Flow, Stability

chapter 3

ct4310 Bed, Bank and Shoreline protection

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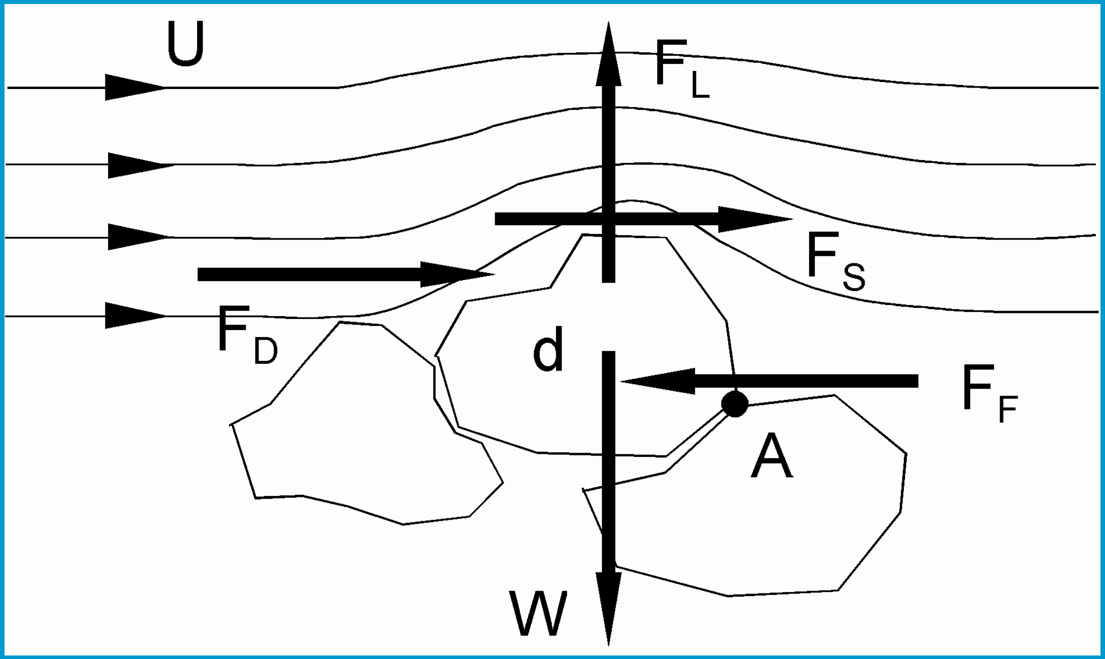
Introduction

- focus on non-cohesive grains
- grains may vary in size from microns to tons
- basic principle not very different
- always turbulent

forces on a grain in flow



forces on a stone

$$\left.\begin{aligned}
&Drag\ \ force:\ F_D = \frac{1}{2} C_D \rho_w u^2 A_D \\
&Shear\ \ force:\ F_S = \frac{1}{2} C_F \rho_w u^2 A_S \\
&Lift\ \ force:\ F_L = \frac{1}{2} C_L \rho_w u^2 A_L
\end{aligned}\right\} \quad F \propto \rho_w u^2 d^2$$

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load and strength relationship

$$u_c^2 \propto \left(\frac{\rho_s - \rho_w}{\rho_w}\right) g\, d = \Delta\, g\, d \rightarrow u_c^2 = K\, \Delta\, g\, d$$

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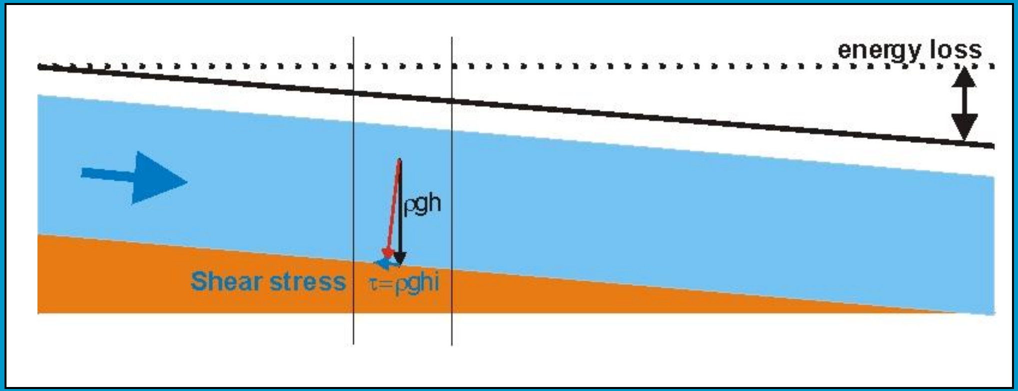
Izbash (1930)

$$u_c = 1.2\sqrt{2\,\Delta\, g\, d} \quad or \quad \frac{u_c}{\sqrt{\Delta\, g\, d}} = 1.7 \quad or \quad \Delta\, d = 0.7\,\frac{u_c^2}{2\,g}$$

- no waterdepth
- no good definition of u_c and d

Approach of Shields (1936)



- Stability of stones depends on (generalized) friction force
- The force of flowing water on bed is:

  F= Area * ρghi (or τ = ρghi)
- Make stability number based on τ and d
- Make this number dimensionless by dividing by g and (ρ_s-ρ_w)
- So:

$$\psi_c = \frac{\tau_c}{(\rho_s - \rho_w) g d} = \frac{\rho_w g h i}{(\rho_s - \rho_w) g d}$$

No velocity in equation
No need to measure velocity

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Comparison of Shields and Izbash

- Both are formulas with stability as function of u^2
- Izbash focuses on the force action on one single grain
- Shields focuses on the average shear stress on the bed
- Shields does not consider individual grains
- Izbash explicitly looks to individual rocks

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Shields (1936)

$$\psi_c = \frac{\tau_c}{(\rho_s - \rho_w)\,gd} = \frac{u_{*c}^2}{\Delta gd} = f(\mathrm{Re}_*) = f\left(\frac{u_{*c}d}{\upsilon}\right)$$

$$u_* = \bar{\bar{u}}\frac{\sqrt{g}}{C}$$

$$\psi_c = \frac{\bar{\bar{u}}^2}{C^2\Delta d}$$

Only valid assuming the Chezy equation is valid

$$v = C\sqrt{hi}$$

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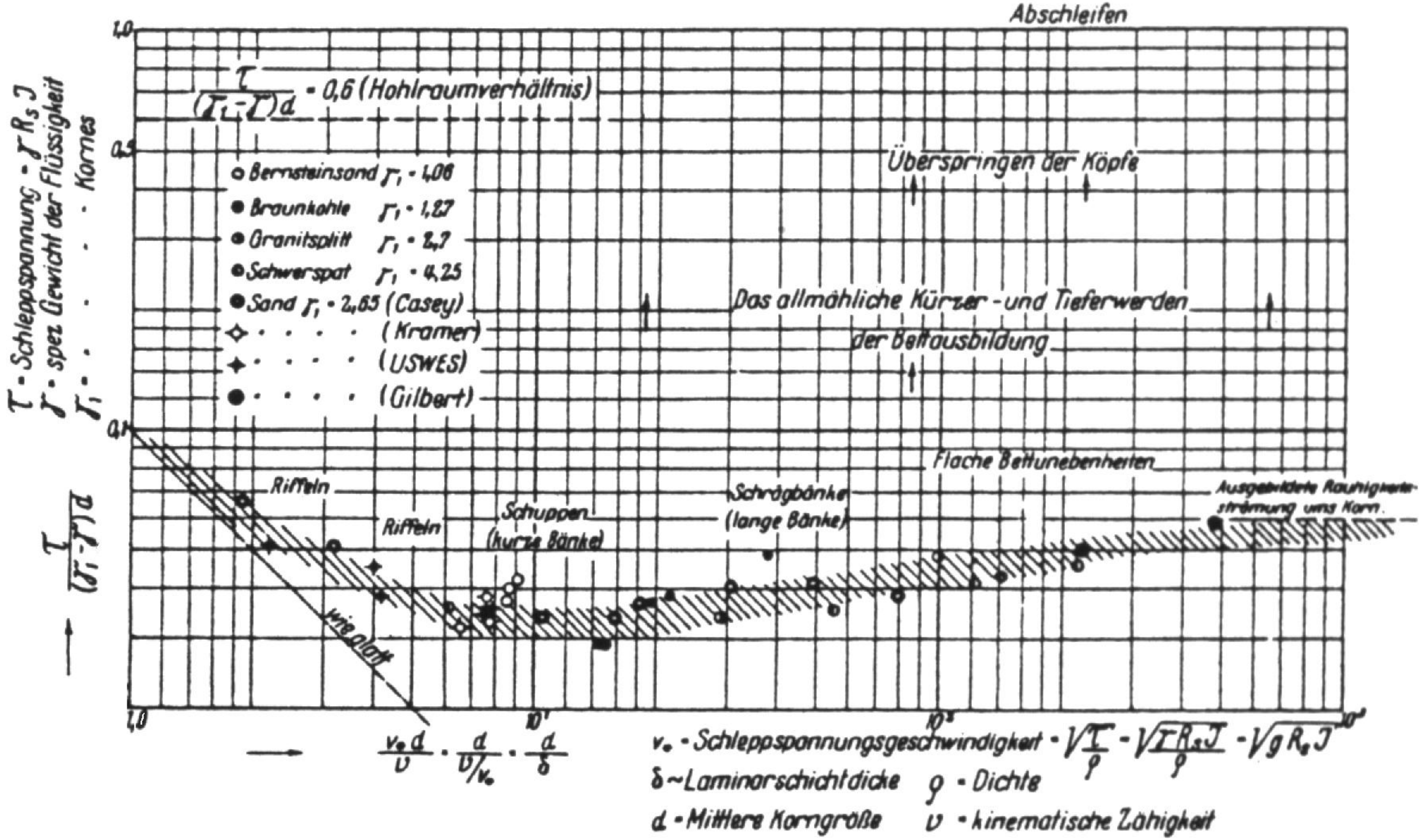
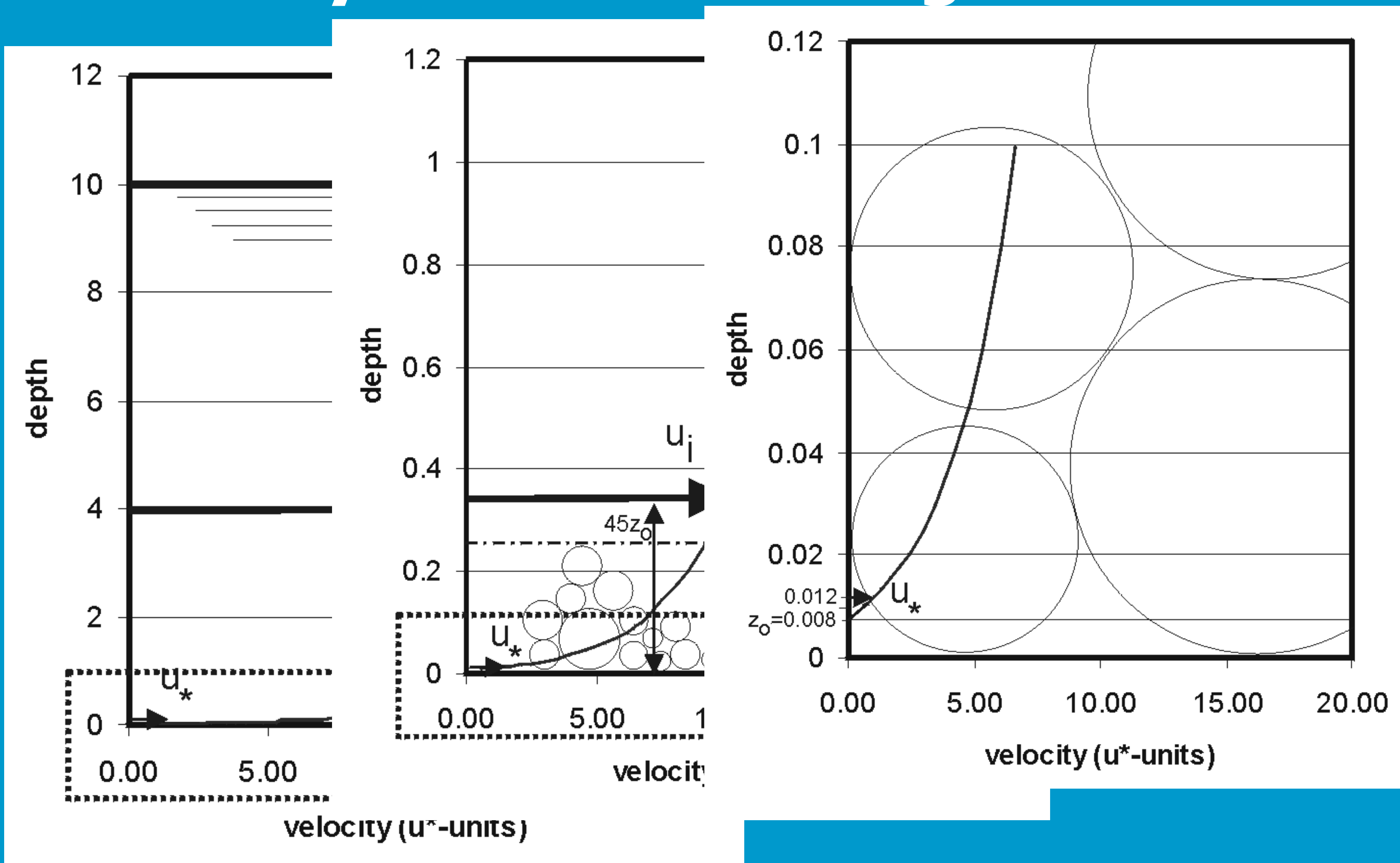


