+k^2r^2} - i\frac{kr}{1+k^2r^2}\right]$$

Acoustic impedance, spherical harmonic waves

$$Z = \frac{p(r,t)}{v(r,t)} = \rho_0 c\left[\frac{k^2r^2}{1+k^2r^2} - i\frac{kr}{1+k^2r^2}\right]$$

$kr >> 1$ ($r >> \lambda$): $Z = \rho_0 c$

$kr << 1$ ($r << \lambda$): $Z = -i\rho_0 ckr$

Spherical wave intensity

$$I = \left\langle \frac{F \partial \eta}{A \partial t} \right\rangle = < pv >$$

$kr >> 1$ ($r >> \lambda$): $Z = \rho_0 c$

Harmonic wave:

$$\frac{p}{v} = \rho_0 c$$

$$I = \left\langle \frac{p^2}{\rho_0 c} \right\rangle = \frac{p_{rms}{}^2}{\rho_0 c}$$

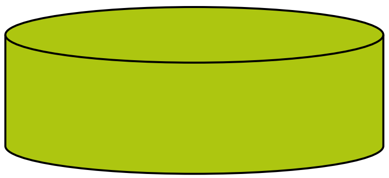
$$p_{rms}^2 = \frac{p_0^2}{2} = \frac{A^2}{2r^2}$$

$$I \sim \frac{1}{r^2}$$

In accordance with the conservation of energy law!

Cylindrical waves

$$I(r) = \frac{p_0^2}{2\rho_0 c}$$



$$P = 2\pi r H \frac{p_o^2}{2\rho_0 c}$$

$$p_0 \sim \frac{1}{\sqrt{r}}$$

Harmonic cylindrical wave:

$$p(r,t) = \frac{A}{\sqrt{r}} e^{i(kr-\omega t)}$$

Geometrical loss

Plane wave

$$p(x,t) = Ae^{i(kx-\omega t)}$$

$$I = \frac{A^2}{2\rho_0 c}$$

Spherical wave

$$p(r,t) = \frac{A}{r}e^{i(kr-\omega t)}$$

$$I = \frac{A^2}{2\rho_0 c r^2}$$

Cylindrical wave

$$p(r,t) = \frac{A}{\sqrt{r}}e^{i(kr-\omega t)}$$

$$I = \frac{A^2}{2\rho_0 c r}$$

Up to now only geometrical loss

Sound absorption

Plane wave

$$p(r,t) = e^{i(kx-\omega t)-\alpha x}$$

$$-20\,{}^{10}\log e^{-\alpha x} = \alpha x(20\,{}^{10}\log e) = 8.686\ \alpha x$$

x = 1 km = 1000 m: α (in dB/km) = 8686 α (m^{-1})

