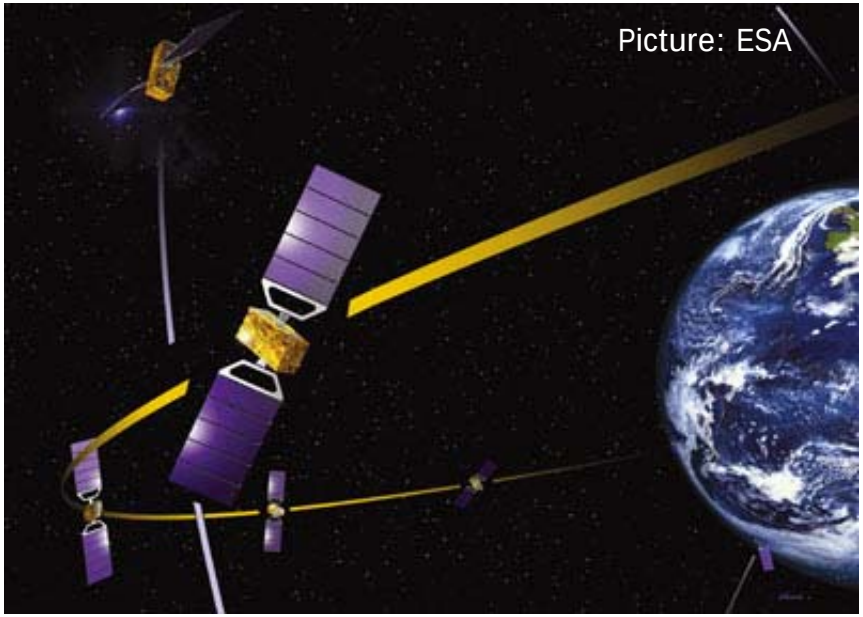
Satellite Navigation
GPS measurements and error sources



AE4E08

Sandra Verhagen

Course 2010 – 2011, lecture 4

Today's topics

- Recap: GPS signal components
- Code Phase measurements → pseudoranges
- Carrier Phase measurements
- GPS measurements: example
- Outlook: error sources

- Book: Section 5.1

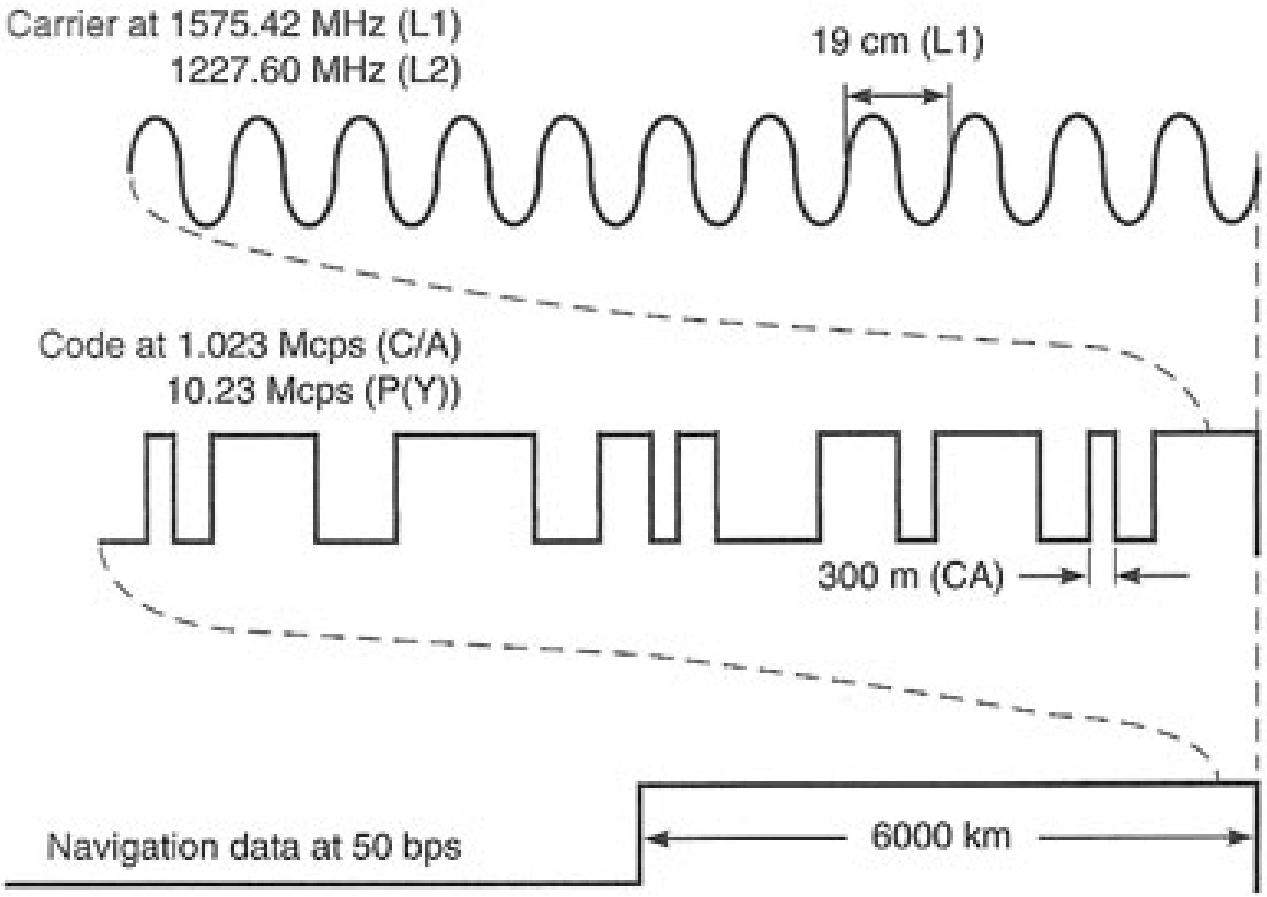
Recap: GPS signal components

- All signals and time information are coherently derived from the same clock with a frequency of **f_0=10.23 MHz**