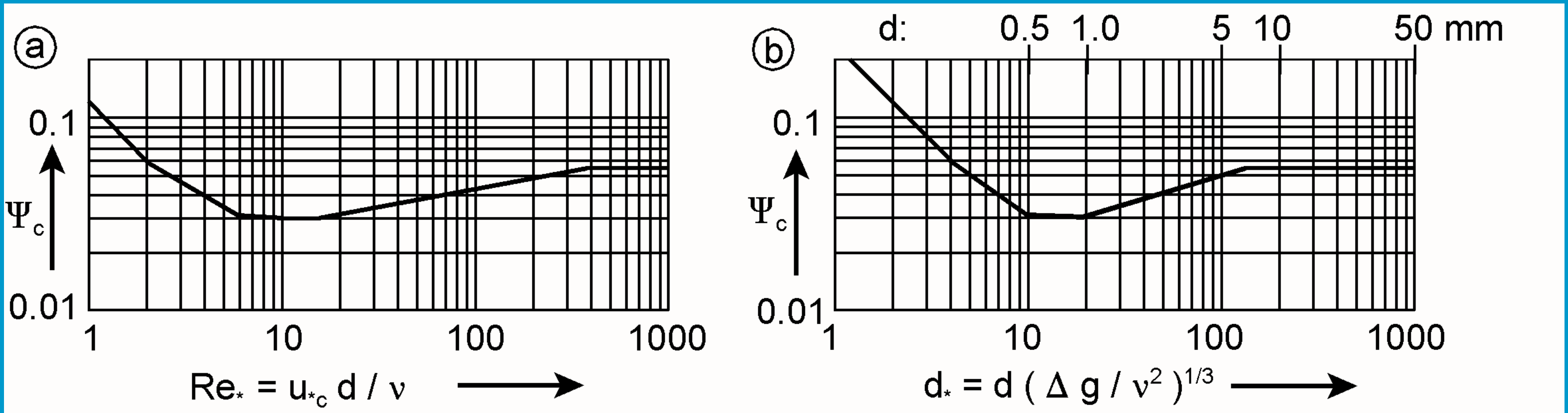
Abb. 6.

Schleppspannungsbeiwert $\frac{\tau}{(\gamma_1-\gamma)\,d}$ gegen die Reynold'sche Zahl des Kornes $\frac{v_* d}{\nu}$.

Velocity at a certain height



critical shear stress according to Shields Van Rijn



$$\psi_c = \frac{\tau_c}{(\rho_s - \rho_w)\ g\ d} = \frac{u_{*c}^2}{\Delta\ g\ d} = f\left(\mathrm{Re}_*\right) = f\left(\frac{u_{*c} d}{\upsilon}\right)$$

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Example for determination of d_* (Shields)

What is u_{*c} for sand of 2 mm??

Wild guess: u_{*c} is 1 m/s

$$\mathrm{Re}_* = \frac{u_{*c} d}{\nu} = \frac{1 \cdot 0.002}{1.33 * 10^{-6}} = 1500$$

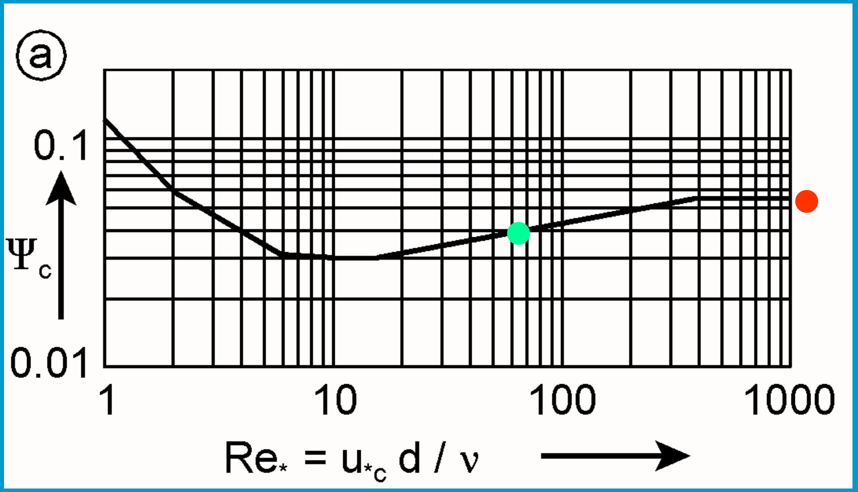
Thus: $\Psi_c = 0.055$

$$\Psi_c = \frac{u^2_{*c}}{\Delta g d} \Rightarrow u_{*c} = \sqrt{\Psi_c \Delta g d}$$

$$= \sqrt{0.055 \cdot 1.65 \cdot 9.8 \cdot 0.002}$$

$$= 0.042\, m/s$$

$$Thus: \mathrm{Re}_* = \frac{0.042 \cdot 0.002}{1.33 * 10^{-6}} = 63 \Rightarrow \Psi_c = 0.04$$



$$u_{*c} = \Psi_c \Delta g d$$

$$= 0.04 \cdot 1.65 \cdot 9.81 \cdot 0.002$$

$$= 0.036\ m/s$$

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Example for determination of d_* (Van Rijn)

d = 2 mm

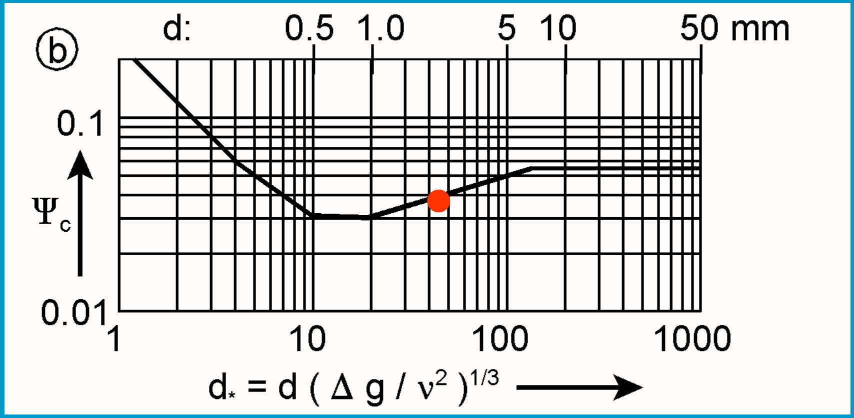
$$d_* = d\sqrt[3]{\frac{\Delta g}{\nu^2}} = 0.002 \cdot \sqrt[3]{\frac{1.65 \cdot 9.81}{\left(1.33 \cdot 10^{-6}\right)^2}} = 42$$

$$\Psi_c = 0.04$$

$$u_{*c} = \Psi_c \Delta g d$$

$$= 0.04 \cdot 1.65 \cdot 9.81 \cdot 0.002$$

$$= 0.036\ m/s$$



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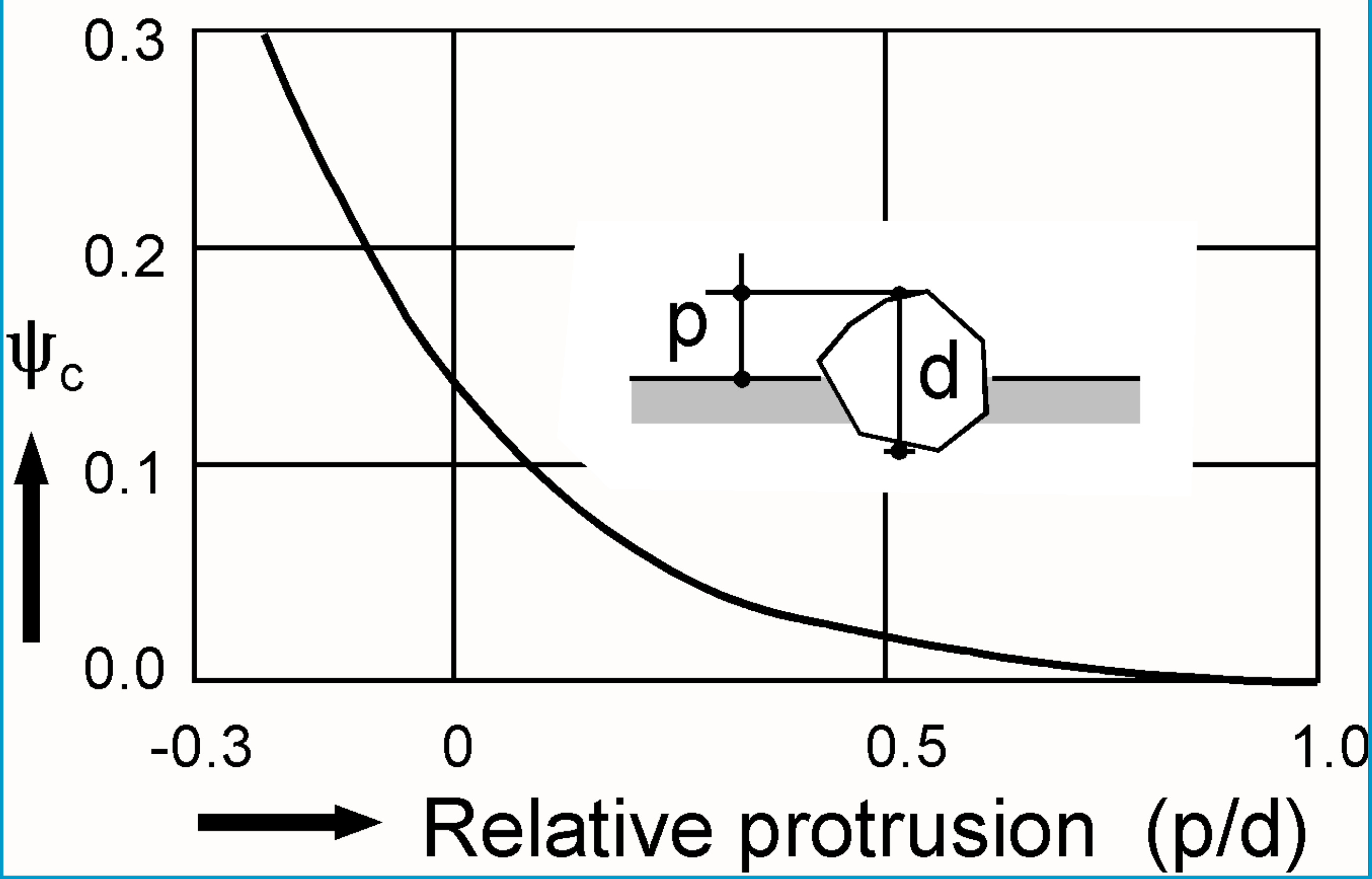
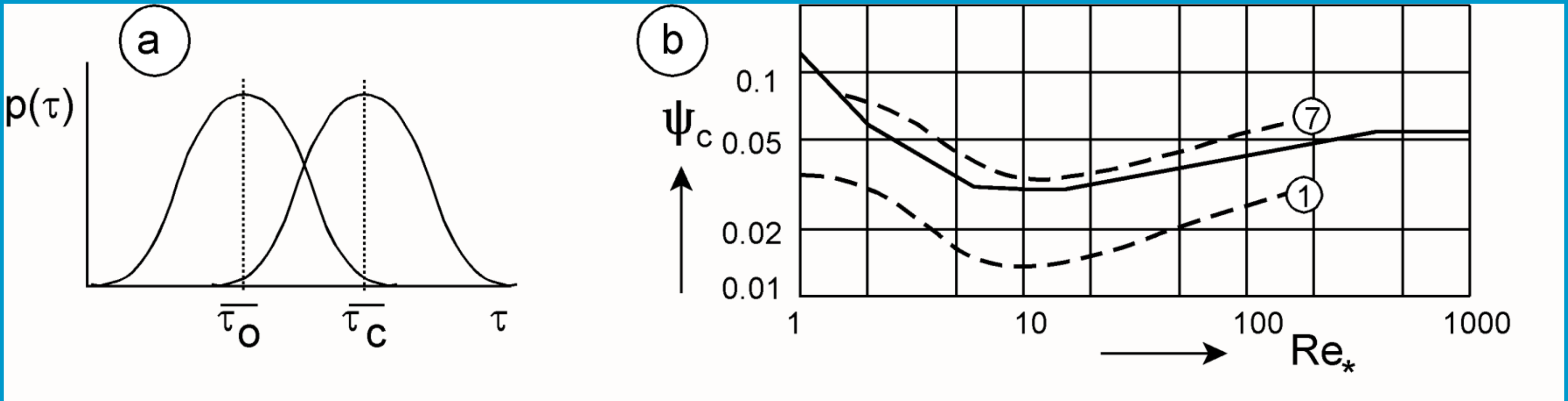
relative protrusion of a grain



Load and strength distribution



0 no movement at all
1 occasional movement at some locations
2 frequent movement at some locations
3 frequent movement at several locations
4 frequent movement at many locations
5 frequent movement at all locations
6 continuous movement at all locations
7 general transport of the grains ← **Shields**