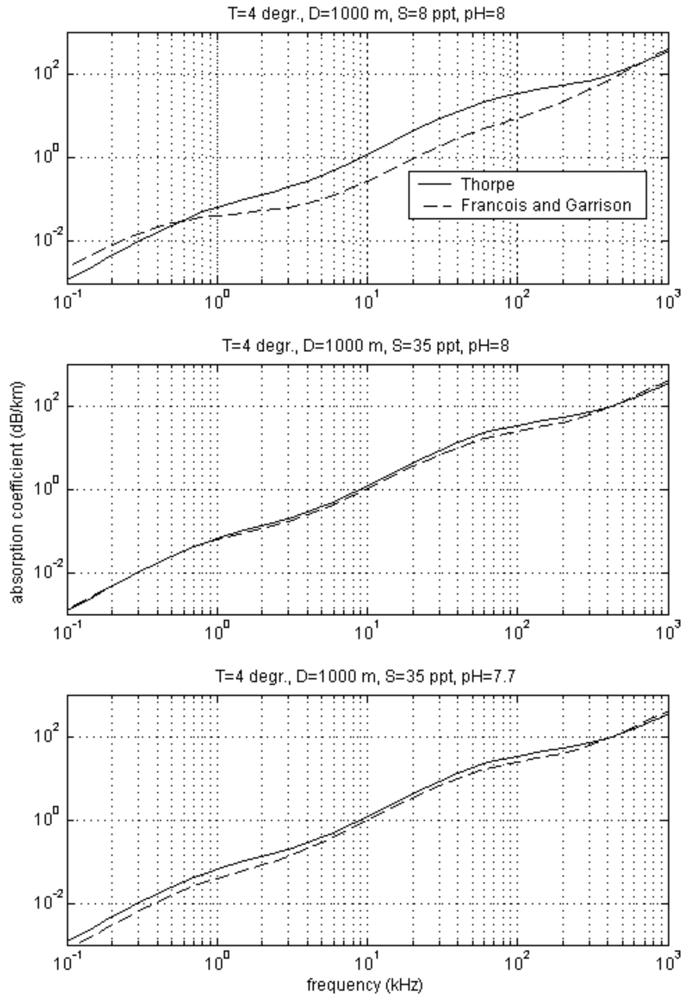
Absorption coefficient

Thorpe:

$$\alpha = \frac{0.11 f^2}{1 + f^2} + \frac{44 f^2}{4100 + f^2} + 0.0003 f^2$$

f = frequency (kHz)
α = absorption coefficient (dB/km)

Absorption coefficient



Sound absorption

Frequency (kHz)	α (dB/km) (Thorpe)	r_{10dB} (km)
0.1	0.0012	8333
1	0.07	143
10	1.2	8.3

Propagation loss

$$PL = 10\,{}^{10}\log\frac{I(1)}{I(r)} = 10\,{}^{10}\log\frac{p_{rms}^2(1)}{p_{rms}^2(r)} = 10\,{}^{10}\log\frac{p_0^2(1)}{p_0^2(r)}$$

- Geometrical spreading loss
- Absorption

$$p_0 \sim \frac{e^{-\alpha r}}{r}$$

$$PL = 20\,{}^{10}\log(re^{\alpha r}) = 20\,{}^{10}\log(r) + \alpha r\, 20\,{}^{10}\log(e)$$

Propagation loss

spherical

$$PL(dB) = 60 + 20\,{}^{10}\log\left[r(\mathrm{km})\right] + \alpha(\mathrm{dB/km})\, r(\mathrm{km})$$

cylindrical

$$PL(dB) = 30 + 10\,{}^{10}\log\left[r(\mathrm{km})\right] + \alpha(\mathrm{dB/km})\, r(\mathrm{km})$$

Environment?

Example: Losses
$f = 6$ kHz

r **(km)**	***PL*** **(dB)**
0.1	**60-20+0.05 = 40.05**
1	**60+ 0+0.5 = 60.5**
10	**60+20+5 = 85**
100	**60+40+50 = 150**

Reflection, refraction and transmission

$$p(x,y,z,t) = p(\vec{r},t) = e^{i(\vec{k}.\vec{r}-\omega t)}$$

with

$$\vec{k}.\vec{r} = k_x x + k_y y + k_z z$$

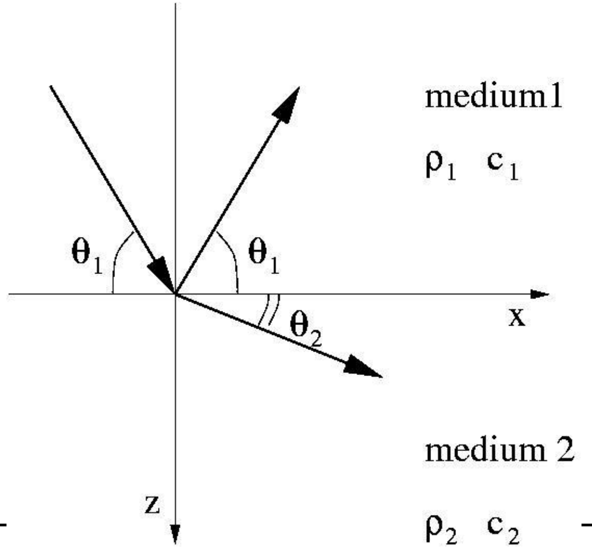
$$p_i = e^{ik_1(x\cos\theta_1 + z\sin\theta_1)}$$