-

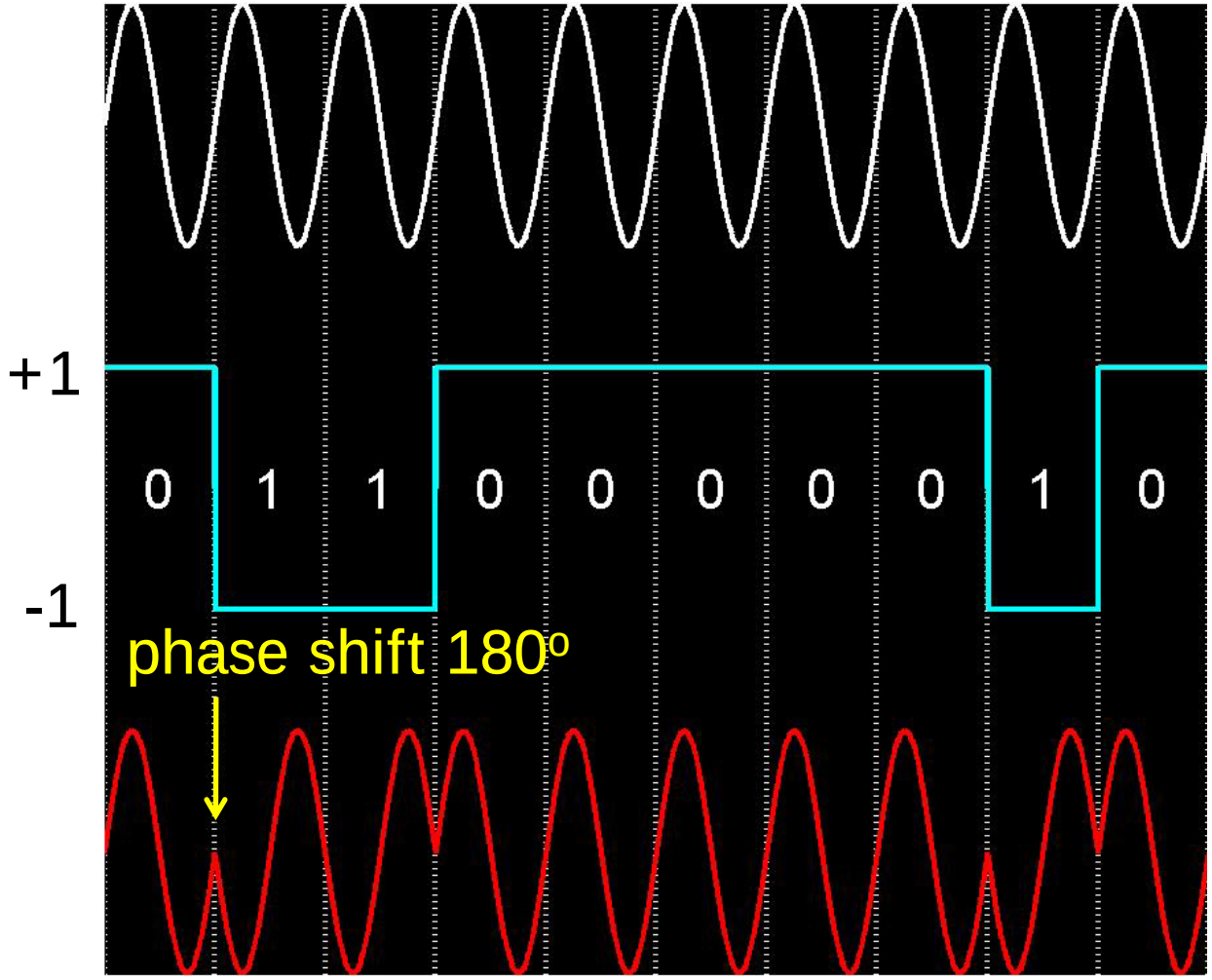
Signal components	Frequency	Wavelength / chiplength
L1 carrier	1575.42 MHz (154*f_0)	19.05 cm
L2 carrier	1227.60 MHz (120*f_0)	24.45 cm
C/A code on L1 with	1.023 Mbits/sec (0.1*f_0)	293 m
P code on L1 and L2	10.23 Mbits/sec (f_0)	29.3 m
Broadcast message	50 bits/sec	

Recap: GPS signal components



From: Misra and Enge

Recap: GPS signal components



carrier
$f(t)$

code and data
$C(t) \oplus D(t)$

signal
$S(t)$

Code Phase measurements

τ ? transit time → 70 to 90 ms

t ? **true** GPS time at which code is received

$t^s(t-\tau)$ emission time (imprinted on signal)