Incipient motion according to Shields

U = 0.60 m/s, Ψ=0.03

U = 0.70 m/s, Ψ=0.04

U = 0.83 m/s, Ψ=0.05

U = 0.90 m/s, Ψ=0.055

U = 0.92 m/s, Ψ=0.06

U = 0.97 m/s, Ψ=0.07

ct4310/1 u=xxenPsi=xx
bb 4310-3: Shieldsxxx

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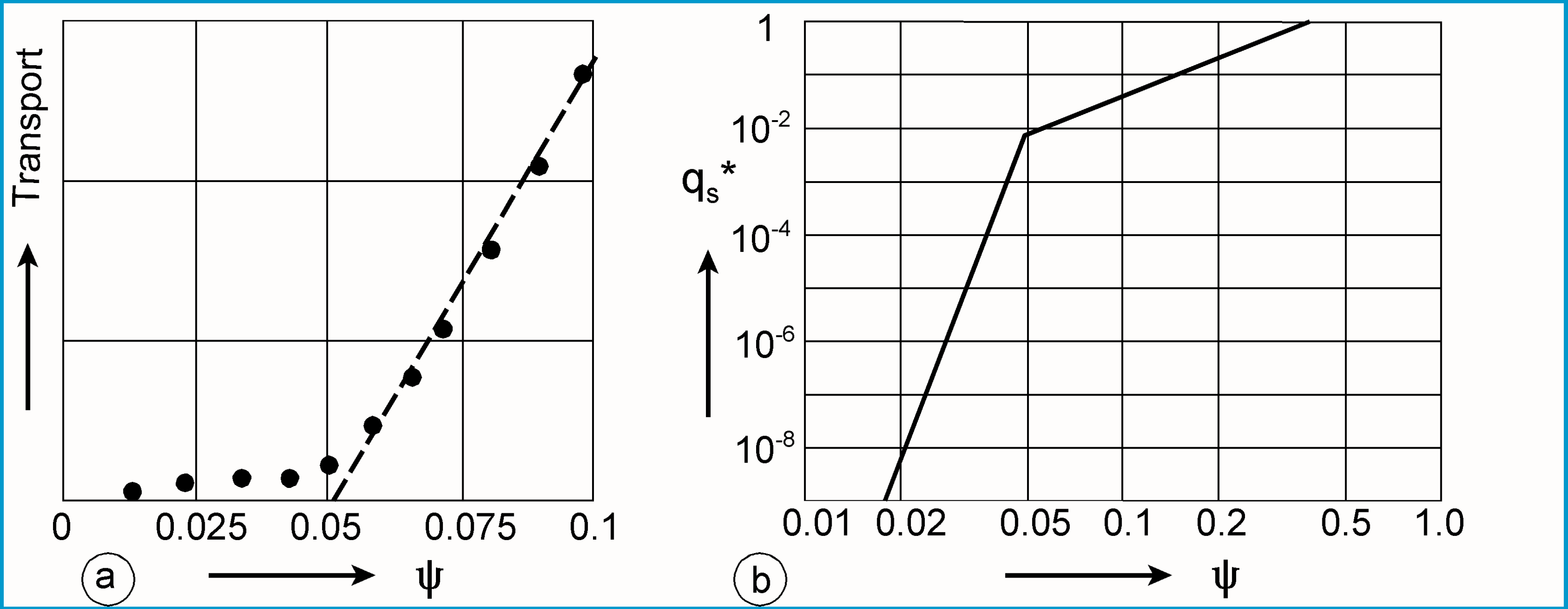
Threshold of motion



extrapolation to zero (Shields)

Paintal

Ψ is a stability parameter

Ψ is a mobility parameter

Paintal

$$\left.\begin{aligned} q_s^* &= 6.56 \cdot 10^{18}\ \psi^{16} \quad (\text{for } \psi < 0.05) \\ q_s^* &= 13\ \psi^{2.5} \quad (\text{for } \psi > 0.05) \end{aligned}\right\} \text{ with } q_s^* = \frac{q_s}{\sqrt{\Delta\ g\ d^3}}$$

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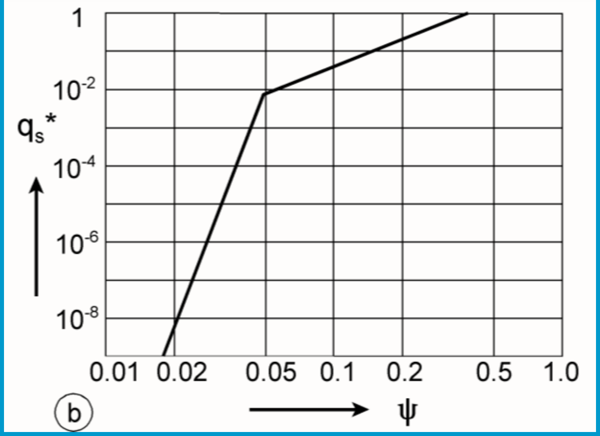
example

- For $\Psi_c = 0.03$ is considered a safe choice
- Assume $d_d = 0.4$ m
- $q_s = 6.56 * 10^{18} \Psi^{16} \sqrt{(\Delta d^3)} = 3*10^{-6}$ $m^3/m/s$
- This is equivalent to 4 stones per day per m width
- Design velocity occurs only exceptional (1 % per year)
- Note: This loss is per m width and not per m^2

After some time transport stops in case $\Psi < 0.06$

In case $\Psi > 0.06$ transport never stops



De Boer, 1998

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nominal diameter

$$d_n = \sqrt[3]{V} = \sqrt[3]{M / \rho}$$

$$d_{n50} \neq d_{50}$$

usually $d_{50} = 1.2\ d_{n50}$

for a sphere $d_{50} = 1.24\ d_{n50}$

influence of waterdepth

$$\frac{\overline{u_c}}{\sqrt{\Delta g d_{n50}}} = \frac{C\sqrt{\psi_c}}{\sqrt{g}}$$

Isbash: $\frac{u_{ic}}{\sqrt{\Delta g d}} = 1.7$

$$C = 18\log\frac{12h}{k_r}$$

Attention:

$$\overline{u_c} \neq u_{ic}$$

roughness
$k_r = 2*d_{50}$
or $k_r = 3*d_{50}$

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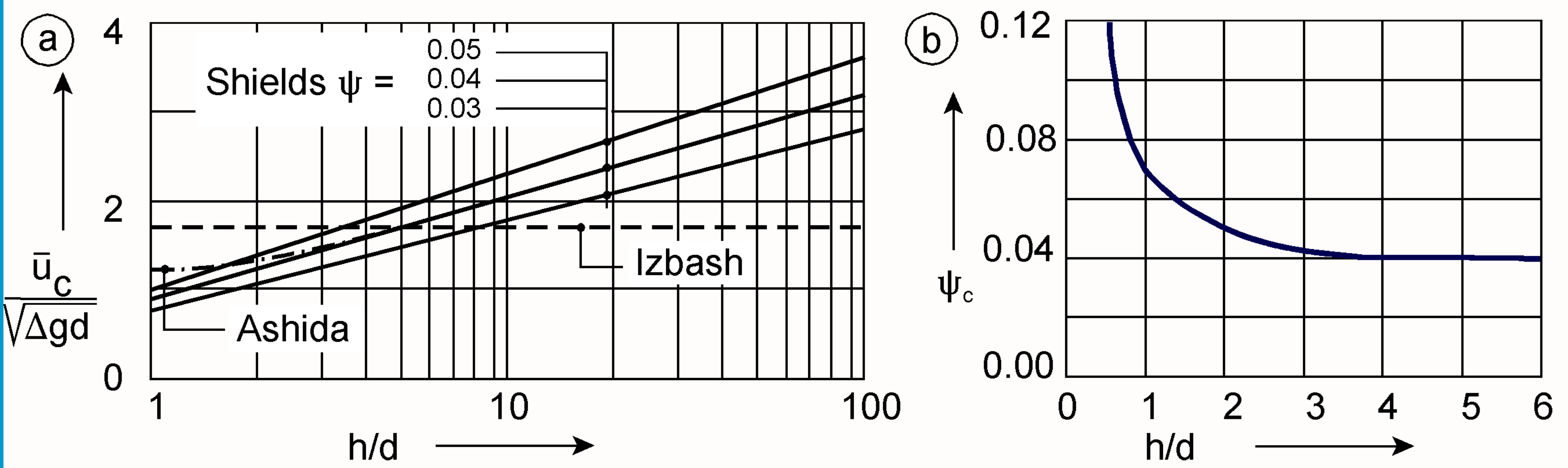
influence water depth on critical velocity



Shields is valid in deep water (h/d>100)
Ashida found a larger Ψ for shallow water (h/d <5)

It is obvious that Izbash gives a horizontal line

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practical application

$$\frac{\overline{u_c}}{\sqrt{\Delta\, g\, d_{n50}}} = \frac{C\sqrt{\psi_c}}{\sqrt{g}} \quad \rightarrow \quad d_{n50} = \frac{\overline{u_c}^2}{\psi_c \Delta C^2}$$

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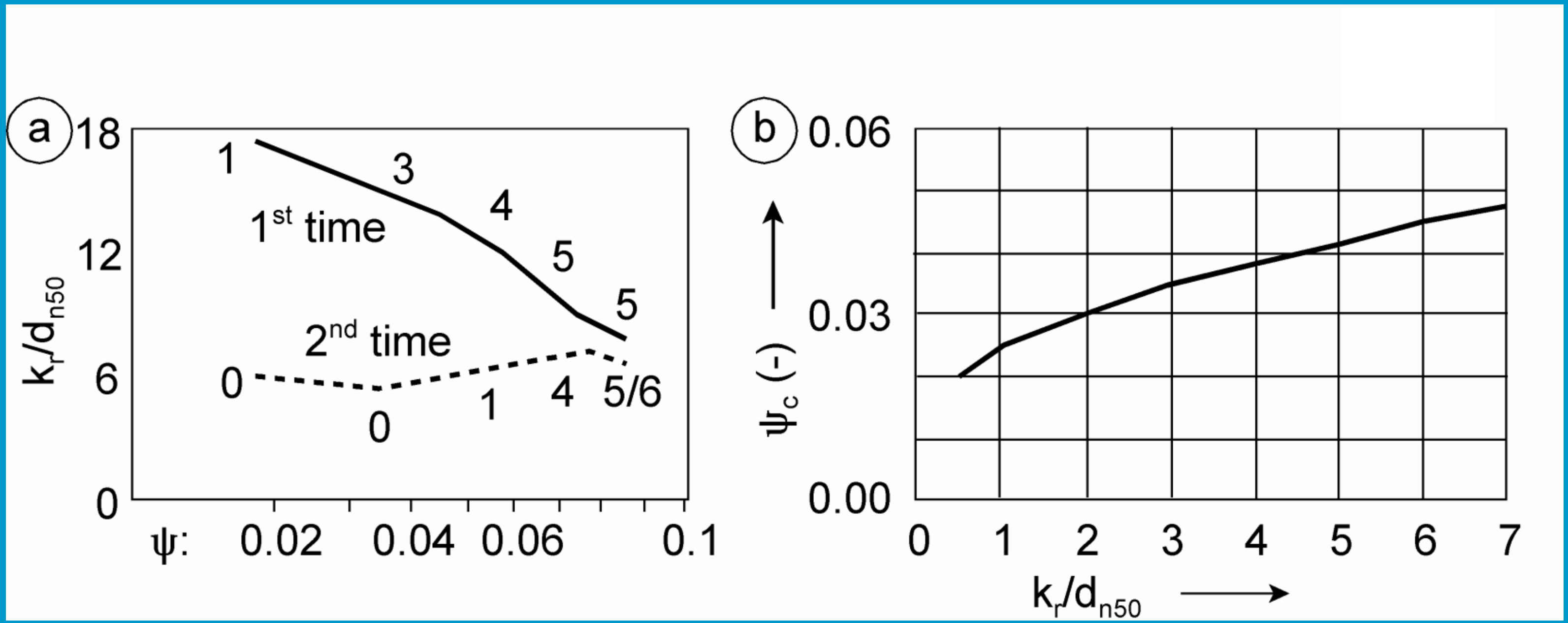
Roughness and threshold of motion



Lammers, 1997

Note the plating-effect (pantsering)

Problem with the choice of Ψ:

do we select Ψ on the safe side or do we use the expected value of Ψ ??

demo influence Ψ

run demo Cress

River structures
Bed protections

Influence of the parameter Ψ on rock size

v vary from 1…3 m/s
Ψ 4 lines 0.03…0.06
h 6 lines 1 ….11
choice 2 lines 1 and 2
Fi 3 lines 30…40

