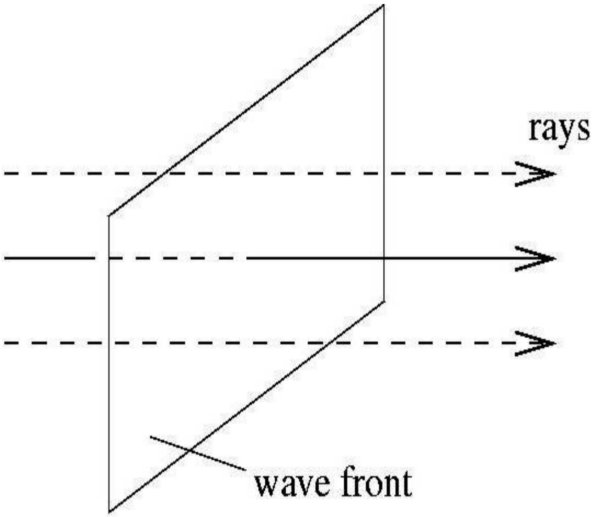
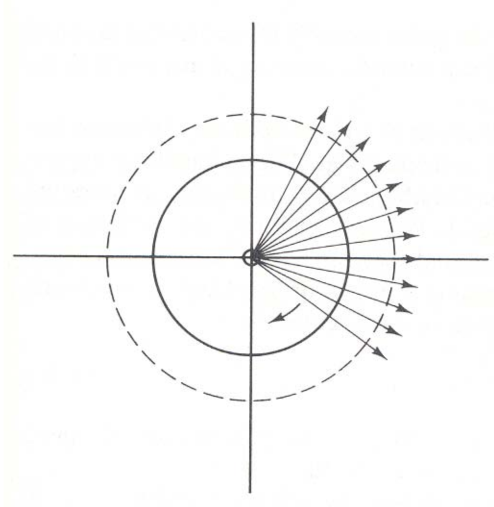
Ray acoustics

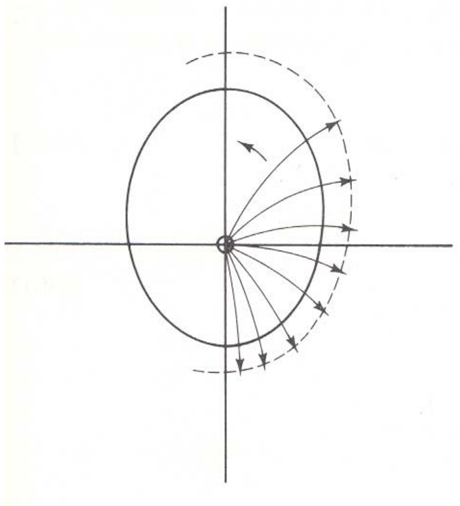
Wave fronts and rays



Plane wave



Spherical wave
homogenous medium



Spherical wave
inhomogeneous medium

Refraction of a sound ray

Sound speed *c* varies linearly with depth *z:*

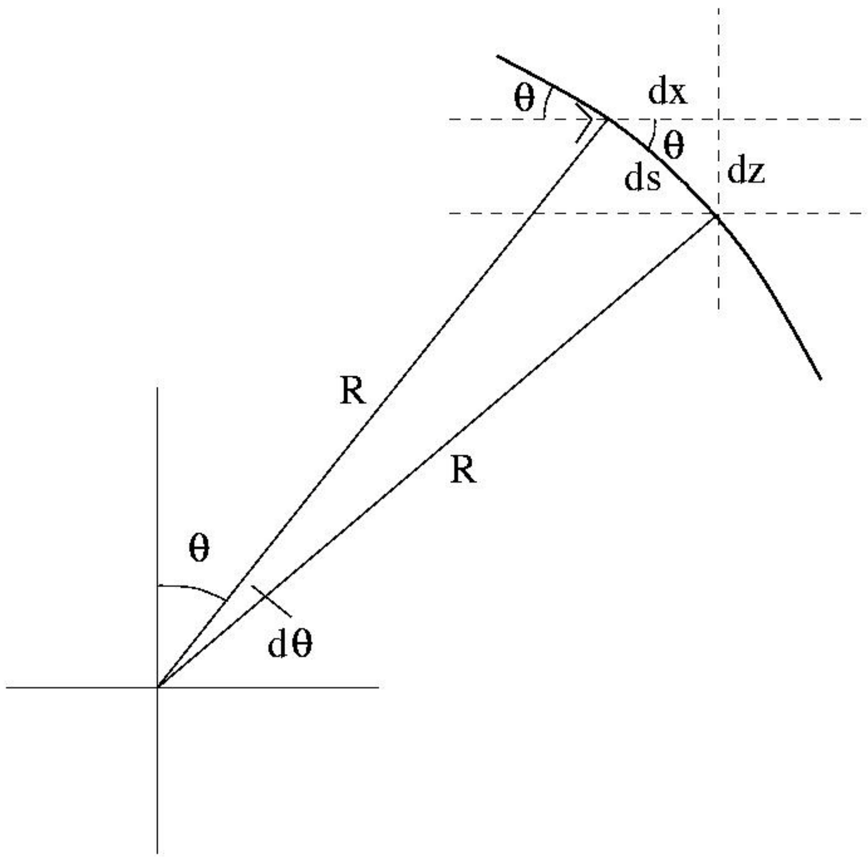
$$c(z) = c_0 + g\,z \Leftrightarrow \frac{dc}{dz} = g$$

Snell's law:

$$\frac{\cos\theta(z)}{c(z)} = \frac{\cos\theta_0}{c_0} = \frac{1}{c_0} \Leftrightarrow \frac{dc}{dz} = -c_0 \sin\theta \frac{d\theta}{dz}$$