$$k_1 = \frac{\omega}{c_1} = \left\|\vec{k}_1\right\|$$

$$p_r = Re^{ik_1(x\cos\theta_1 - z\sin\theta_1)}$$

$$p_t = Te^{ik_2(x\cos\theta_2 + z\sin\theta_2)}$$

$$k_2 = \frac{\omega}{c_2} = \left\|\vec{k}_2\right\|$$



Boundary conditions

$Z = 0$:

(I) *continuity of pressure* $p_i + p_r = p_t$

$$(1+R) = Te^{i(k_2\cos\theta_2 - k_1\cos\theta_1)x}$$

$$k_2\cos\theta_2 - k_1\cos\theta_1 = 0$$

Snell's law: $$\frac{\cos\theta_2}{c_2} = \frac{\cos\theta_1}{c_1}$$

$$(1+R) = T$$

Boundary conditions

Recall:
$$\rho_0 \frac{\partial^2 \eta}{\partial t^2} = \rho_0 \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x}$$
$$\frac{\partial v}{\partial t} = -i\omega v$$

$Z = 0$:

Giving:
$$v = \frac{1}{i\omega\rho_0}\frac{\partial p}{\partial x}$$

(II) *continuity of the normal component of the particle velocity*

$$\frac{1}{i\omega\rho_1}\frac{\partial(p_i + p_r)}{\partial z} = \frac{1}{i\omega\rho_2}\frac{\partial p_t}{\partial z}$$

$$\rightarrow \quad 1 - R = T\frac{\rho_1 c_1 \sin\theta_2}{\rho_2 c_2 \sin\theta_1}$$

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$T = \frac{2Z_2}{Z_2 + Z_1}$$

$$Z_1 = \frac{\rho_1 c_1}{\sin\theta_1} \qquad Z_2 = \frac{\rho_2 c_2}{\sin\theta_2}$$

Typical values for geo-acoustic parameters

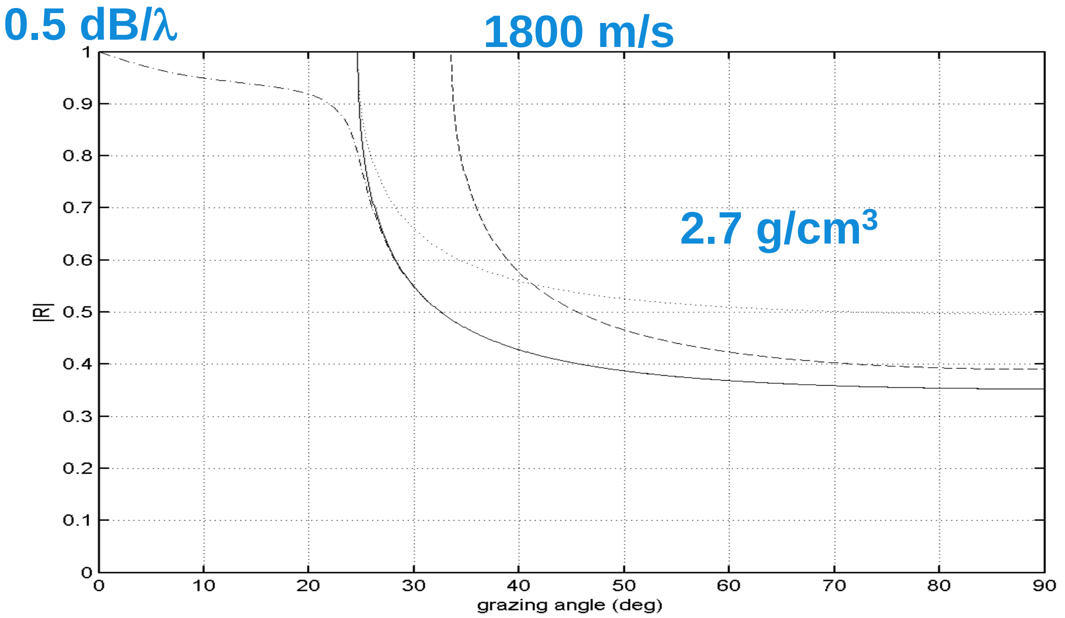
Bottom type	porosity (%)	ρ (g/cm^3)	c (m/s)
clay	80	1.2	1470
Clayey silt	70	1.5	1515
Fine sand	45	1.95	1725
siltstone	-	2.4	3800
basalt	-	2.5	4800