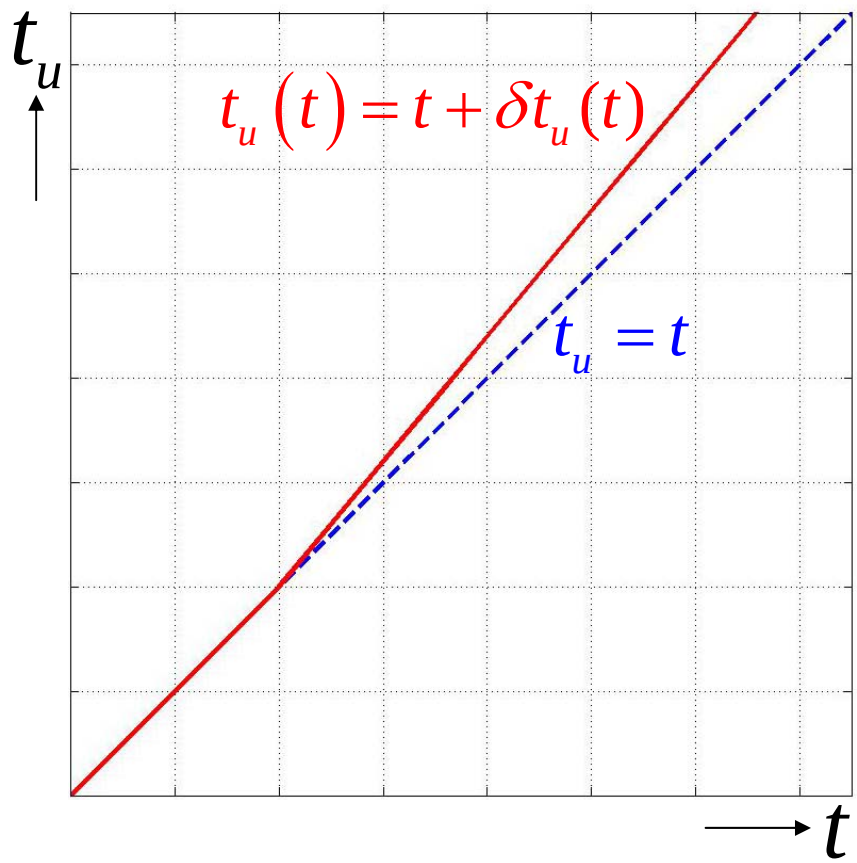
$t_u(t)$ measured arrival time (clock reading)

$$\rho(t) = c\left[t_u(t) - t^s(t-\tau)\right]$$ pseudorange

Code Phase measurements

$$t_u(t) = t + \delta t_u(t)$$

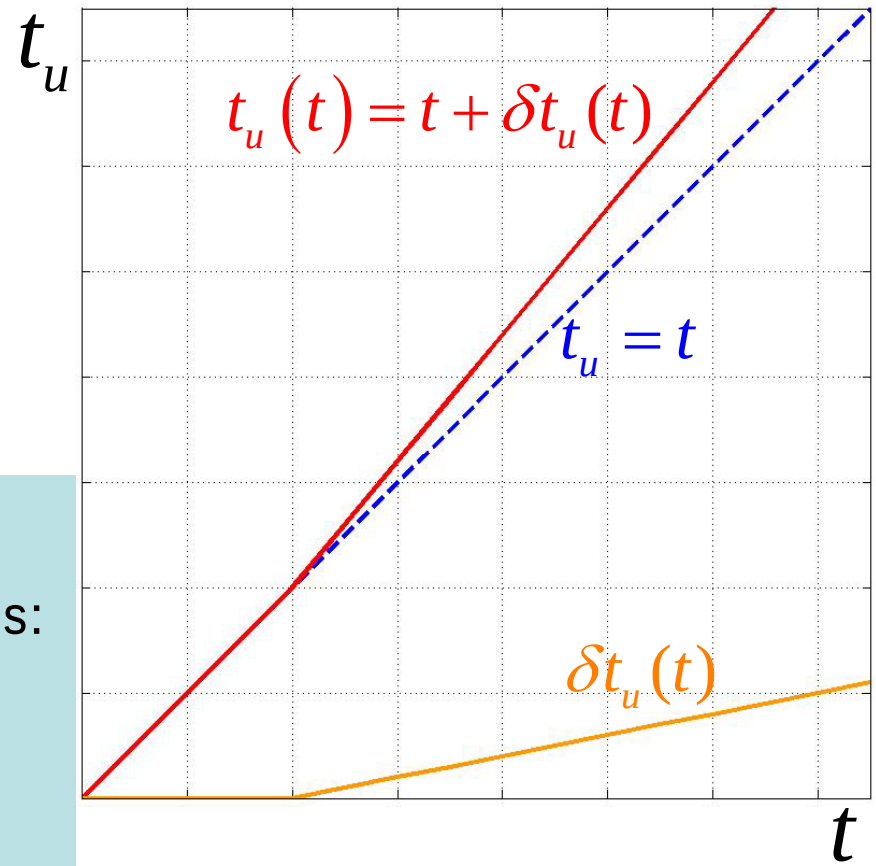
receiver clock bias



Code Phase measurements

$$t_u(t) = t + \delta t_u(t)$$

receiver clock bias



Receiver clocks: drift!

Deviation from GPS time limited to ±1 ms:

- continuous clock steering
- reset (clock jump!) when certain threshold is reached

Code Phase measurements

$$t_u(t) = t + \delta t_u(t)$$

receiver clock bias

$$t^s(t-\tau) = (t-\tau) + \delta t^s(t-\tau)$$

satellite clock bias

estimated by control segment

Code Phase measurements

$$t_u(t) = t + \delta t_u(t)$$

$$t^s(t-\tau) = (t-\tau) + \delta t^s(t-\tau)$$

Unmodeled effects and errors

$$\begin{aligned}\rho(t) &= c\left[t_u(t) - t^s(t-\tau)\right] + \varepsilon_\rho(t) \\ &= c\left[t + \delta t_u(t) - (t-\tau) - \delta t^s(t-\tau)\right] + \varepsilon_\rho(t) \\ &= c\tau + c\left[\delta t_u(t) - \delta t^s(t-\tau)\right] + \varepsilon_\rho(t)\end{aligned}$$

pseudorange

$$\rho(t) = c\tau + c\left[\delta t_u(t) - \delta t^s(t-\tau)\right] + \varepsilon_\rho(t)$$

clock biases

noise + errors

distance traveled by signal

pseudorange

$$\rho(t) = c\tau + c\left[\delta t_u(t) - \delta t^s(t-\tau)\right] + \varepsilon_\rho(t)$$

clock biases

noise + errors

distance traveled by signal

$$c\tau = r(t, t-\tau) + I_\rho(t) + T_\rho(t)$$

geometric range

ionosphere and troposphere delays

pseudorange

$$\rho = r + I_\rho + T_\rho + c\left[\delta t_u - \delta t^s\right] + \varepsilon_\rho$$

pseudorange measurement = biased and noisy measurement of the **geometric range *r***