$$dz = \frac{-c_0 \sin\theta}{g} d\theta$$

Motion of a point mass on a circle



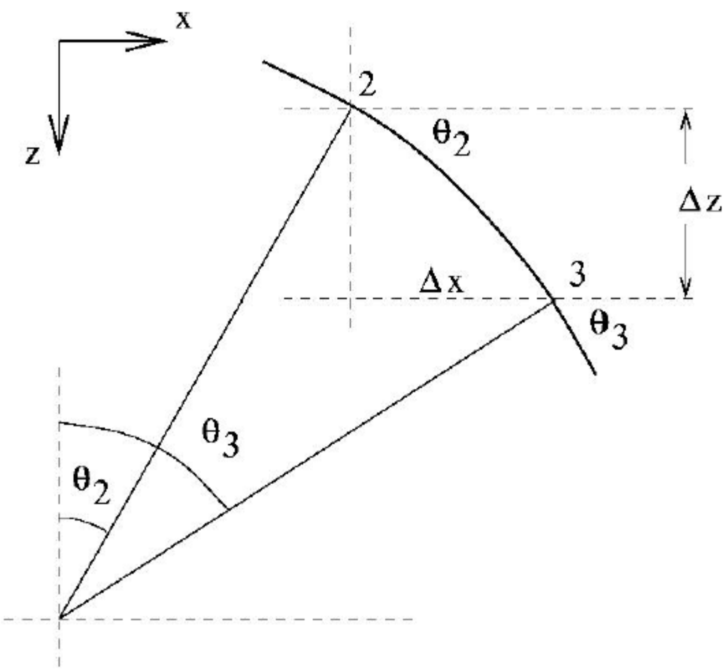
$$dz = R\sin\theta d\theta$$

Combine with

$$dz = \frac{-c_0 \sin\theta}{g} d\theta$$

$$R = -\frac{c_0}{g} = -\frac{c(z)}{g\cos\theta(z)}$$

Additional formulas



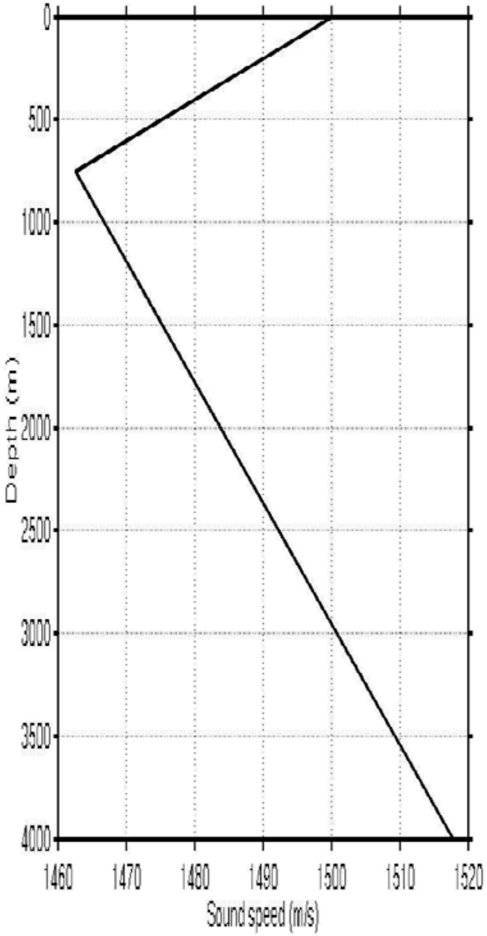
$$dz = R\sin\theta\, d\theta$$

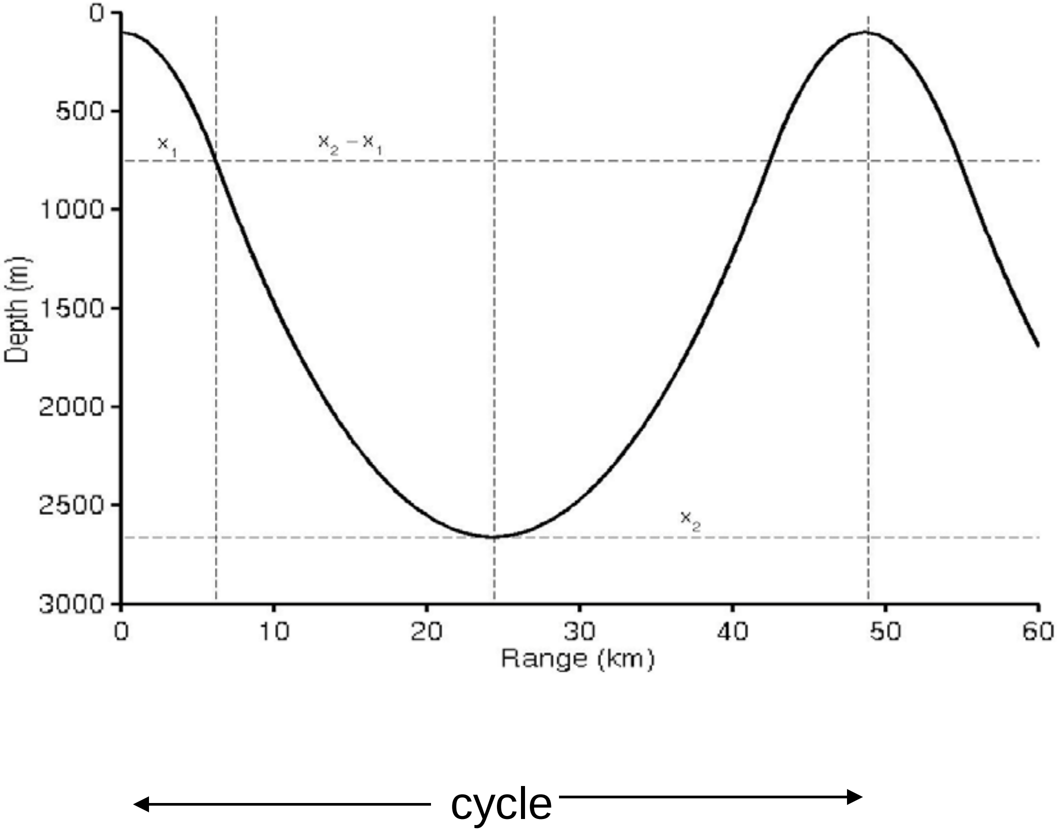
$$dx = R\cos\theta\, d\theta$$

$$\Delta z = \int_{\theta_2}^{\theta_3} dz = \int_{\theta_2}^{\theta_3} R\sin\theta\, d\theta = R\left(\cos\theta_2 - \cos\theta_3\right)$$

$$\Delta x = \int_{\theta_2}^{\theta_3} dx = \int_{\theta_2}^{\theta_3} R\cos\theta\, d\theta = R\left(\sin\theta_3 - \sin\theta_2\right)$$