Important note: for ocean bottom materials a good approximation is that the absorption coefficient increases linearly with frequency

$$\alpha(dB/\lambda) = 8.686\lambda(m)\alpha(m^{-1})$$ **: independent on frequency!**

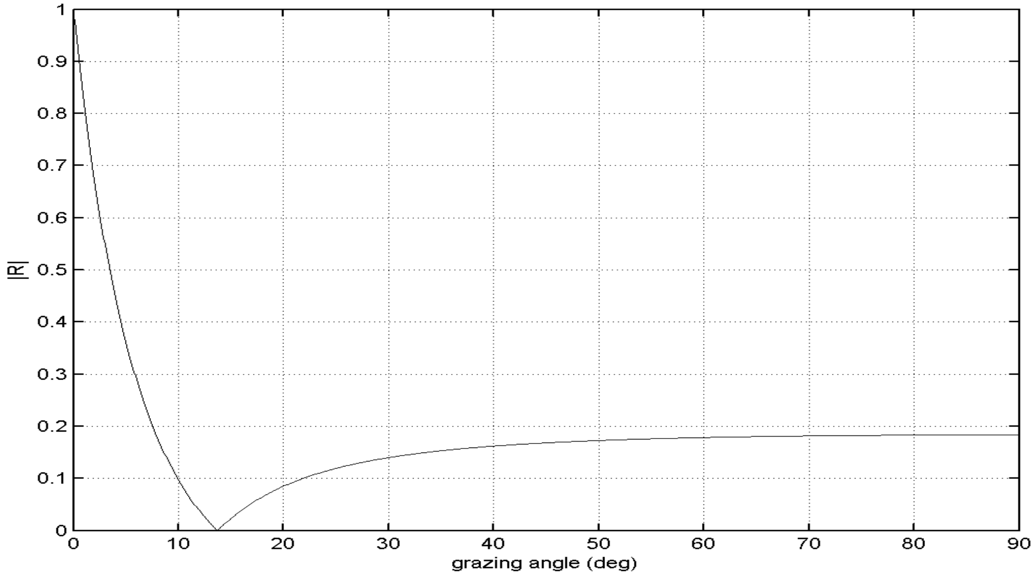
Example:

$$\rho_2 = 1.9 \text{ g/cm}^3,\ \alpha = 0 \text{ dB}/\lambda,\ c_2 = 1650 \text{ m/s}$$



Example

$$\rho_2 = 1.5\ \mathrm{g/cm^3},\ \alpha = 0\ \mathrm{dB}/\lambda,\ c_2 = 1450\ \mathrm{m/s}$$