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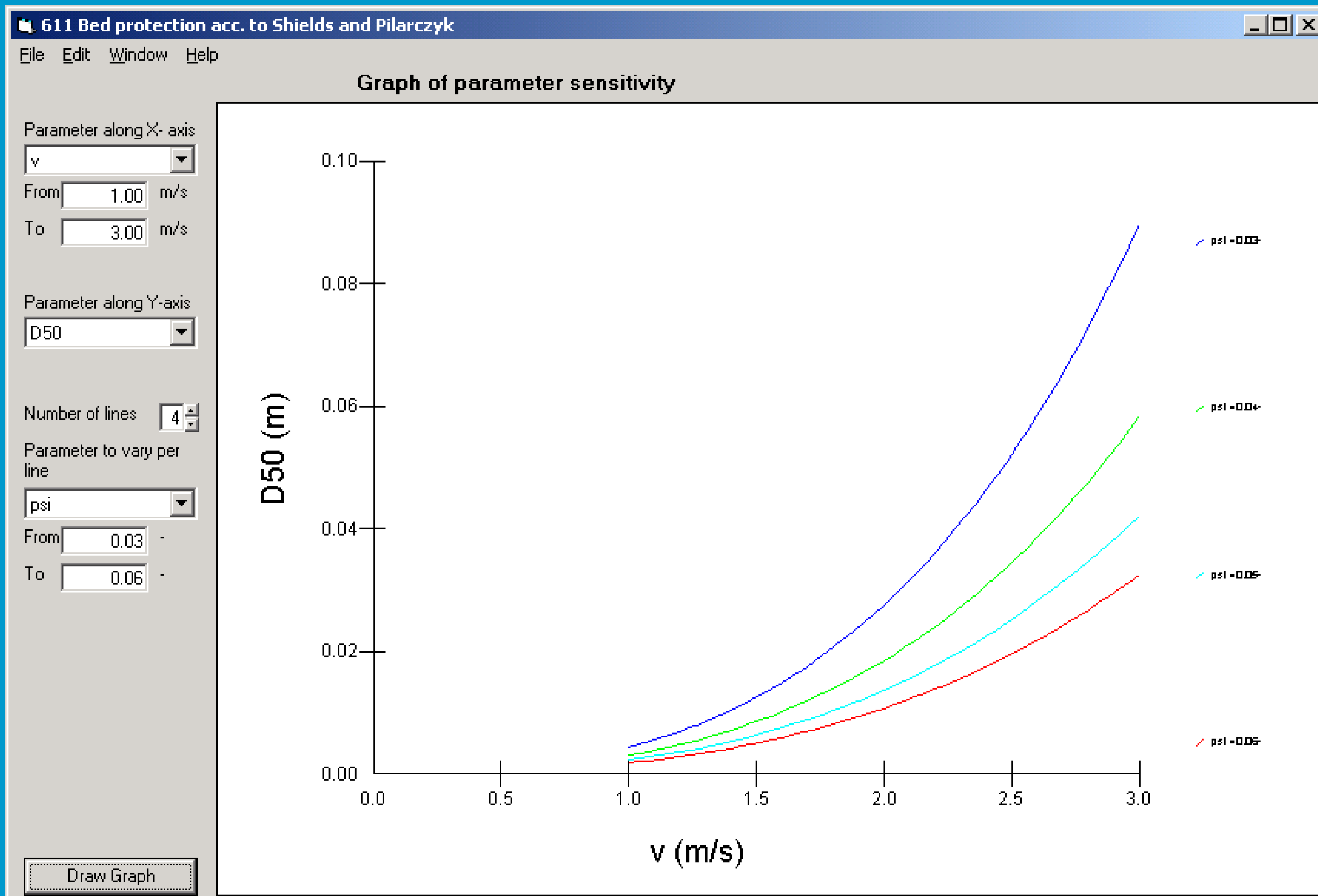
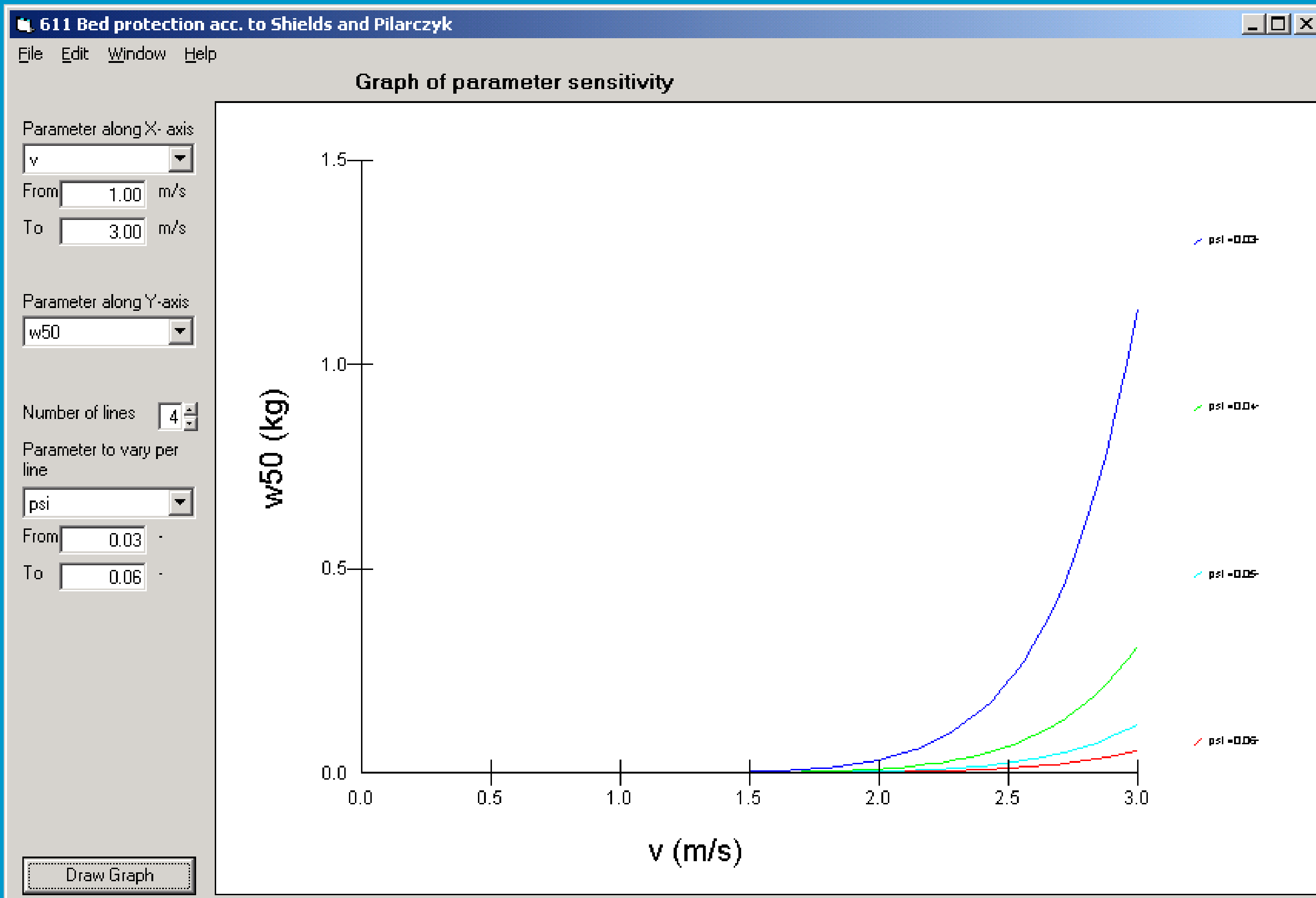
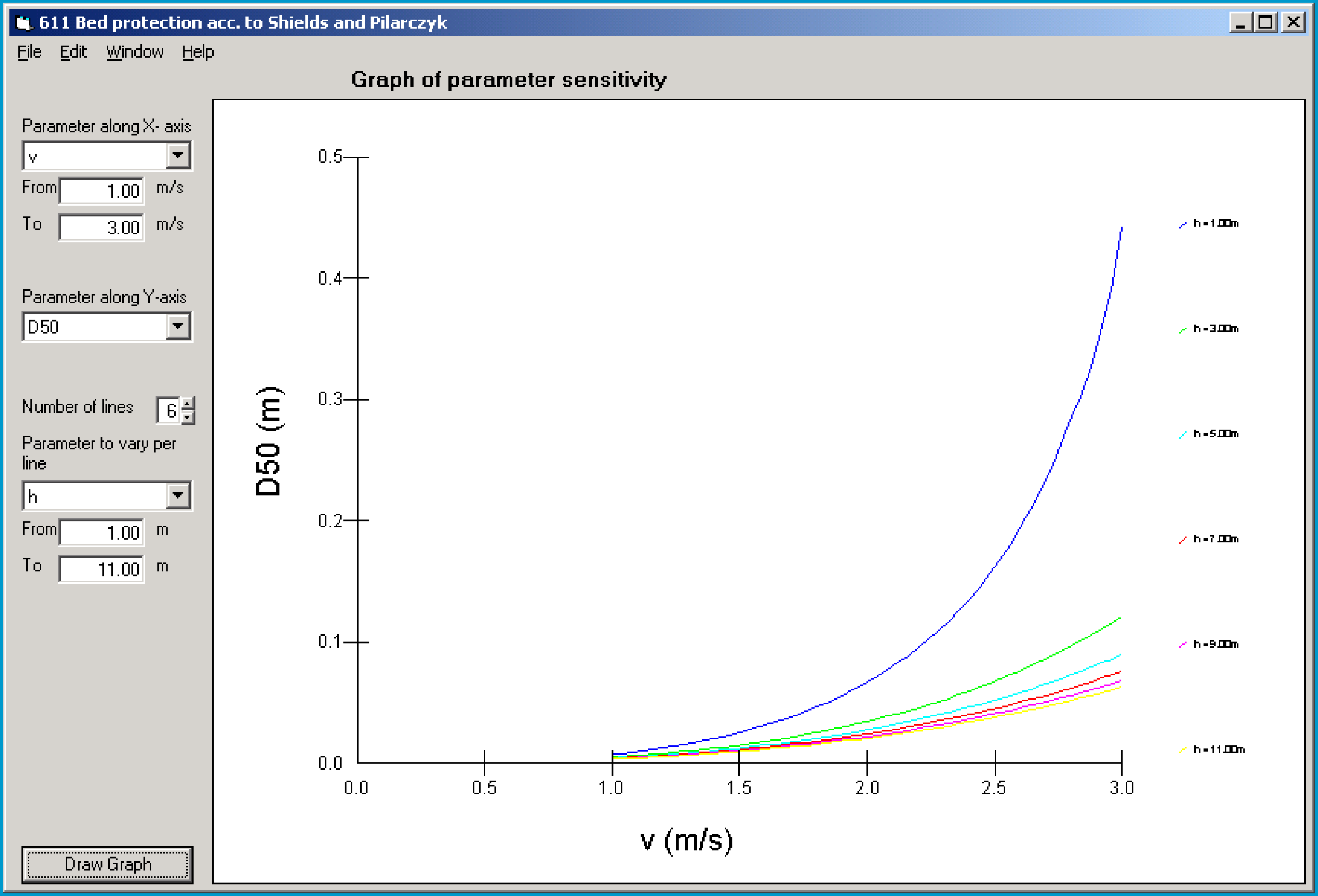
Variation of Ψ (output D_{50})



Variation of Ψ (output W_{50})



Variation of h



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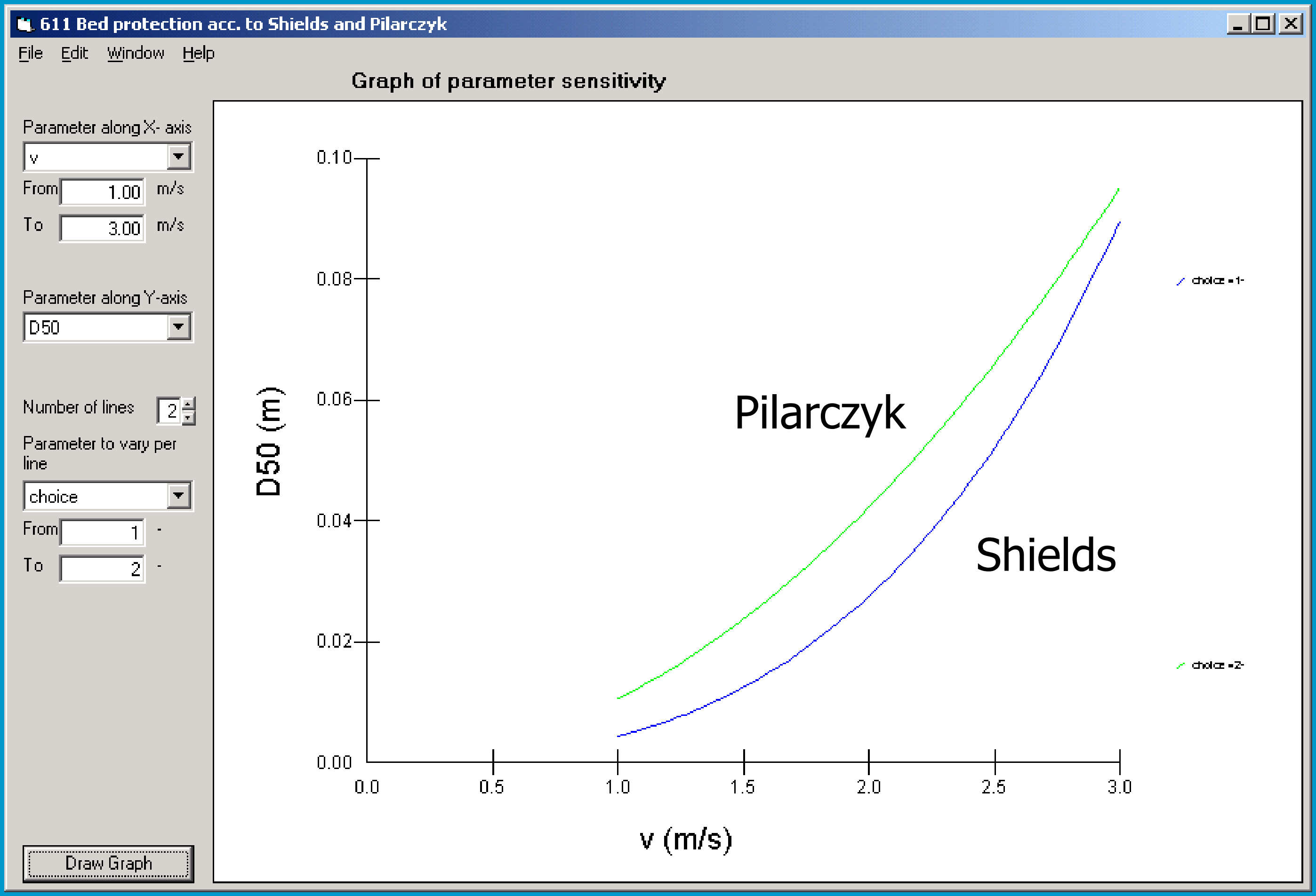
Comparison Shields and Pilarczyk