Not to be studied: part on “Constructing pseudorange measurements” in Section 5.1.1

pseudorange measurements: example

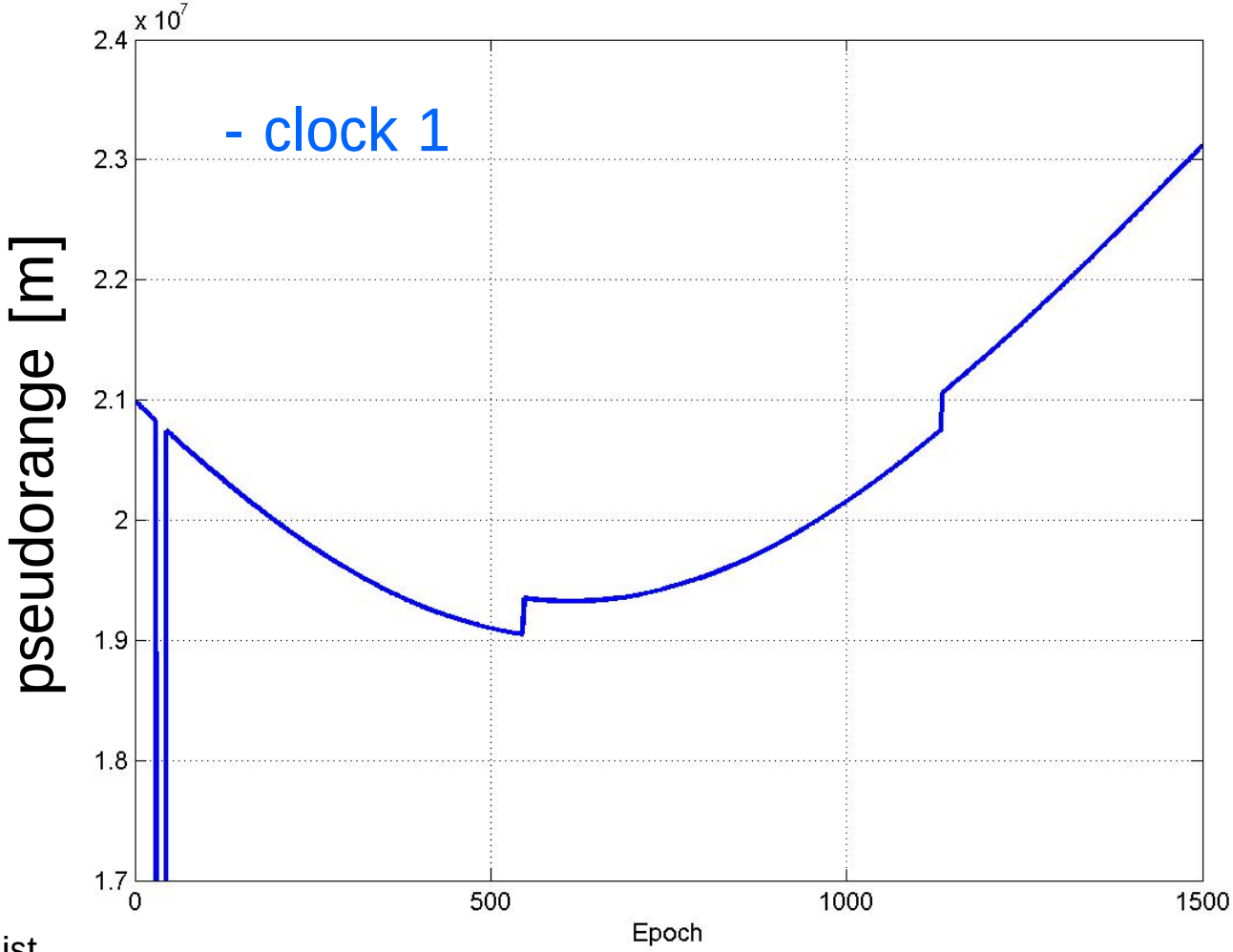


Figure: Peter Buist