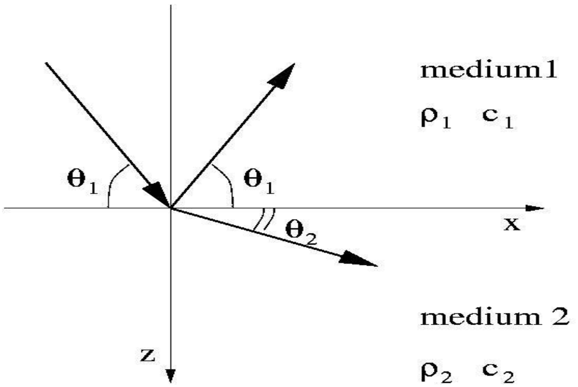


Critical angle: total reflection

If $c_2 > c_1$:

$|R| = 1$ for $0 < \theta < \theta_c$

$R < 1$ for $\theta > \theta_c$



$$\theta_c = \arccos\left(\frac{c_1}{c_2}\right)$$

Angle of intromission

***R* = 0**

$$\theta_0 = \arctan\sqrt{\frac{1-\left(\dfrac{c_2}{c_1}\right)^2}{\left(\dfrac{\rho_2 c_2}{\rho_1 c_1}\right)^2-1}}$$

θ_0 **exists if**

$\rho_2 c_2 > \rho_1 c_1$ and $c_2 < c_1$ muddy ocean bottoms;

$\rho_2 c_2 < \rho_1 c_1$ and $c_2 > c_1$ never occurs, is not physical.

General wave phenomena

- Diffraction
- Reflection
- Scattering
- Refraction

Exercises

1
- Harmonic plane wave
- *f* = 1000 Hz
- Sound pressure level = 0 dB
- Undisturbed density = 1.21 kg/m3, c = 343 m/s

Calculate the particle displacement

2 Plane harmonic waves, frequency of 1000 Hz, in air and in water. Intensities are identical.

Sound pressure level of the wave in air = 120 dB.

Calculate the sound pressure level in water.

3 Calculate the intensity at the pain threshold in air

4 Consider an echosounder at 100 kHz, 4 km of water depth. What is the loss expected due to absorption?

Exercise 1

- Harmonic plane wave
- F = 1000 Hz
- Sound pressure level = 0 dB
- Undisturbed density = 1.21 kg/m^3, c = 343 m/s

Estimate the particle displacement

$$p_{rms} = 10^{\frac{0}{20}} \cdot p_{ref} = p_{ref} = 2 \cdot 10^{-5}\ \mathrm{Pa}$$

$$\eta_{rms} = \frac{p_{rms}}{\rho_0 c \omega} = \frac{2 \cdot 10^{-5}}{1.21 \cdot 343 \cdot 2\pi \cdot 1000} \approx 10^{-11}\ \mathrm{m}$$

Ten times smaller than the radius of an atom!

Exercise 2

Plane harmonic wave, frequency of 1000 Hz, in air and in water.
Intensities are identical.

Sound pressure level of the wave in air = 120 dB.

Calculate the sound pressure level in water.

$$\left(\frac{p_{rms}{}^2}{\rho_0 c}\right)_{air} = \left(\frac{p_{rms}{}^2}{\rho_0 c}\right)_{water}$$

$$p_{rms,water} = \sqrt{\frac{1000 \cdot 1500}{1.21 \cdot 343}}\, p_{rms,air} = 60\, p_{rms,air}$$

$$120 = 20\,{}^{10}\log\left(\frac{p_{rms}}{p_{ref}}\right)_{air} \Leftrightarrow p_{rms,air} = 10^6\, p_{ref,air} = 20 \text{ Pa}$$

$$p_{rms,water} = 1200\, Pa$$

$$20\,{}^{10}\log\left(\frac{1200}{10^{-6}}\right) = 182 \text{ dB}$$

Exercise 3

Calculate the intensity at the pain threshold in air

$$\frac{p^2}{\rho_0 c} = \frac{100^2}{1.21 \cdot 343} = 24\,\frac{\mathrm{Watt}}{\mathrm{m}^2}$$

Exercise 4

At 100 kHz: absorption coefficient of 35 dB/km

Total loss due to absorption: 4*2*35 = 280 dB