Movement of marine gravel (tests by Delft Hydraulics)

Flow velocity 0.65 m/s
Flow velocity 1.05 m/s
Flow velocity 1.35 m/s
Flow velocity 1.43 m/s
Flow velocity 1.53 m/s
Flow velocity 1.70 m/s
Flow velocity 1.80 m/s
Flow velocity 2.10 m/s

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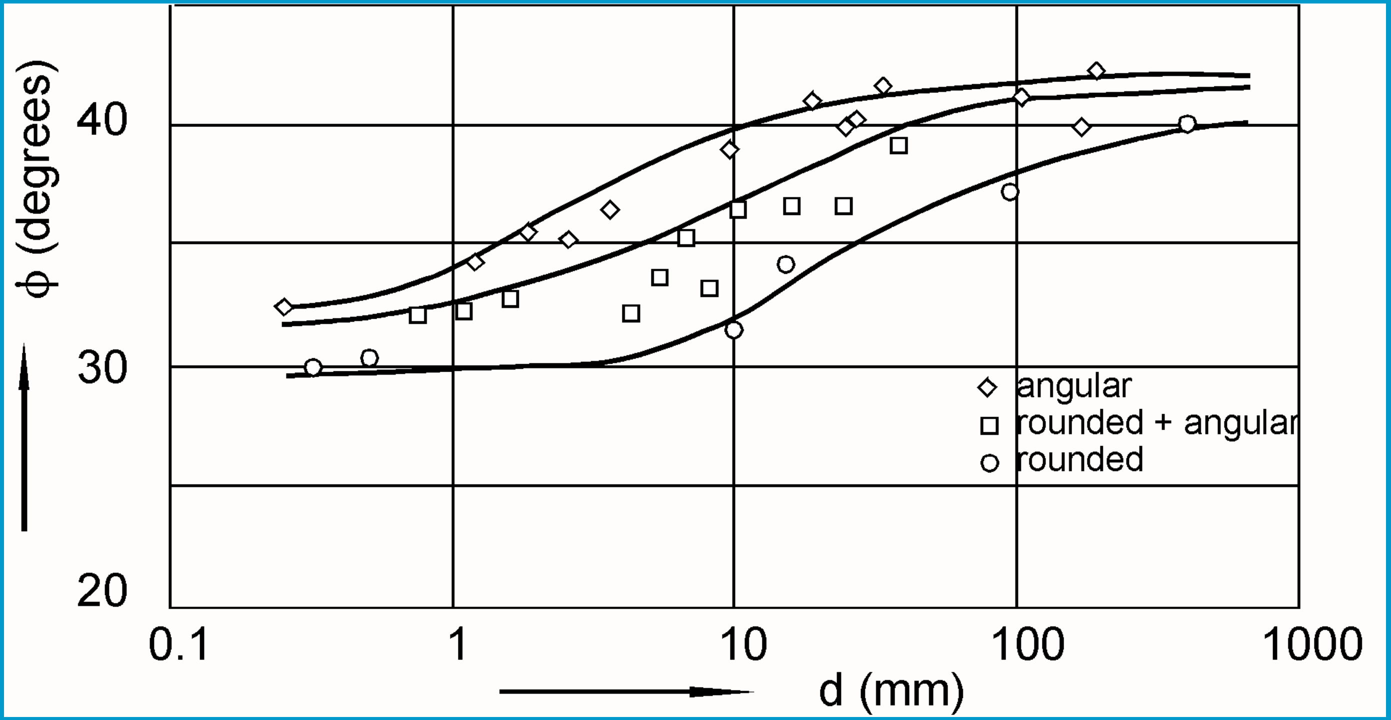
angles of repose for non-cohesive materials



1:1

1:1.5

1:2

1:3

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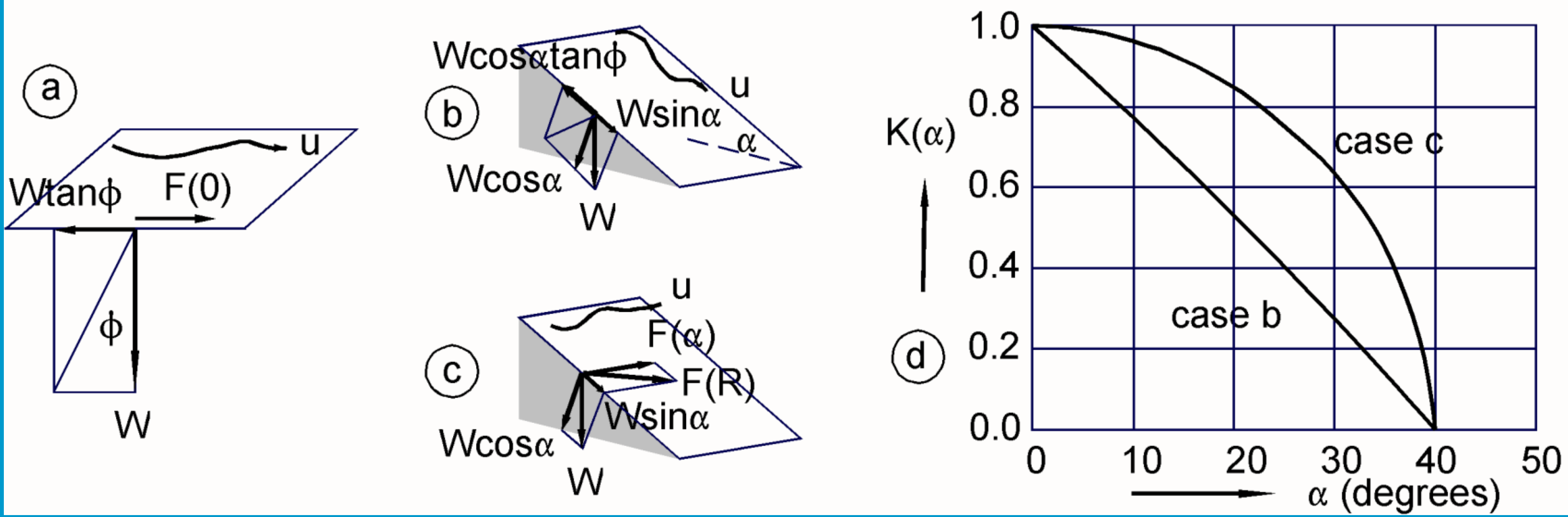
influence of slope on stability



b = parallel flow
c = perpendicular flow

slope parallel to current

$$K(\alpha_{//}) = \frac{F(\alpha_{//})}{F(0)} = \frac{W\cos\alpha\tan\phi - W\sin\alpha}{W\tan\phi} =$$

$$= \frac{\sin\phi\cos\alpha - \cos\phi\sin\alpha}{\sin\phi} = \frac{\sin(\phi - \alpha)}{\sin\phi}$$

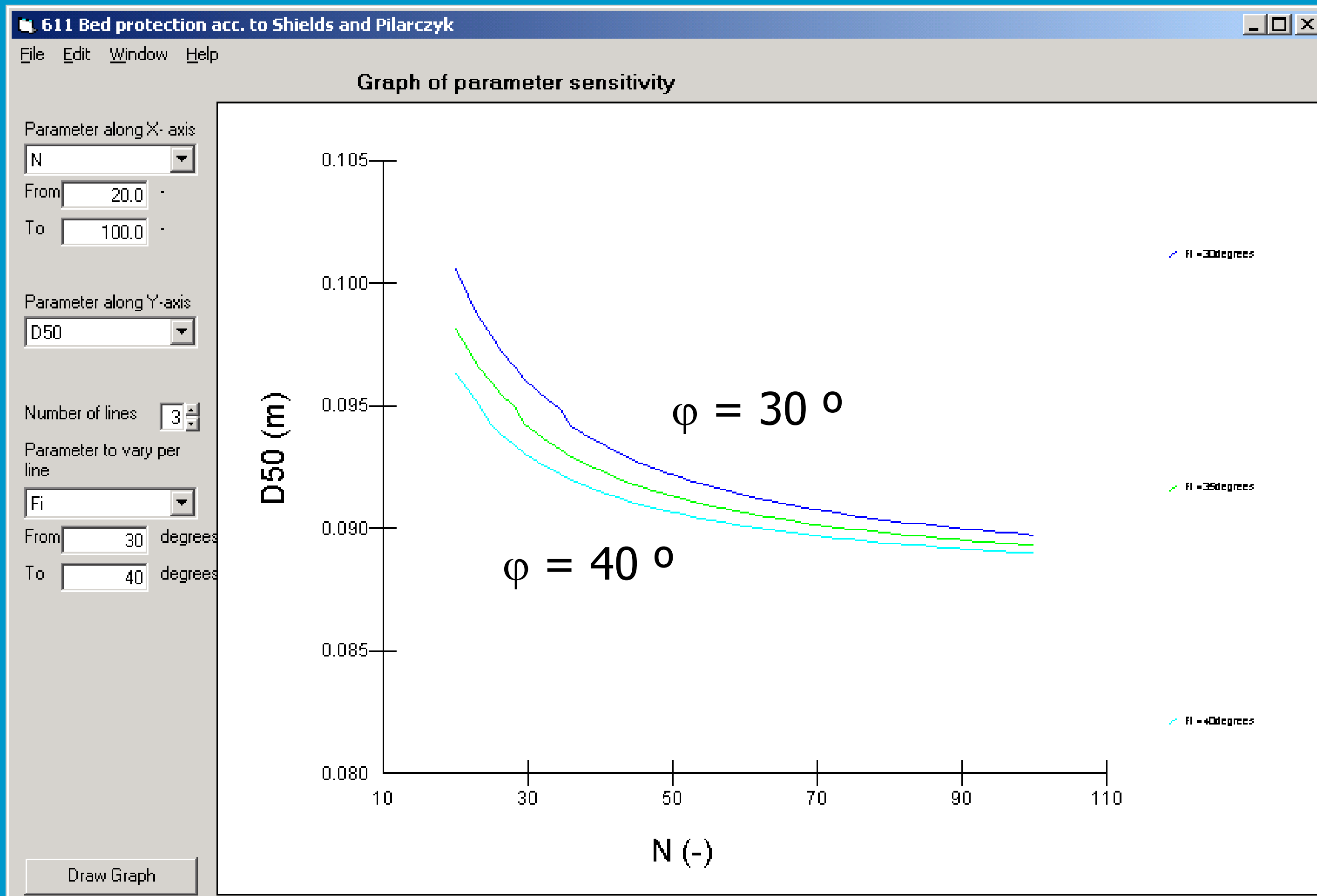
slope perpendicular to current

$$K(\alpha) = \frac{F(\alpha)}{F(0)} = \sqrt{\frac{\cos^2\alpha \tan^2\phi - \sin^2\alpha}{\tan^2\phi}} =$$

$$= \cos\alpha \sqrt{1 - \frac{\tan^2\alpha}{\tan^2\phi}} = \sqrt{1 - \frac{\sin^2\alpha}{\sin^2\phi}}$$

paral n from 1:20.. 1:100
perp. nfrom 1:2 .. 1:10
3 lines fi 30..40

Variation of bed slope (parallel flow)



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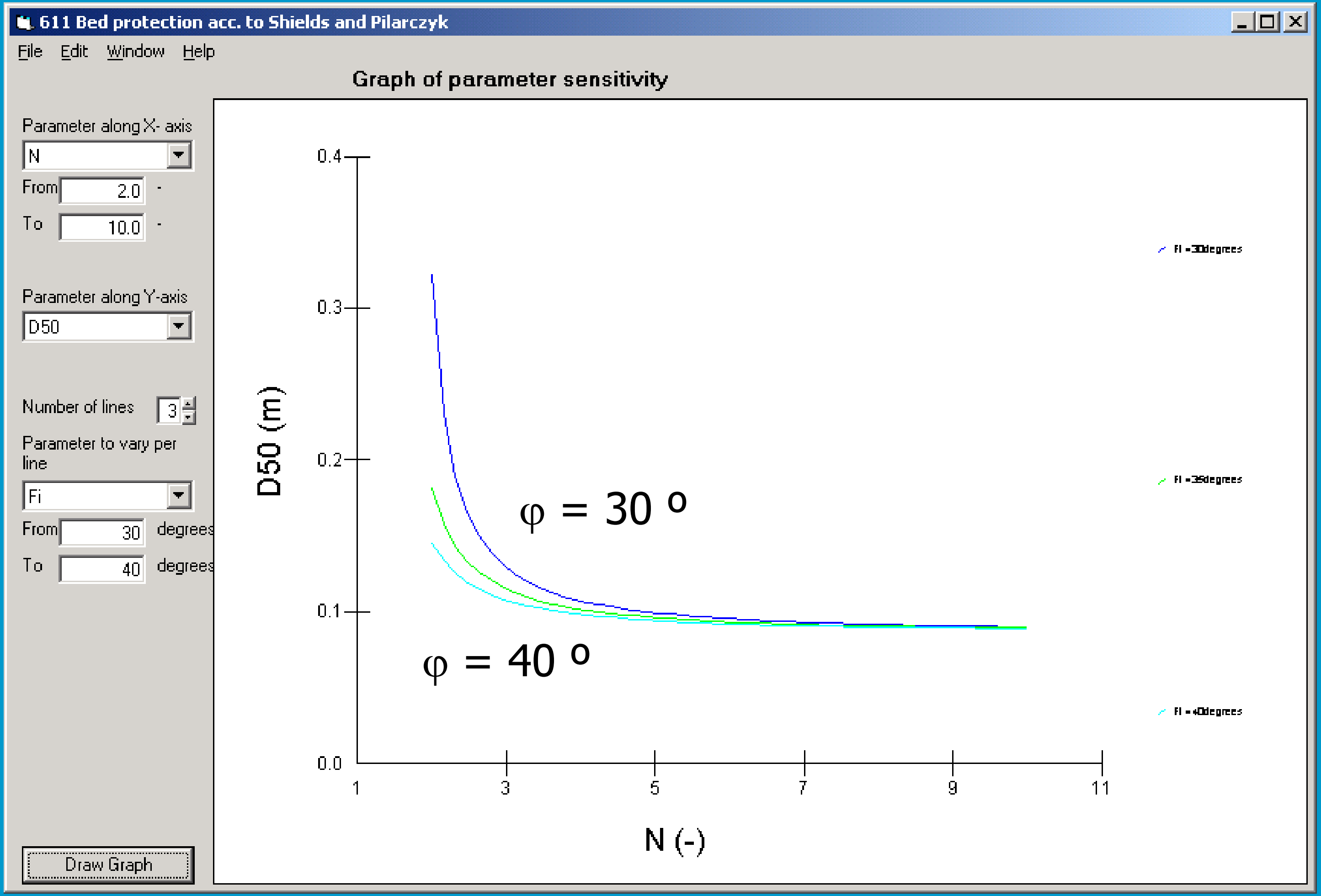
Variation of bed slope (perpendicular)