pseudorange measurements: example

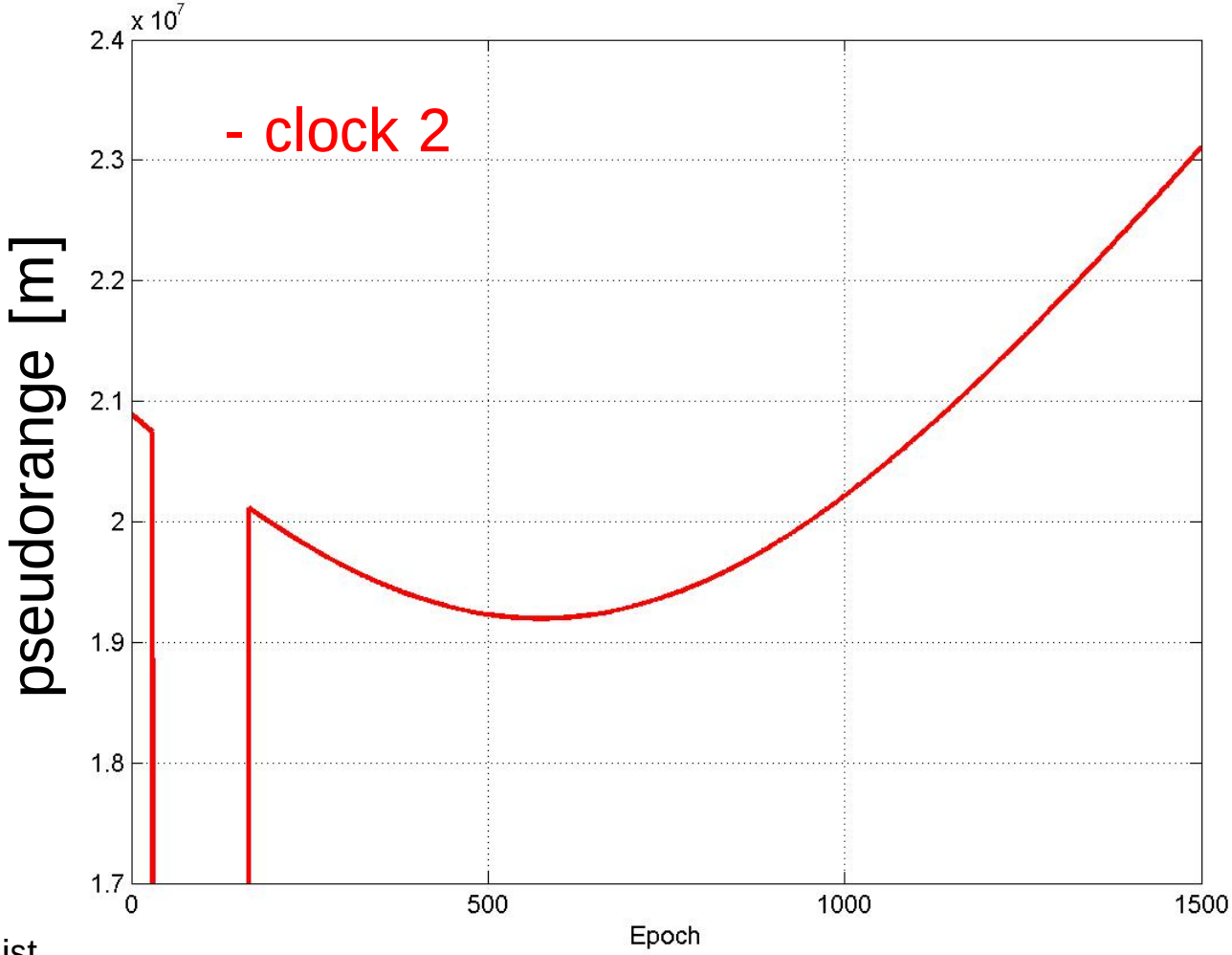


Figure: Peter Buist

pseudorange measurements: example

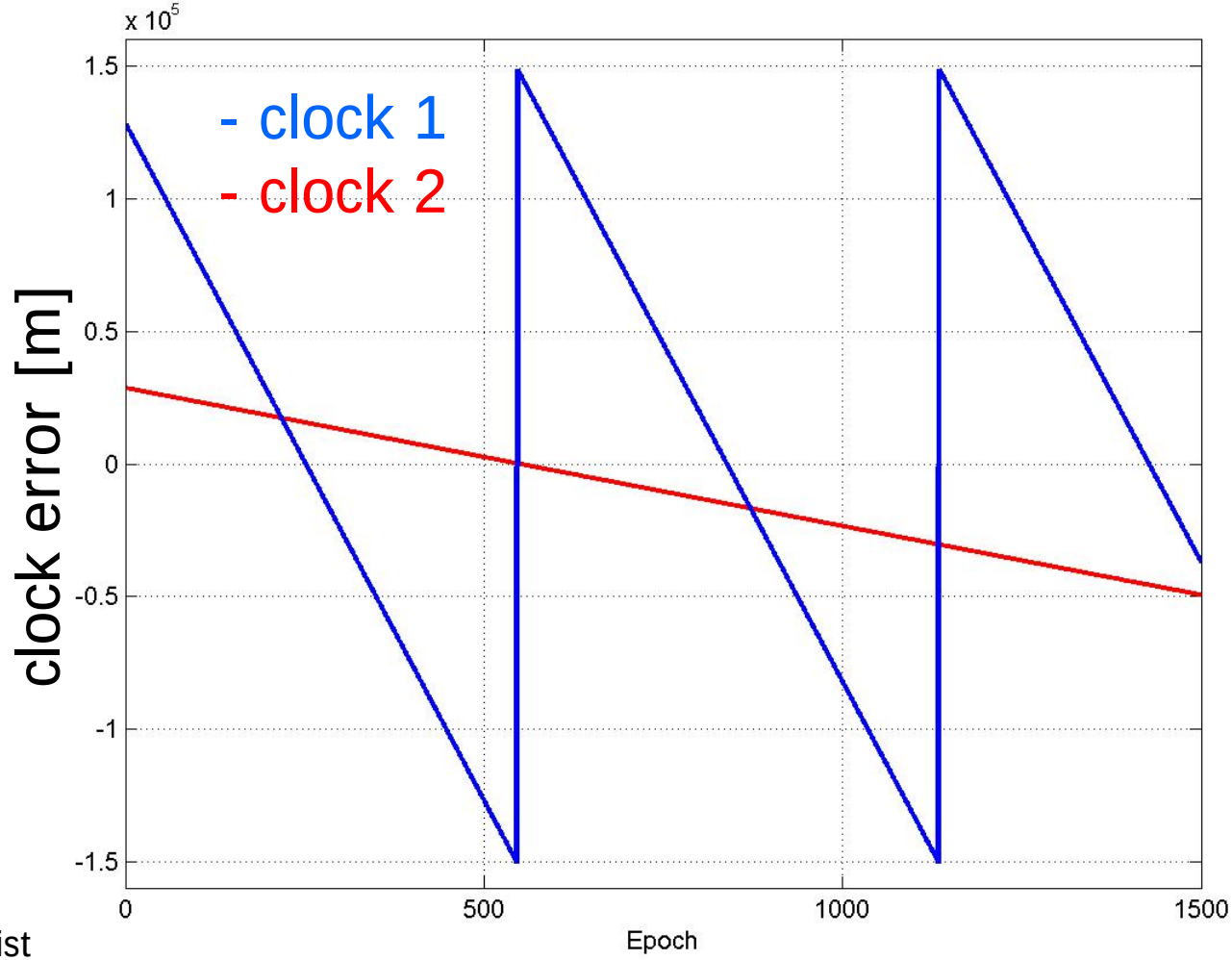
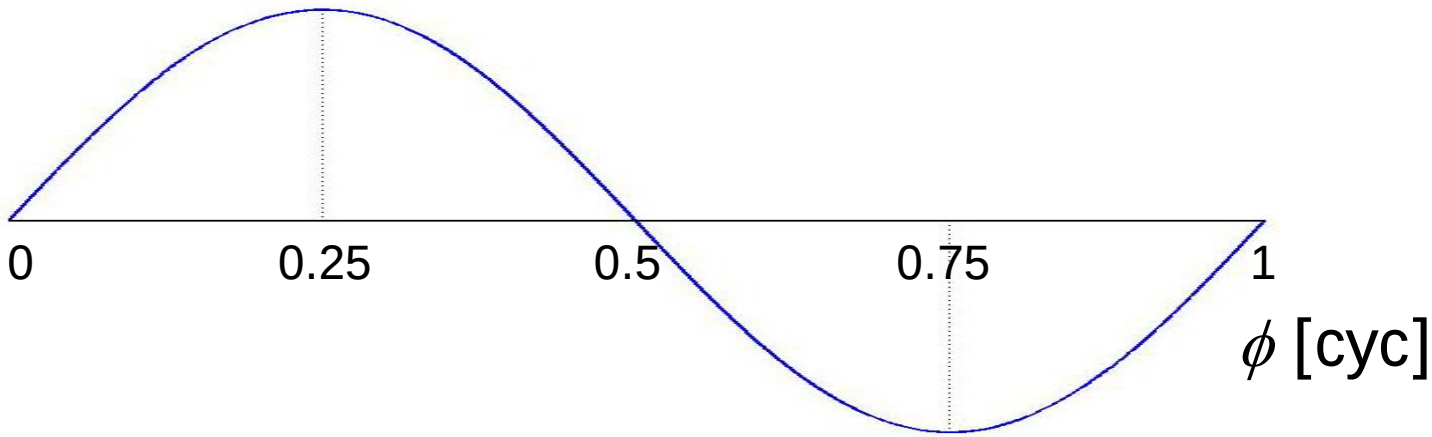