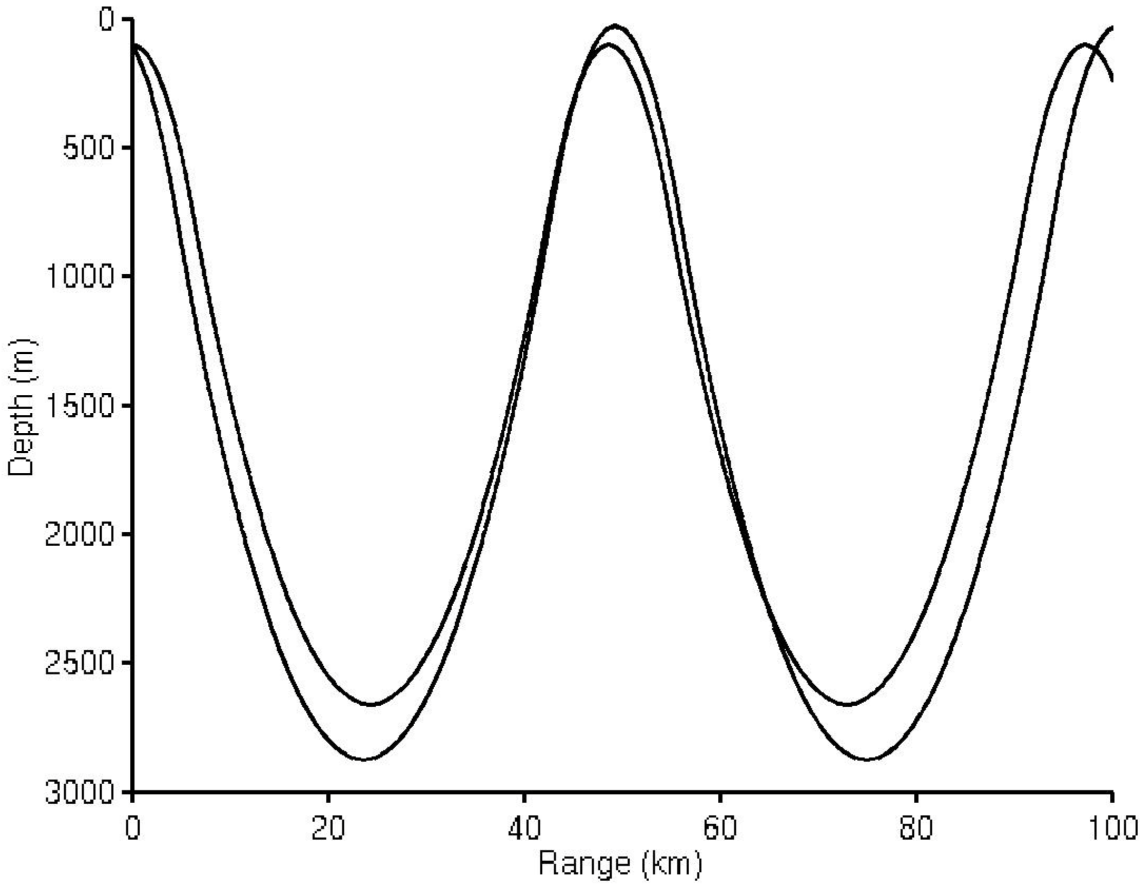
Example: a ray calculation





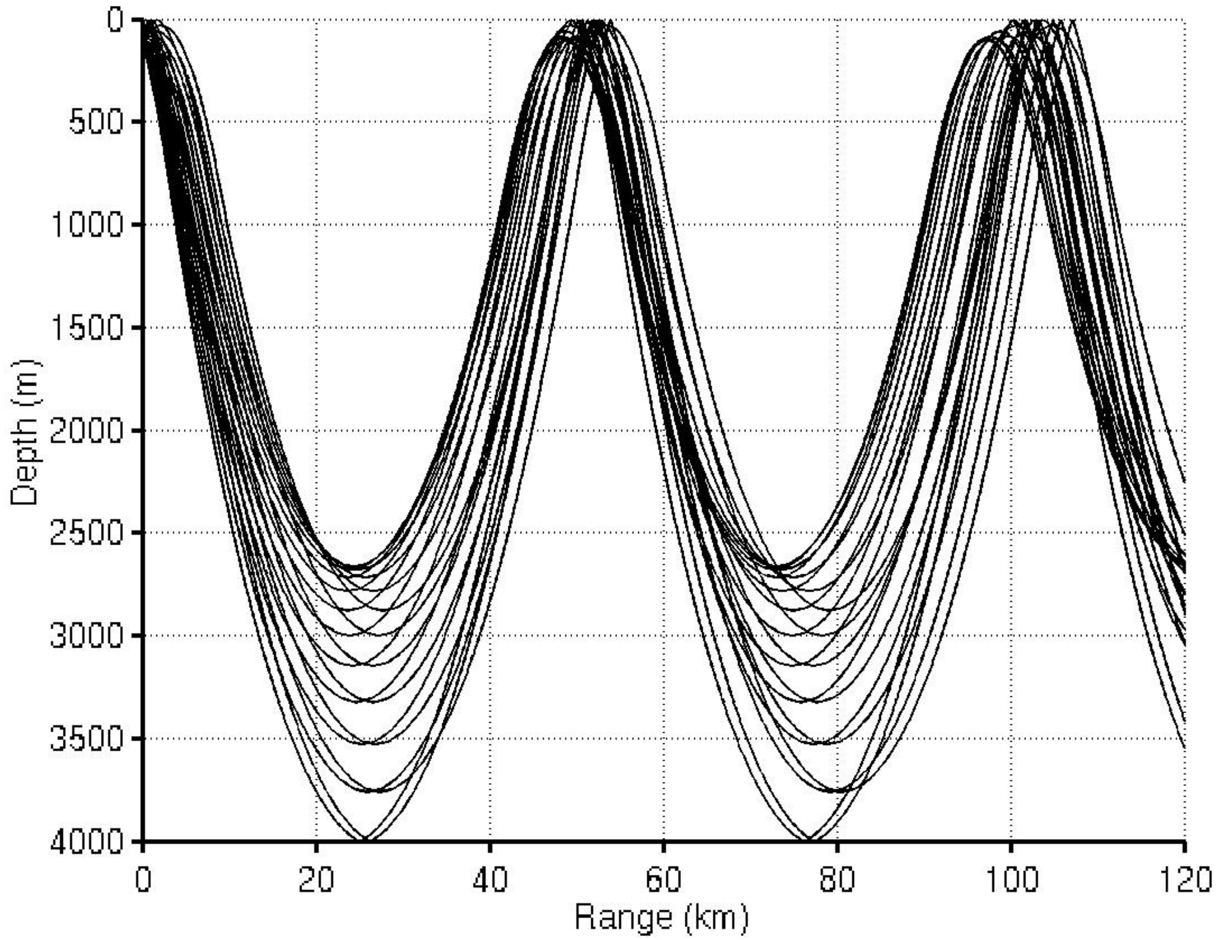
Example: calculate a second ray
(Propagation loss)