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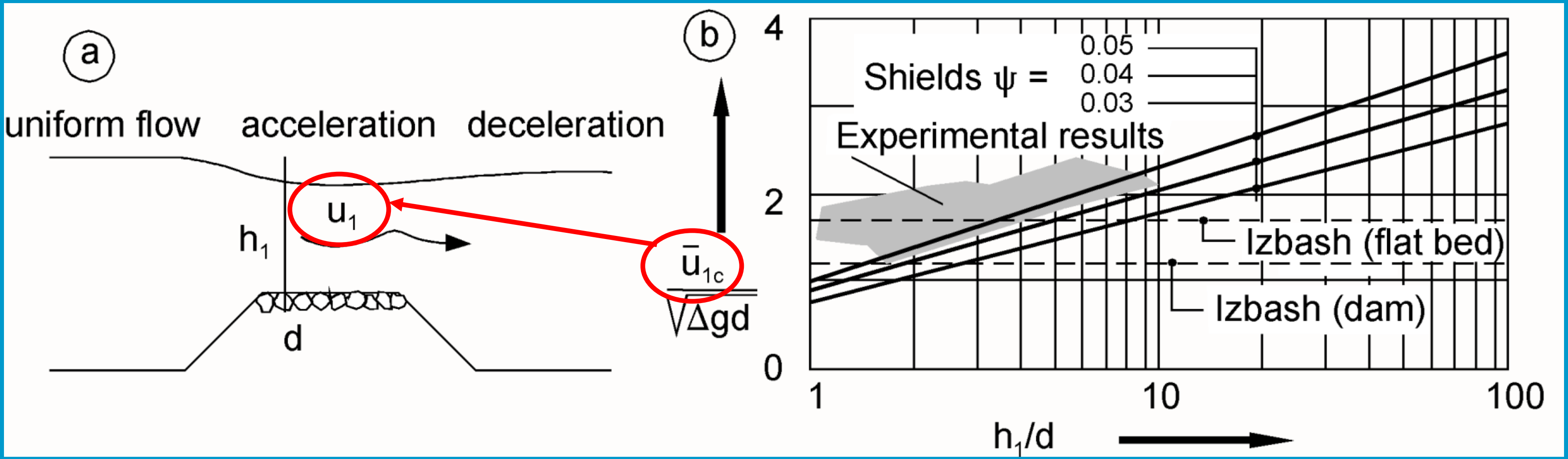
stability on top of sill

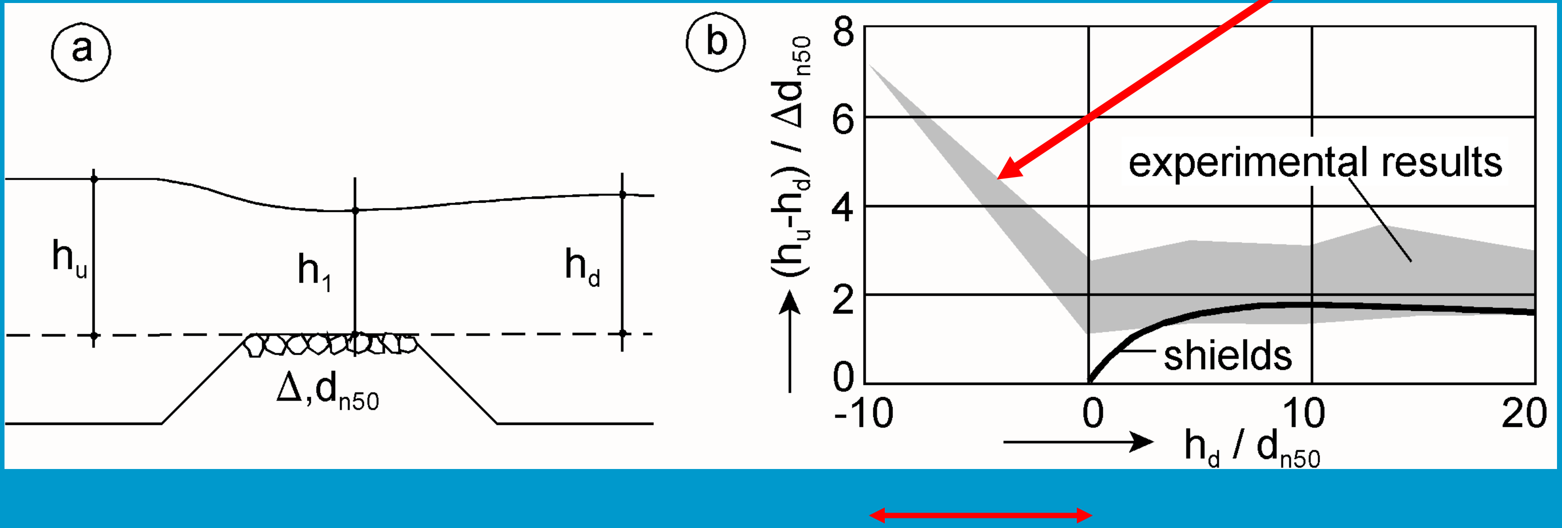


use velocity above threshold

Important aspect for closure works
much research has been done in the framework of the Deltaworks

stability and head difference

Shields is useless here because Shields contains waterdepth



waterlevel downstream is below the top of the dam

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shields for flow over sill

$$u_1^2 = \mu^2 \, 2g(h_u - h_d) = (0.5 + 0.04\frac{h_d}{d_{n50}}) \, 2g(h_u - h_d)$$

discharge coefficient

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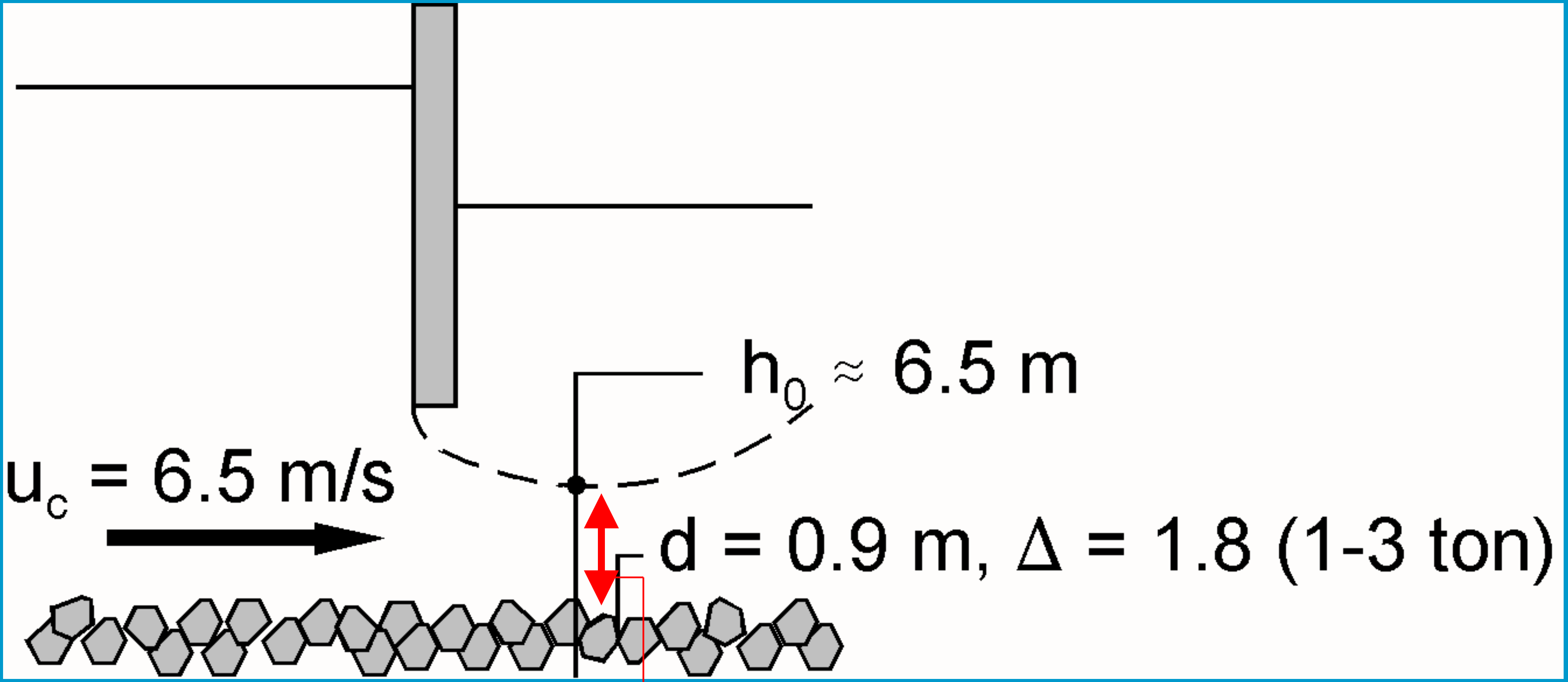
stability flow under weir



Note that this cross section is less than the gap width

Shields in horizontal constriction

(horizontal closure with trucks)

$$\frac{\overline{u_{gap}}}{\sqrt{\Delta g d_{n50}}} = C \sqrt{\frac{\psi_c}{g}} \qquad \sqrt[4]{1 - \frac{\sin^2 \alpha}{\sin^2 \phi}} = 4.5 \log\left(\frac{3h}{d_{n50}}\right)\sqrt{\psi_c}$$

General formula

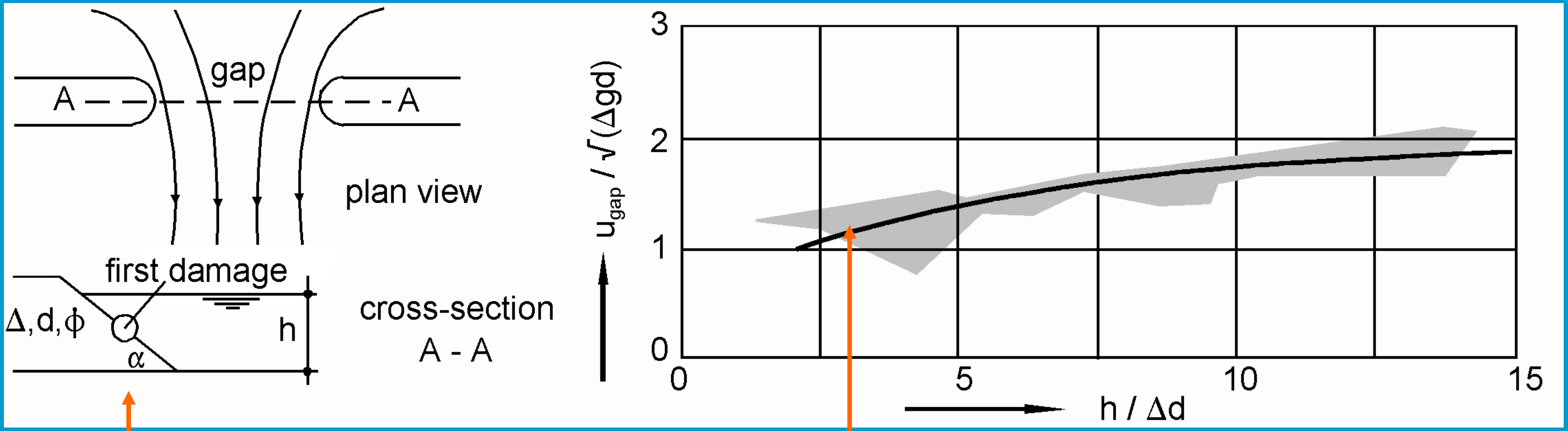
Correction for horizontal closure

α slope of construction

(see also next slide)

φ angle of repose (internal stability)

stability on head of dam



α from equation

in book: h/d

$\Psi_c = 0.04$

deceleration

$$K_v = \frac{u_c\, uniform\ flow}{u_c\, with\, load\ increase}$$

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relation between K and turbulence level

$$\left(1+3r_{cu}\right)\bar{u}_{cu} = \left(1+3r_{cs}\right)\bar{u}_{cs} \longrightarrow K_v = \frac{\bar{u}_{cu}}{\bar{u}_{cs}} = \frac{1+3r_{cs}}{1+3r_{cu}}$$

u_{cu} = vertically averaged critical velocity in uniform flow
u_{cs} = velocity in case with a structure
r_{cu} = turbulence intensity in uniform flow
r_{cs} = vertically averaged turbulence intensity