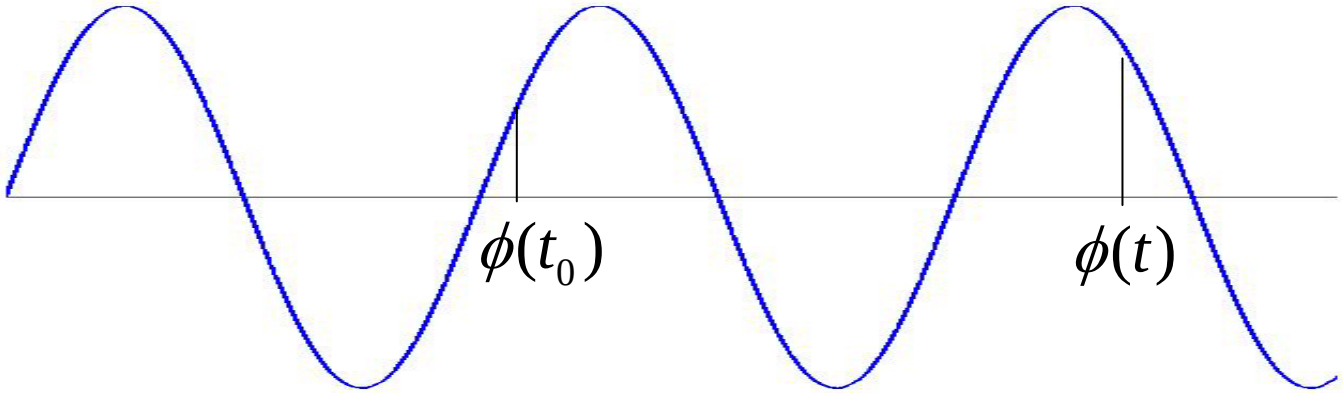
Figure: Peter Buist

Carrier Phase measurements



Very precise!

Carrier Phase measurements

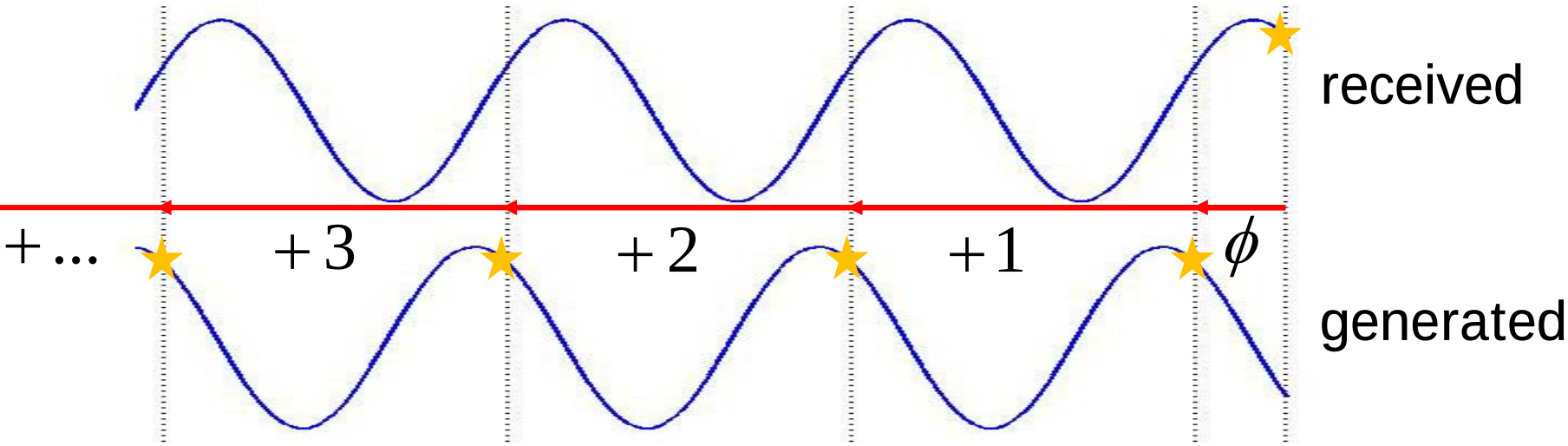


ϕ [cyc]