Launch angles:
$\theta_0 = 0°$
$\theta_0 = -4°$

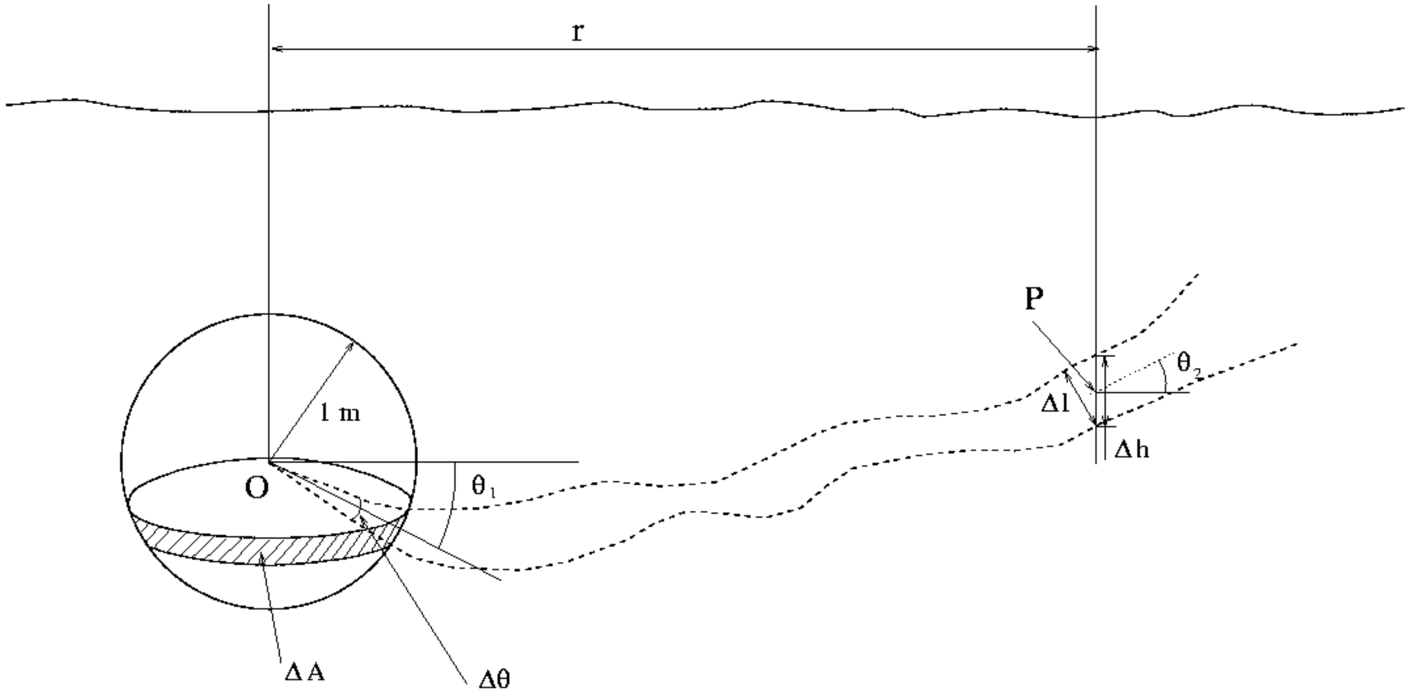


Example: ray tracing

Launch angles:
θ_0 = -10° → 10°



Propagation loss



In general:

$$PL(r,z) = 10\,{}^{10}\log\left(\frac{r\Delta h}{\Delta\theta}\right)$$

Spherical spreading:

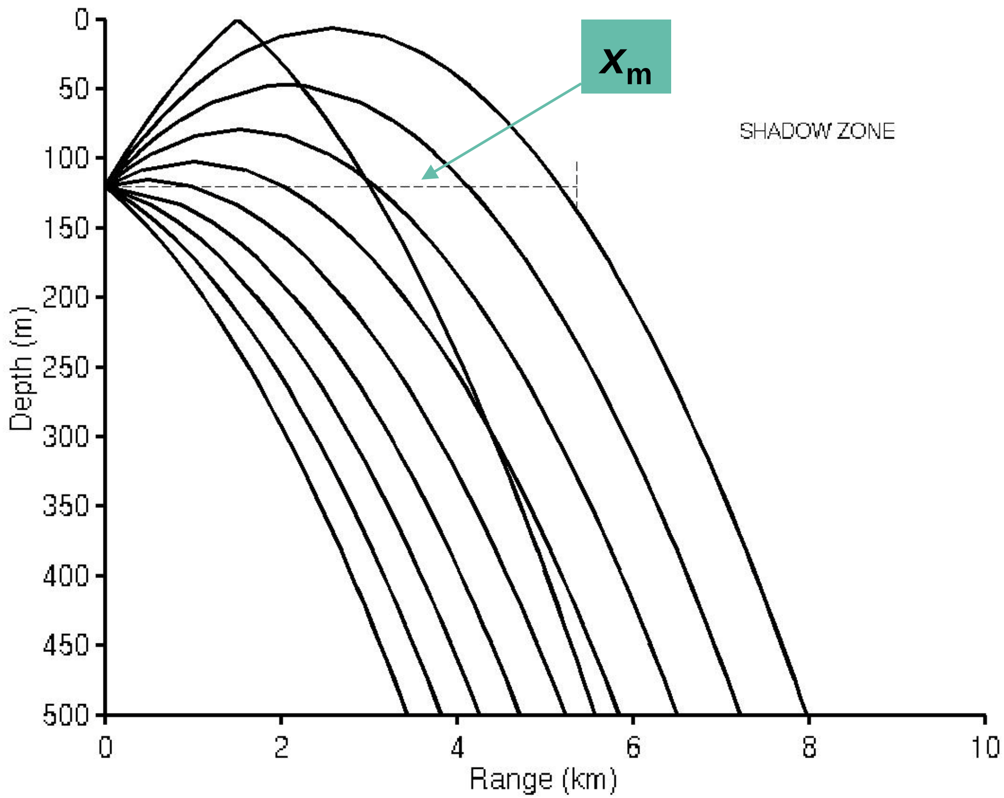
$$PL = 20\,{}^{10}\log r$$

because $\Delta h = r\Delta\theta$

Case study 1: Deep water, negative g below sea surface

H = 4000 m, g = -0.05 s^{-1}, c_0 = 1500 m/s, z_0 = 120 m

Launch angles:
θ_0 = -3° → 6°



$$z_s = R(1-\cos\theta_m)$$

with

$$R = -\frac{c(z_s)}{g\cos\theta_m} = -\frac{c_0}{g}$$

Hence

$$\theta_m = \arccos\left(1+\frac{z_s g}{c_0}\right)$$

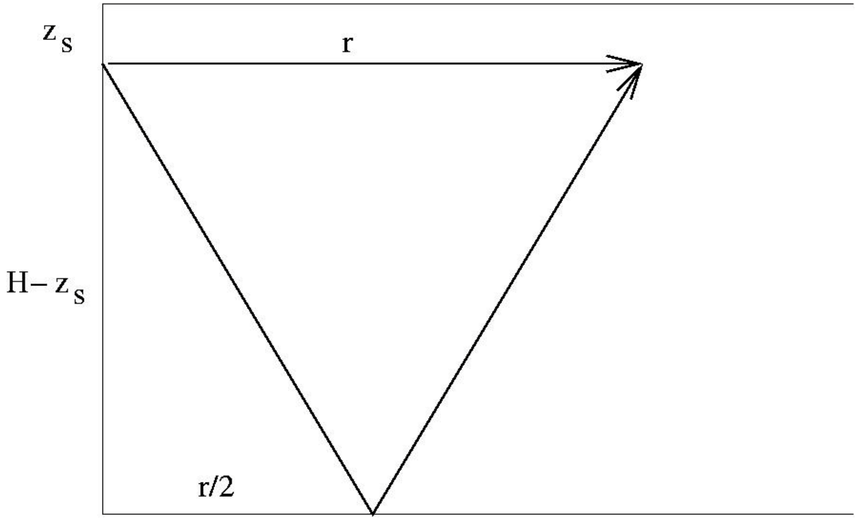
and

$$x_m = 2R\sin\theta_m$$

Case study 1: Propagation loss

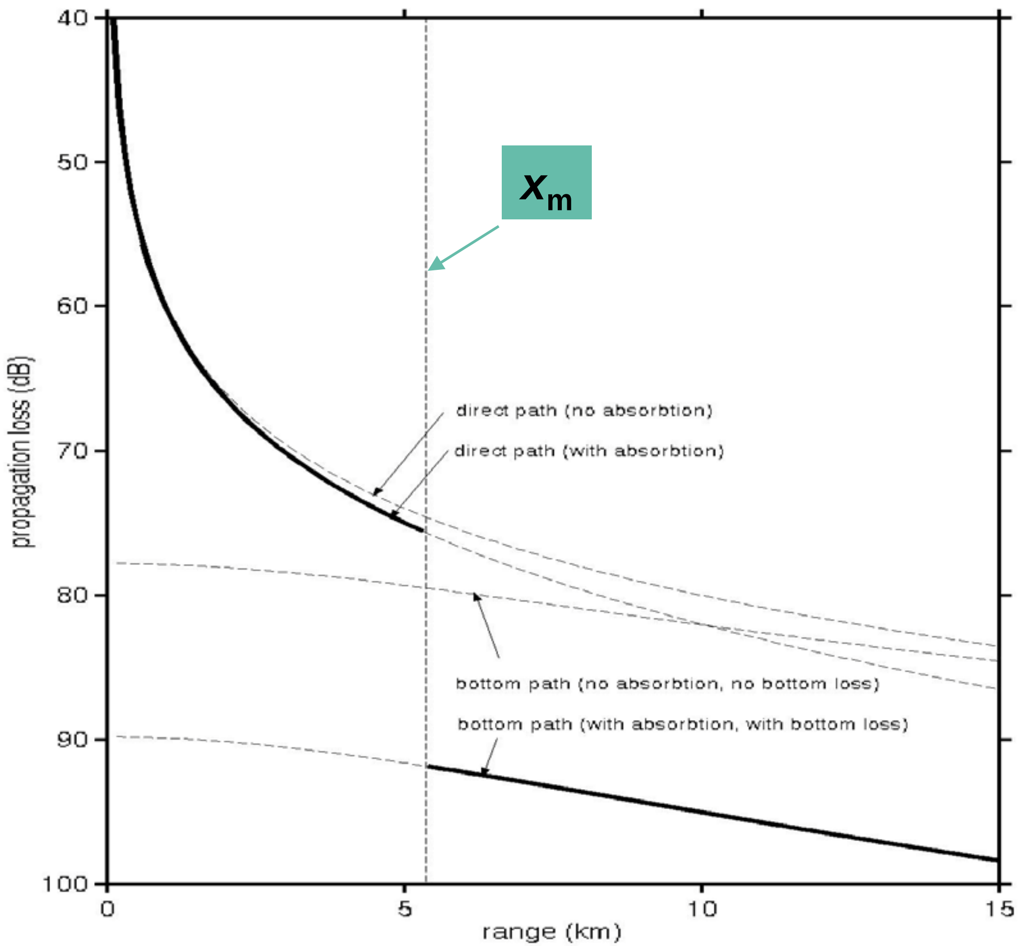
$r < x_m$ $$PL = 60 + 20\,{}^{10}\log r + \alpha r$$

$r > x_m$ $$PL = 60 + 20\,{}^{10}\log s + \alpha s + BL$$



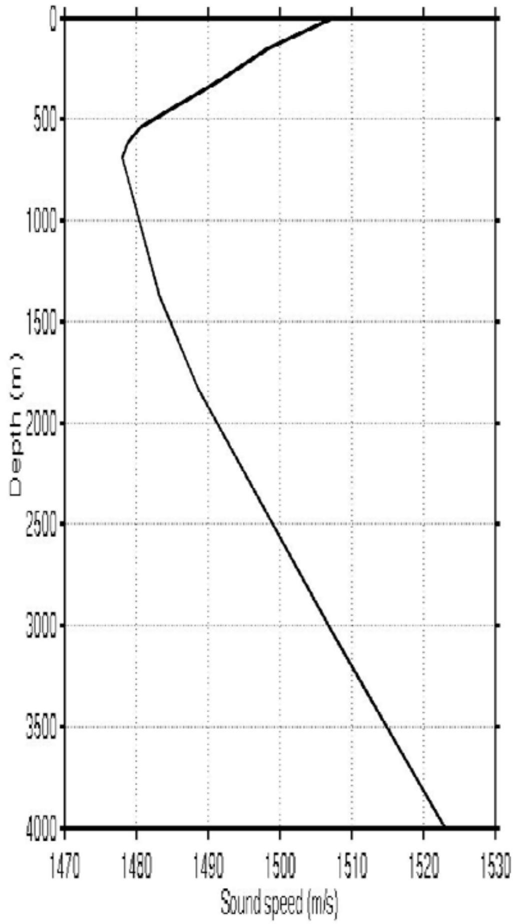
$$s = 2\sqrt{(H - z_s)^2 + \left(\frac{r}{2}\right)^2}$$