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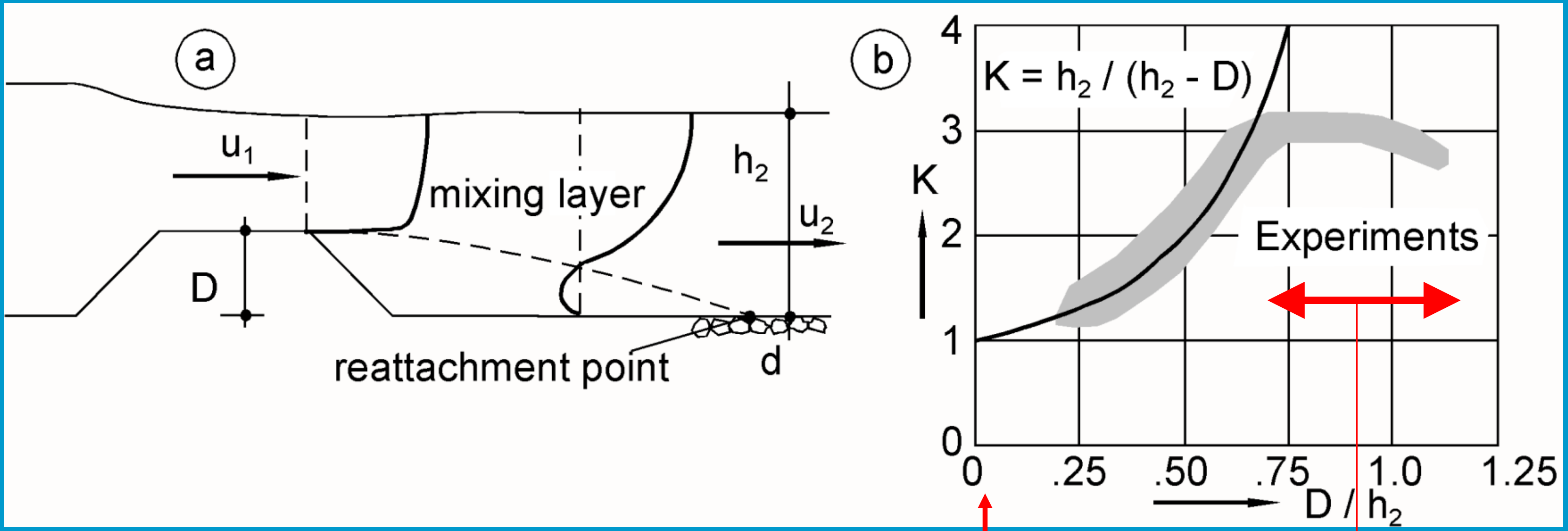
stability downstream of a sill



no dam

high dam

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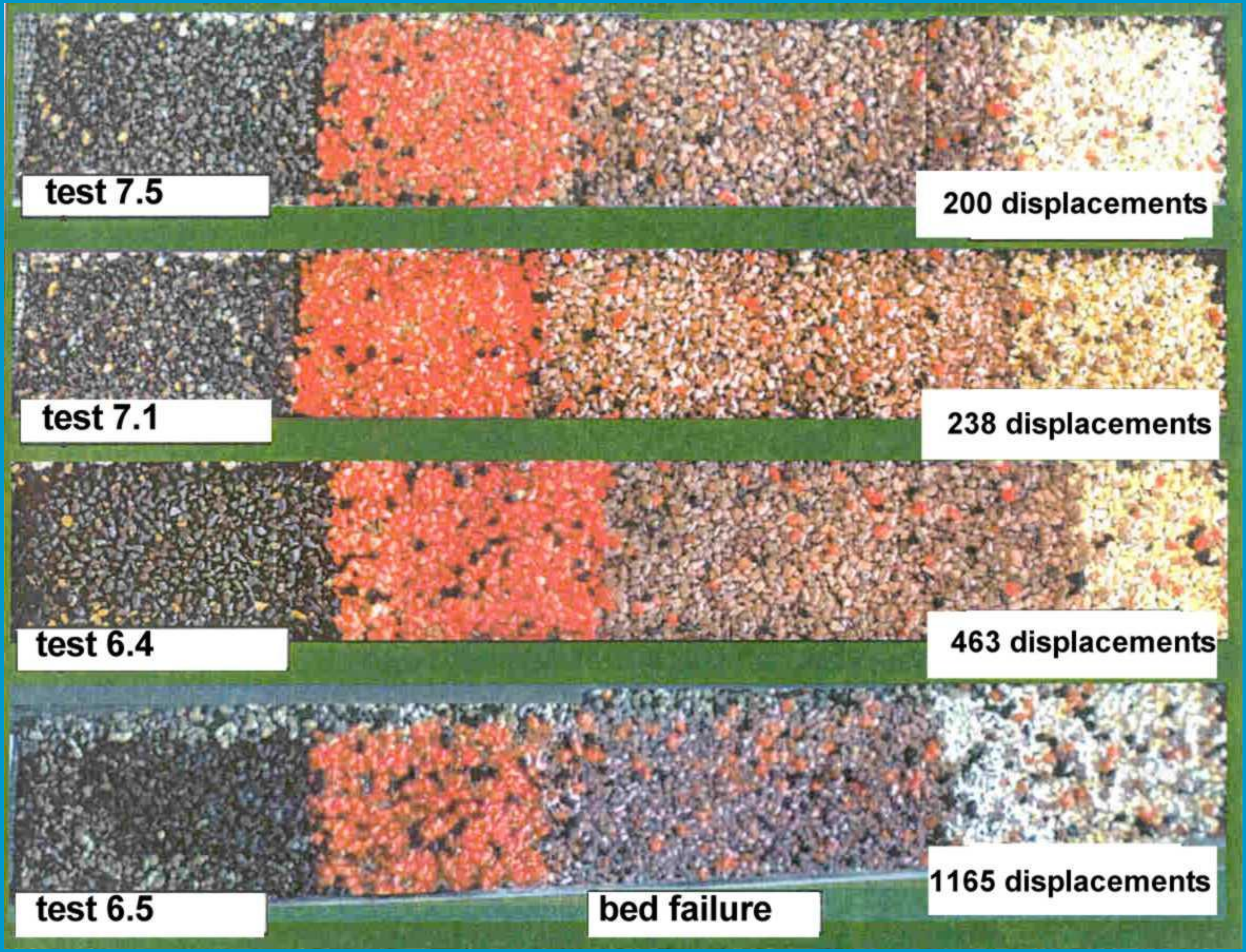
damage after some time



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K in vertical constriction

$$u_1(h_2 - D) = u_2 h_2 \rightarrow u_1 = \frac{h_2}{h_2 - D} u_2 \rightarrow K \propto \frac{h_2}{h_2 - D}$$

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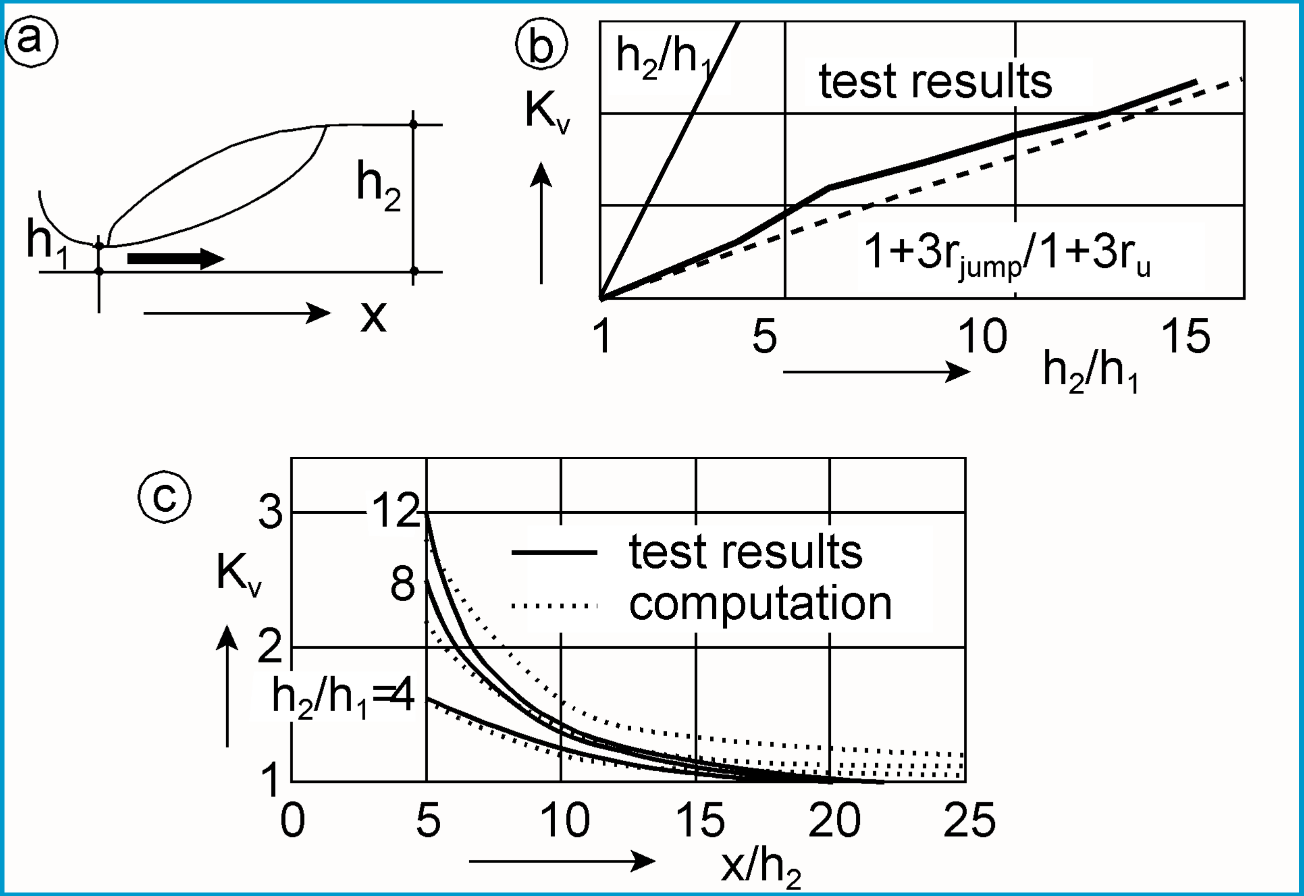
stone stability downstream of a hydraulic jump