$f \cdot (t - t_0)$ = number of cycles since starting point of interval

Carrier phase : $\phi(t) = \phi(t_0) + f \cdot (t - t_0)$

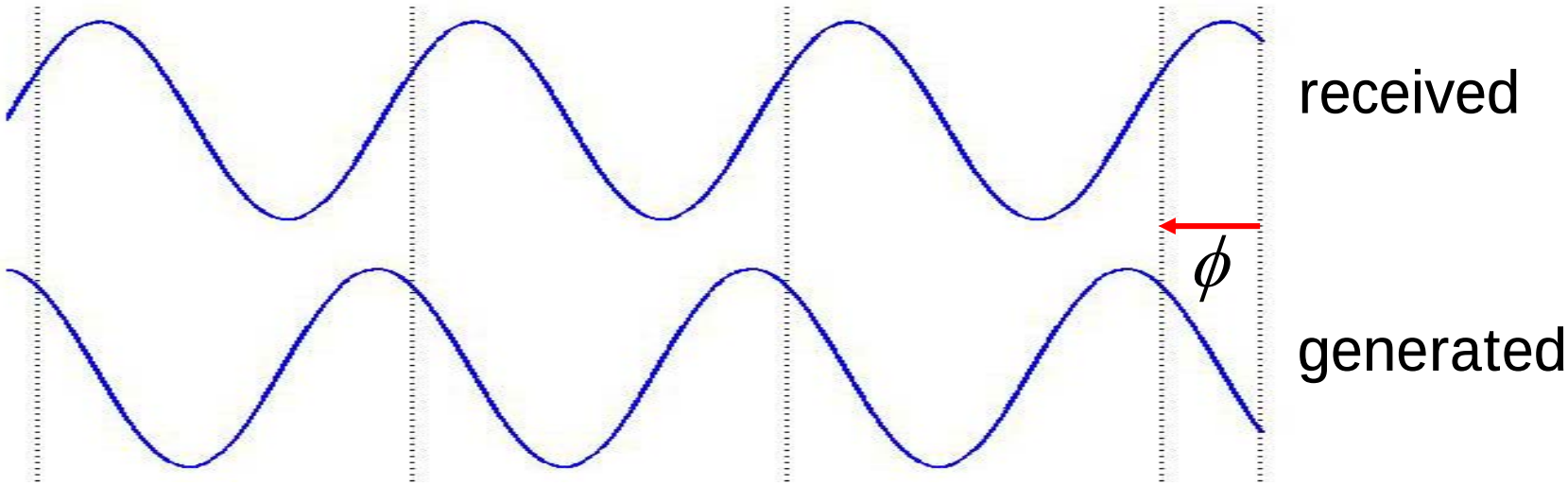
Carrier Phase measurements



Carrier phase measurement:

- Difference between phases of receiver-generated carrier signal and received carrier signal

Carrier Phase measurements



Carrier phase measurement:

- Difference between phases of receiver-generated carrier signal and received carrier signal
- Phase measurement + whole number of cycles traveled → range
- Change in phase continuously measured (incl. full cycles)

Carrier Phase measurements

$$\phi(t) = \phi_u(t) - \phi^s(t-\tau) + N + \varepsilon_\phi$$

Recall: $\phi(t) = \phi(t_0) + f \cdot (t - t_0)$

$$\phi_u(t) = \phi_u(t_0) + f \cdot (t - t_0) + f \cdot \underbrace{(\delta t_u(t) - \delta t_u(t_0))}_{\text{clock biases}}$$

$$\phi^s(t-\tau) = \phi^s(t_0) + f \cdot (t - \tau - t_0) + f \cdot \overbrace{(\delta t^s(t-\tau) - \delta t^s(t_0))}^{\text{clock biases}}$$

Carrier Phase measurements

$$\phi(t) = \phi_u(t) - \phi^s(t-\tau) + N + \varepsilon_\phi$$

$$\phi_u(t) = \phi_u(t_0) + \cancel{f \cdot (t - t_0)} + f \cdot \left(\delta t_u(t) - \delta t_u(t_0)\right)$$

$$\phi^s(t-\tau) = \phi^s(t_0) + f \cdot (\cancel{t} - \tau - \cancel{t_0}) + f \cdot \left(\delta t^s(t-\tau) - \delta t^s(t_0)\right)$$

$$\phi(t) = f \cdot \tau$$

$$+ f \cdot \left(\delta t_u(t) - \delta t^s(t-\tau)\right)$$ - clock biases

$$+\phi_u(t_0) - \phi^s(t_0)$$ - initial phases

$$- f \cdot \left(\delta t_u(t_0) - \delta t^s(t_0)\right)$$ - clock biases at t_0

$$+N$$ - integer ambiguity (constant)

$$\Big\} A$$

$$+\varepsilon_\phi$$ - noise and errors

Carrier Phase measurements

$$\boxed{f = \frac{c}{\lambda}}$$

$$\begin{aligned}\phi(t) = {} & f \cdot \tau \\ & + f \cdot \left(\delta t_u(t) - \delta t^s(t-\tau)\right) \\ & + A \\ & + \varepsilon_\phi\end{aligned}$$