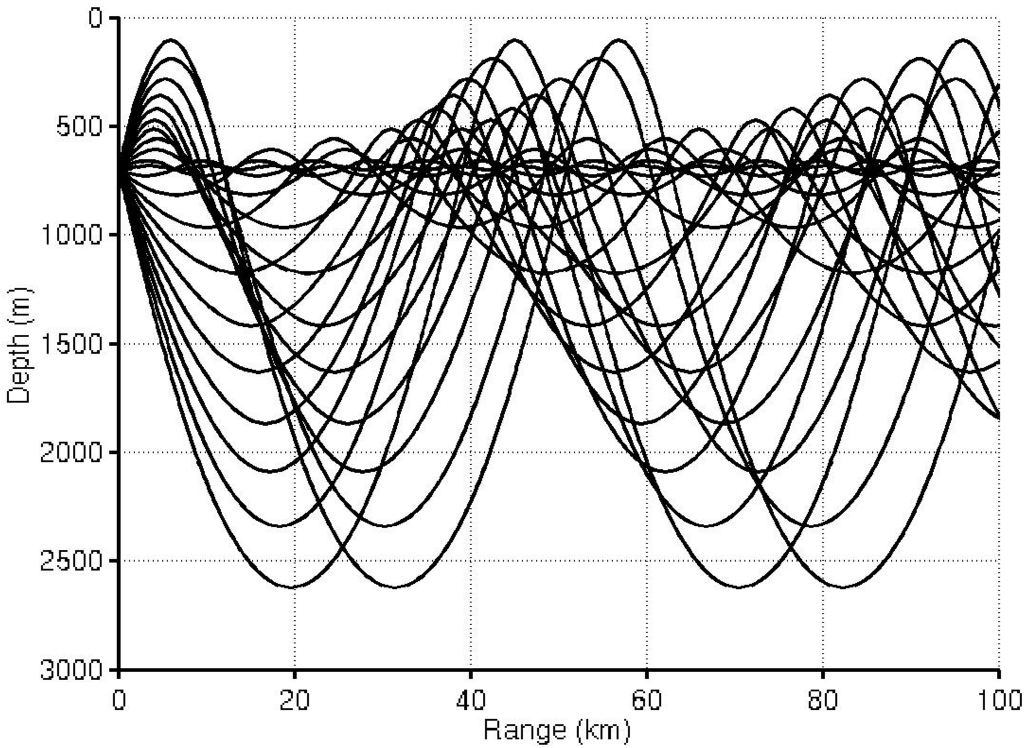
Case study 1: Propagation loss, continued



H = 4000 m
f = 3000 Hz
(α = 0.2 dB/km)
$z_s = z_r$ = 120 m
BL = 10.5 dB (sand)

Case study 2: Sound channel propagation





z_s = 700 m

Case study 2: Propagation loss

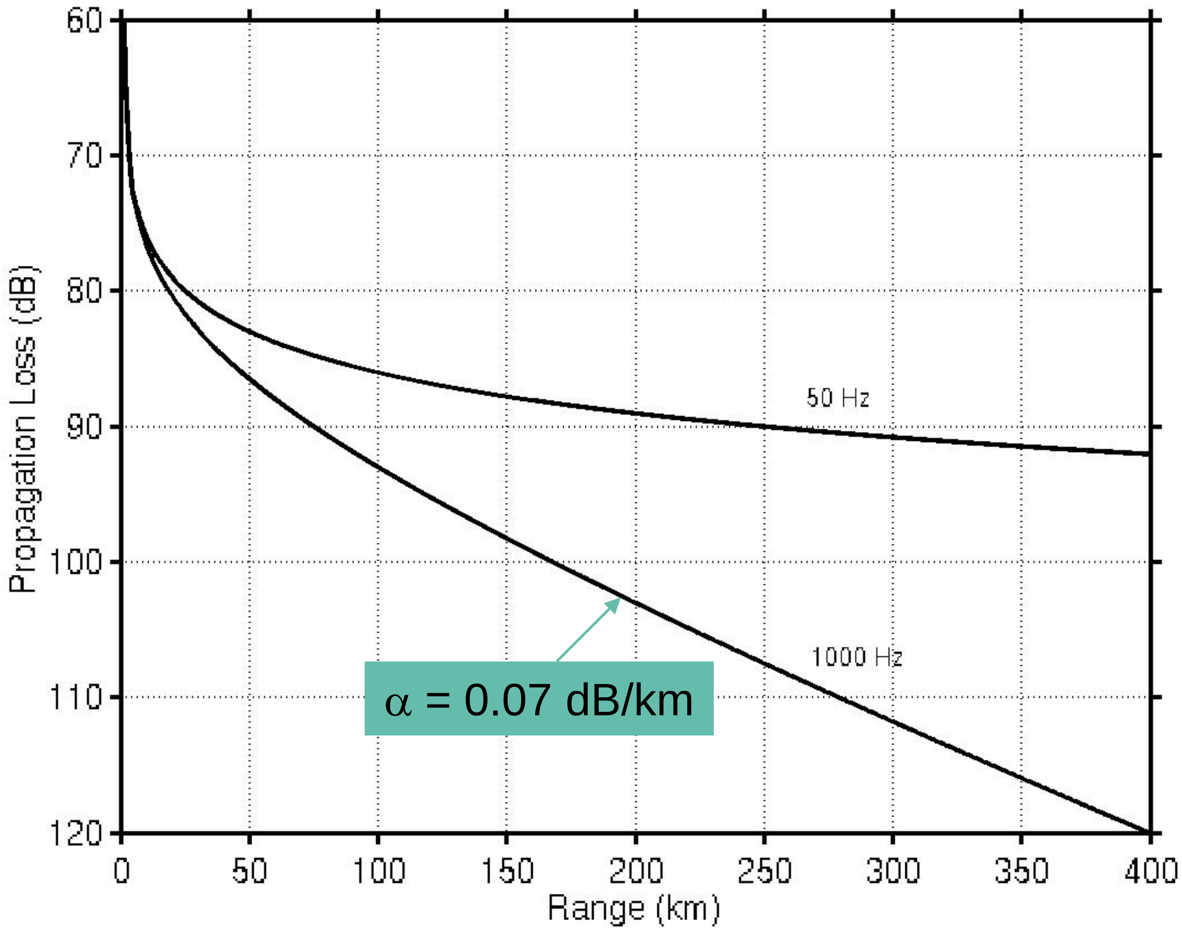
$r < r_0$ $$PL = 20\,{}^{10}\log r + \alpha r$$ 'Spherical'

$r > r_0$ $$PL = 10\,{}^{10}\log r_0 + 10\,{}^{10}\log r + \alpha r$$ 'Cylindrical'