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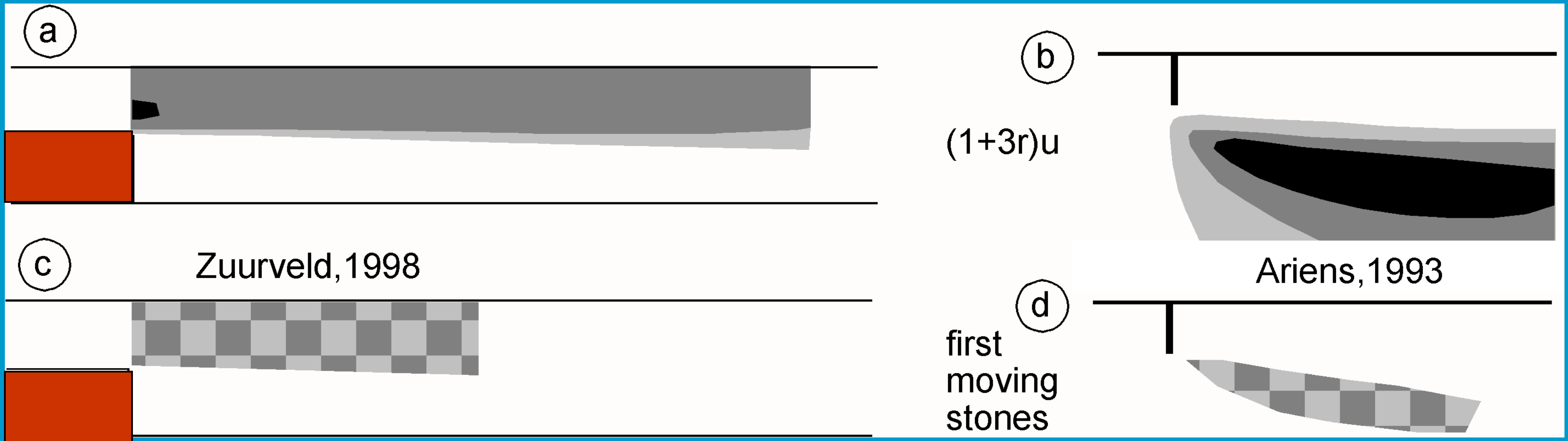
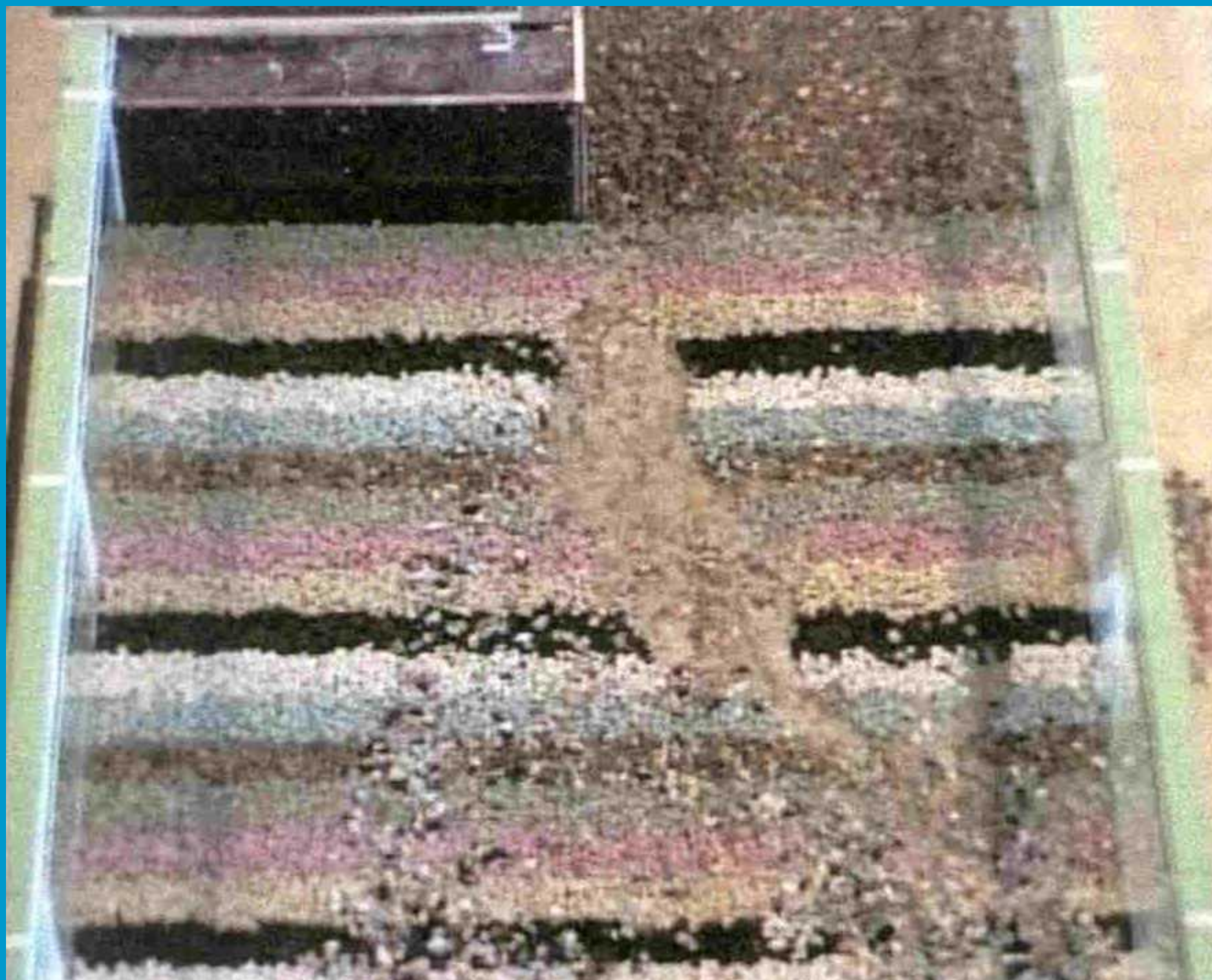
peak velocities and incipient motion in horizontal constriction



damage after constriction



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bb 4310-2 Expansion sbability

zuurveld,
1998

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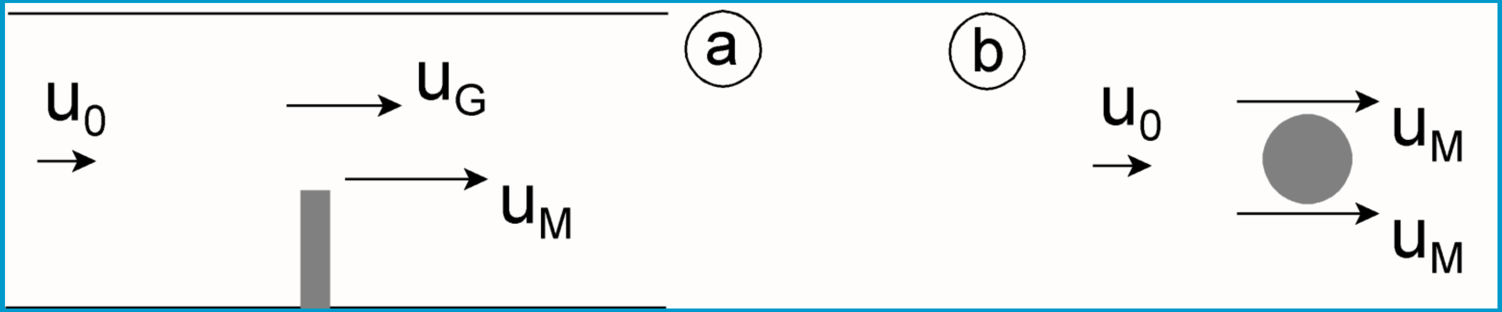
definition of velocities



groyne

vertical pole

K_v - factors for various structures

Structure	Shape	K_{v0}	K_{vG}	K_{vM}
Groyne	Rect-angular	b_0*K_{vG}/b_G	1.3 - 1.7	1.1 - 1.2
	Trape-zoidal	$b_0*K_v b_G$	1.2	1
Abutment	Rect-Angular	b_0*K_v/b_G	1.3 - 1.7	1.2
	Round	b_0*K_v/b_G	1.2 - 1.3	1.2
	Stream Lined	b_0*K_v/b_G	1 - 1.1	1 - 1.1
Pier	Round	b_0*K_v/b_G ⊗ $2*K_v$	1.2 - 1.4 ⊗	1 - 1.1
	Rect-Angular	b_0*K_v/b_G ⊗ $2*K_v$	1.4 - 1.6 ⊗	1.2 - 1.3
Outflow	Abruptly	--	1	--
	Stream Lined	--	0.9	--
Sill	Top	Section 3.6.1	Section 3.6.1	Section 3.6.1
	Down Stream	Fig 3.13	Fig 3.13	Fig 3.13

⊗ *For many piers in a river the first expression for K_v is appropriate. The second is valid for a detached pier in an infinitely wide flow, where K_G is not defined.*

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combined equation

$$d = \frac{K_v^2 \cdot \overline{u_c}^{\,2}}{K_s \psi_c \Delta C^2}$$

K_v = reduction for constriction, etc.
K_s = reduction for slope (parallel, perpendicular)

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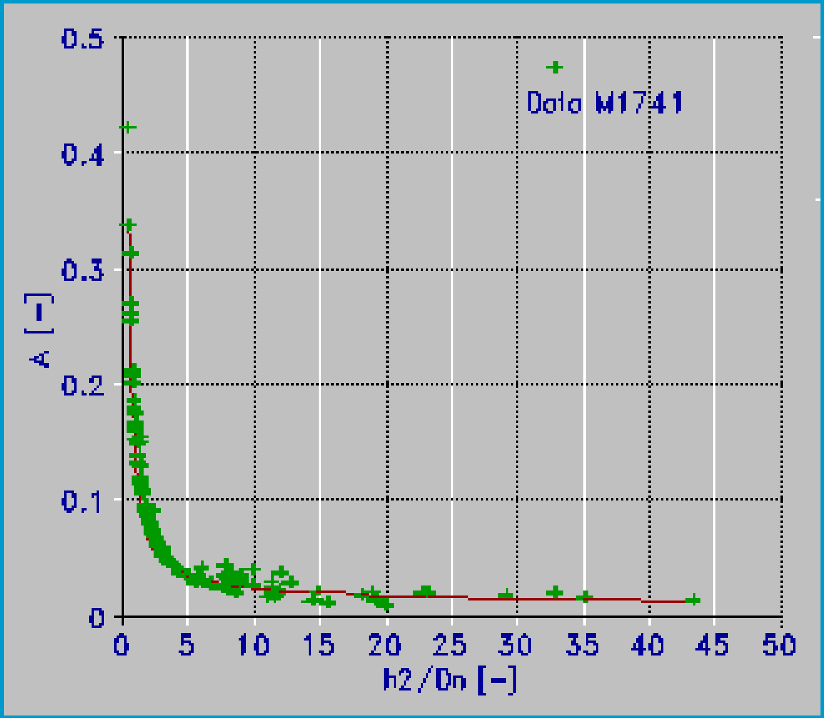
practical application (1)

$$d = \frac{K_v^2 \cdot \overline{u_c}^{\,2}}{K_s \psi_c \Delta C^2}$$

However, in practice $K_v^2 / K_s \cong 1$

$$\Delta D_n = A \bullet u_c^2$$



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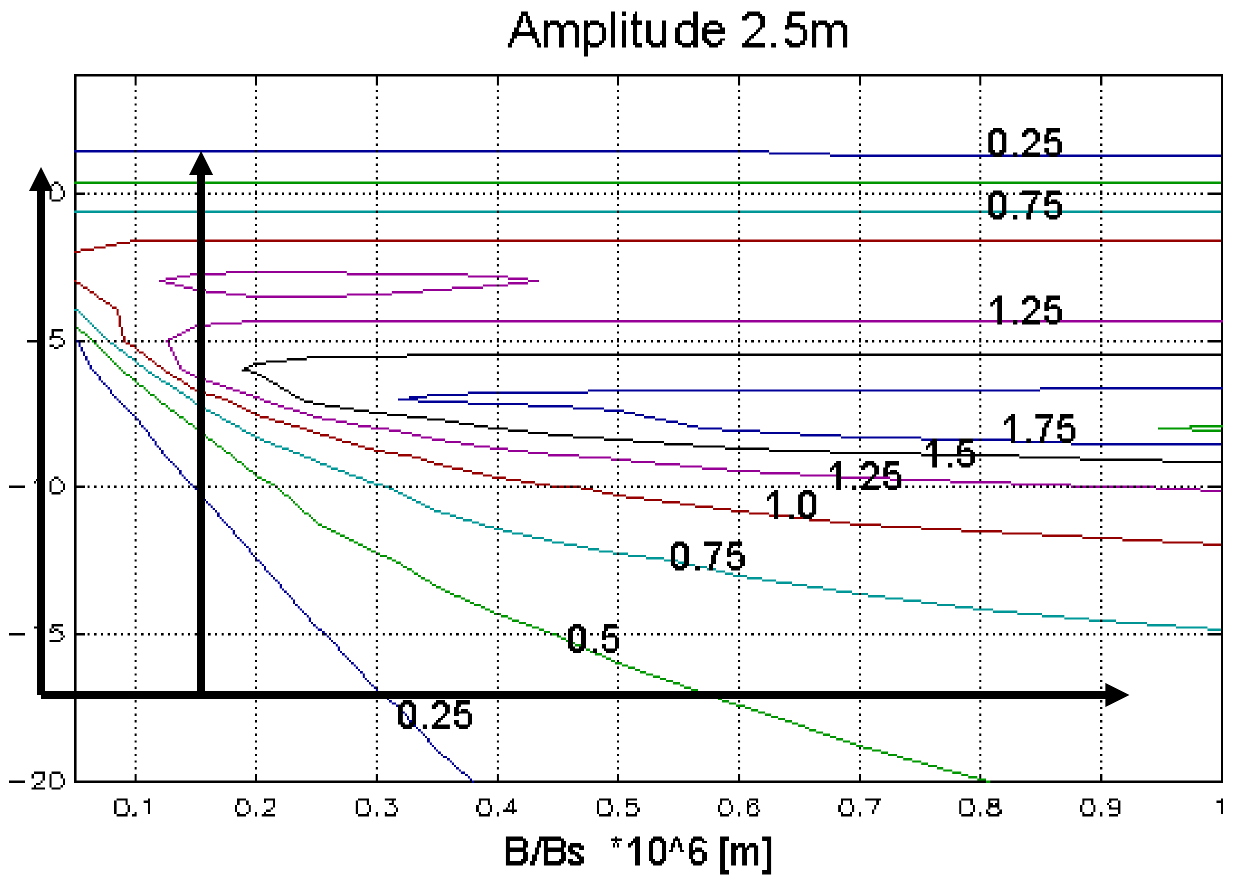
practical application (2)



Values of ΔD_N as a function of B/B_s and d' (on vertical axis)

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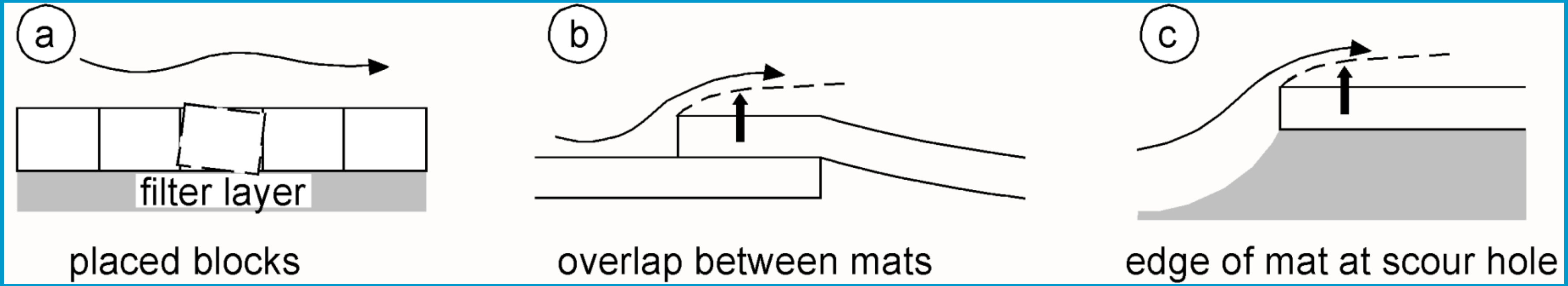
placed blocks



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