With $r_0 \approx 4$ km the transition range

Case study 2: Propagation loss, continued

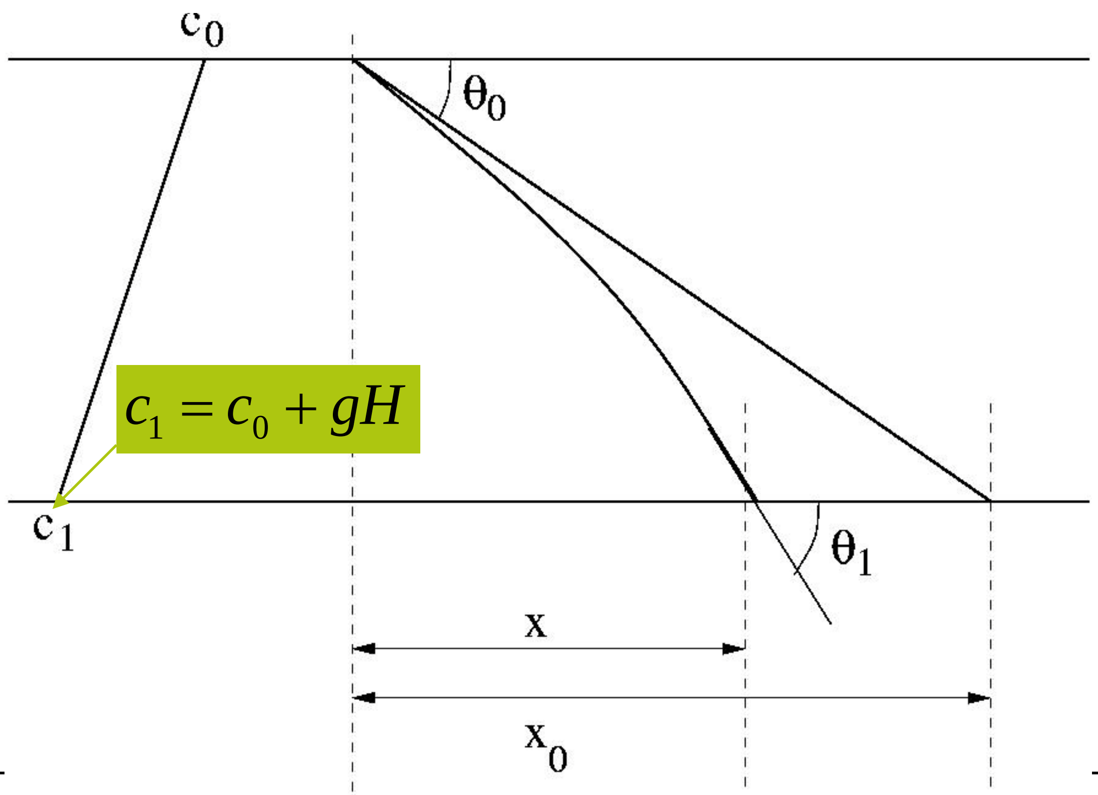
$z_s = z_r = 700$ m



Case study 3: Refraction effect MBES

Radius of curvature of the ray launched at an angle θ_0:

$$R_0 = -\frac{c_0}{g\cos\theta_0}$$



$$c_1 = c_0 + gH$$

Apply Snell's law for calculating θ_1:

$$\cos\theta_1 = \frac{c_1}{c_0}\cos\theta_0$$

Horizontal range:

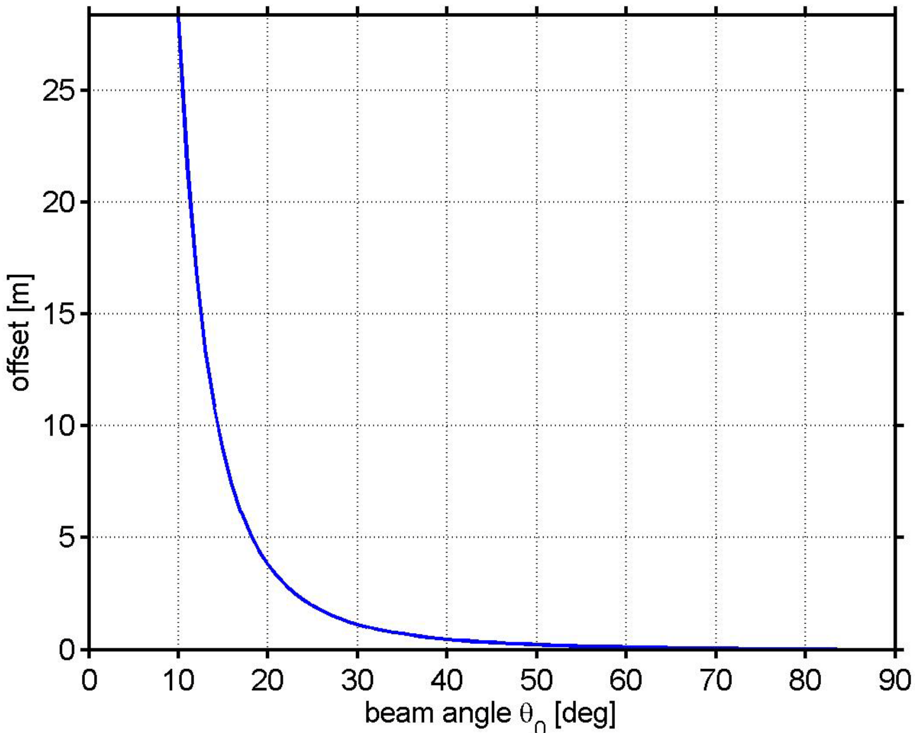
$$x = R_0(\sin\theta_1 - \sin\theta_0)$$

Compare with horizontal range without refraction:

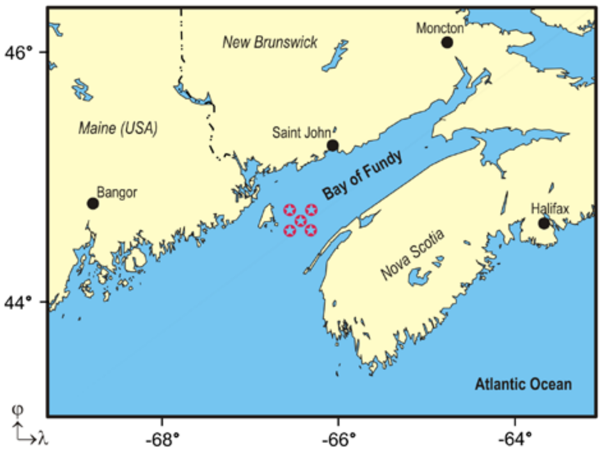
$$x_0 = \frac{H}{\tan\theta_0}$$

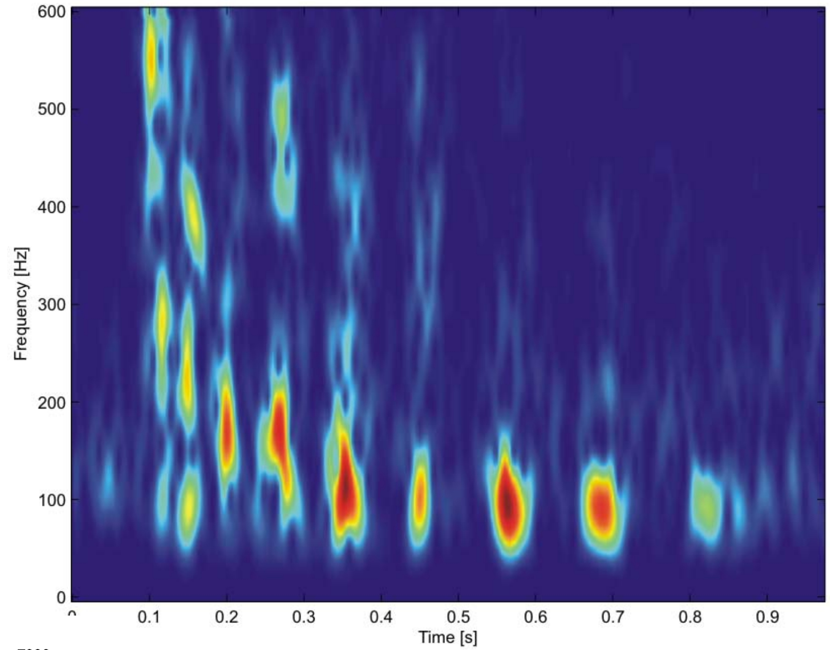
Case study 3: Refraction effect MBES numerical example

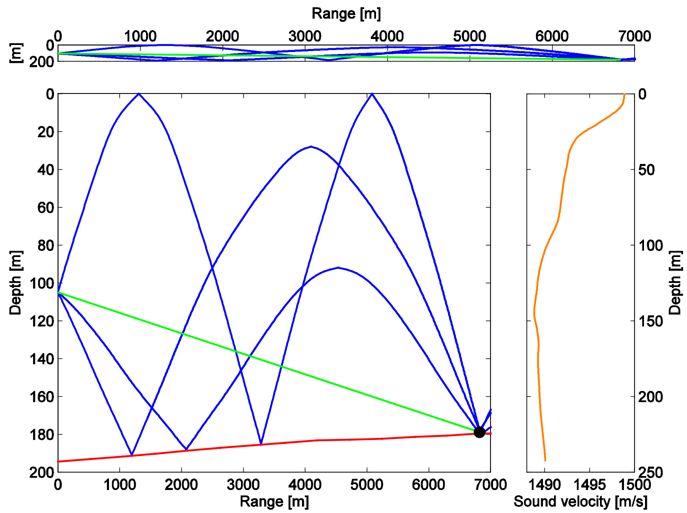
θ_0 [degrees]	θ_1 [degrees]	X_0 [m]	X [m]	Offset x_0-x [m]
45	45.2	100	99.7	0.3
20	20.5	274.7	270.9	3.8

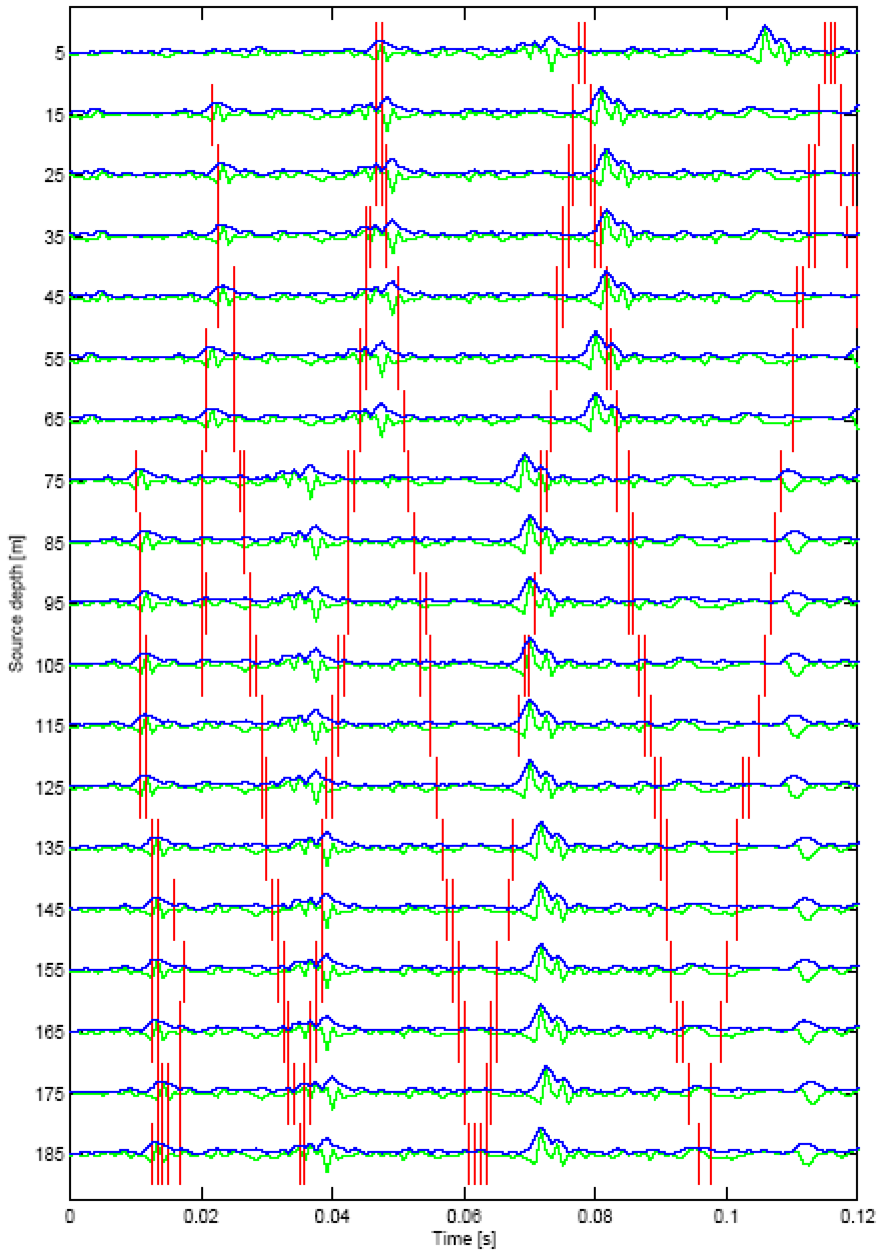


c_0 = 1500 m/s
H = 100 m
g = -0.05 s^{-1}


New Brunswick
Moncton
Maine (USA)
Saint John
Bay of Fundy
Bangor
Halifax
Nova Scotia
Atlantic Ocean
46°
44°
-68°
-66°
-64°


Frequency [Hz]
600
500
400
300
200
100
0
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Time [s]


Range [m]
0 1000 2000 3000 4000 5000 6000 7000
[m]
0
200
Depth [m]
0 20 40 60 80 100 120 140 160 180 200
0 1000 2000 3000 4000 5000 6000 7000
Range [m]
1490 1495 1500
Sound velocity [m/s]
0 50 100 150 200 250



Source depth [m]
5
15
25
35
45
55
65
75
85
95
105
115
125
135
145
155
165
175
185
0
0.02
0.04
0.06
0.08
0.1
0.12
Time [s]