$$\begin{aligned}\lambda \cdot \phi(t) = {} & c \cdot \tau \\ & + c \cdot \left(\delta t_u(t) - \delta t^s(t-\tau)\right) \\ & + \lambda \cdot A \\ & + \lambda \cdot \varepsilon_\phi\end{aligned}$$

cycles

meters

Carrier Phase measurements

$$\lambda \cdot \phi(t) = c \cdot \tau + c \cdot \left(\delta t_u(t) - \delta t^s(t-\tau) \right) + \lambda \cdot A + \lambda \cdot \varepsilon_\phi$$

$$\Phi = r + I_\phi + T_\phi + c \cdot \left(\delta t_u - \delta t^s \right) + \lambda \cdot A + \varepsilon_\Phi$$

Ambiguities must be resolved to take advantage of high precision phase measurements

GPS measurements: example

Dataset June 1st, 2005 - GPS week 1325
00:00-23:59 (GPS-time)
10 seconds interval
Trimble 4700 receiver
dual-frequency GPS (L1 & L2)
choke-ring antenna
at GNSS-observatory in Delft

RINEX: Receiver Independent Exchange format

data (delf1520) provided by H. van der Marel
plots by Q. Le, photo by R. Kremers

Rinex version 2

```
 2.10           OBSERVATION DATA    G (GPS)                 RINEX VERSION / TYPE
teqc  2002Mar14     Automatic GPS proces20050608 13:15:24UTCPGM / RUN BY / DATE
Linux 2.0.36|Pentium II|gcc|Linux|486/DX+                   COMMENT
BIT 2 OF LLI FLAGS DATA COLLECTED UNDER A/S CONDITION       COMMENT
DELFT-16                                                    MARKER NAME
13502M004                                                   MARKER NUMBER
H. VAN DER MAREL    AGRS.NL (KAD,MD,TUD)                    OBSERVER / AGENCY
96517               TRIMBLE 4700        N1.30/S0.00         REC # / TYPE / VERS
93258               TRM29659.00     UNAV                    ANT # / TYPE
  3924687.7080   301132.7690  5001910.7700                  APPROX POSITION XYZ
        0.0000        0.0000        0.0000                  ANTENNA: DELTA H/E/N
     1     1                                                WAVELENGTH FACT L1/2
     7    L1    L2    C1    P2    D1    S1    S2            # / TYPES OF OBSERV
    10.0000                                                 INTERVAL
------------------------------------------------------------COMMENT
AGRS.NL - Active GPS Reference System for the Netherlands   COMMENT
E-mail: H.vanderMarel@geo.tudelft.nl                        COMMENT
------------------------------------------------------------COMMENT
The coordinates in the RINEX header are given in the        COMMENT
system ETRS89 and were based on the ITRF96 solution.        COMMENT
 SNR is mapped to RINEX snr flag value [1-9]                COMMENT
  L1: 3 -> 1; 8 -> 5; 40 -> 9                               COMMENT
  L2: 1 -> 1; 5 -> 5; 60 -> 9                               COMMENT
  2005     6     1     0     0    0.0000000     GPS         TIME OF FIRST OBS
                                                            END OF HEADER

 05  6  1  0  0  0.0000000  0  9G 5G 1G 4G 2G14G30G 6G25G 9
 -16228703.44749 -12554107.65847  22235768.1704   22235762.2514      -2897.9544
        49.0004         37.2004
 -15372176.02649 -11906719.62447  22815508.4054   22815503.6974      -979.5314
        47.0004         31.7004
```

Rinex version 2

```
-15372176.02649 -11906719.62447  22815508.4054   22815503.6974
       47.0004         31.7004
 -2432703.65449  -1855027.24046  24936961.0994   24936957.1284      -2881.3924
       42.2004         20.6004
-17522896.17149 -13749343.25547  22474622.7494   22474615.3724       -318.9404
       48.9004         36.4004
-10837107.98749  -8426009.04946  23605266.1414   23605261.8764      -3120.3954
       41.6004         24.6004
-27882285.32349 -21701357.56948  20155231.9294   20155226.3334       -727.7394
       53.8004         46.0004
-24017070.96249 -18566420.94948  20876774.7764   20876770.1794       1512.9354
       52.4004         43.2004
-18795031.67049 -14636319.62747  22364805.4964   22364799.8824       2024.9074
       49.4004         37.5004
 3056947.27049   2374829.76246  25785833.8964   25785829.7424      -4117.8074
       39.2004         16.9004
 05  6  1  0  0 10.0000000  0  9G 5G 1G 4G 2G14G30G 6G25G 9
-16199710.86349 -12531516.03947  22241284.8934   22241279.5294      -2900.5524
       49.4004         37.2004
-15362349.52949 -11899062.62147  22817378.4744   22817373.8774       -985.8304
       47.8004         32.5004
 -2403869.02749  -1832558.68946  24942446.2624   24942444.1874      -2885.6404
       37.5004         19.6004
-17519670.26049 -13746829.55547  22475236.5794   22475229.4414       -326.2264
       45.5004         30.6004
-10805888.10549  -8401681.84846  23611207.0794   23611202.8874      -3123.5264
       41.6004         22.3004
-27874983.73749 -21695668.02248  20156621.4464   20156615.6674       -732.5684
       53.7004         45.9004
-24032176.01649 -18578191.10848  20873900.3734   20873895.5104       1508.0864
       52.2004         42.7004
-18815259.15049 -14652081.30047  22360956.9864   22360950.5734       2020.0884
```

date

time

observed GPS satellites

L1 [cyc]

L2 [cyc]

C1 [m]

P2 [m]

D1 [Hz]

S1 [dB-Hz]

S2 [dB-Hz]

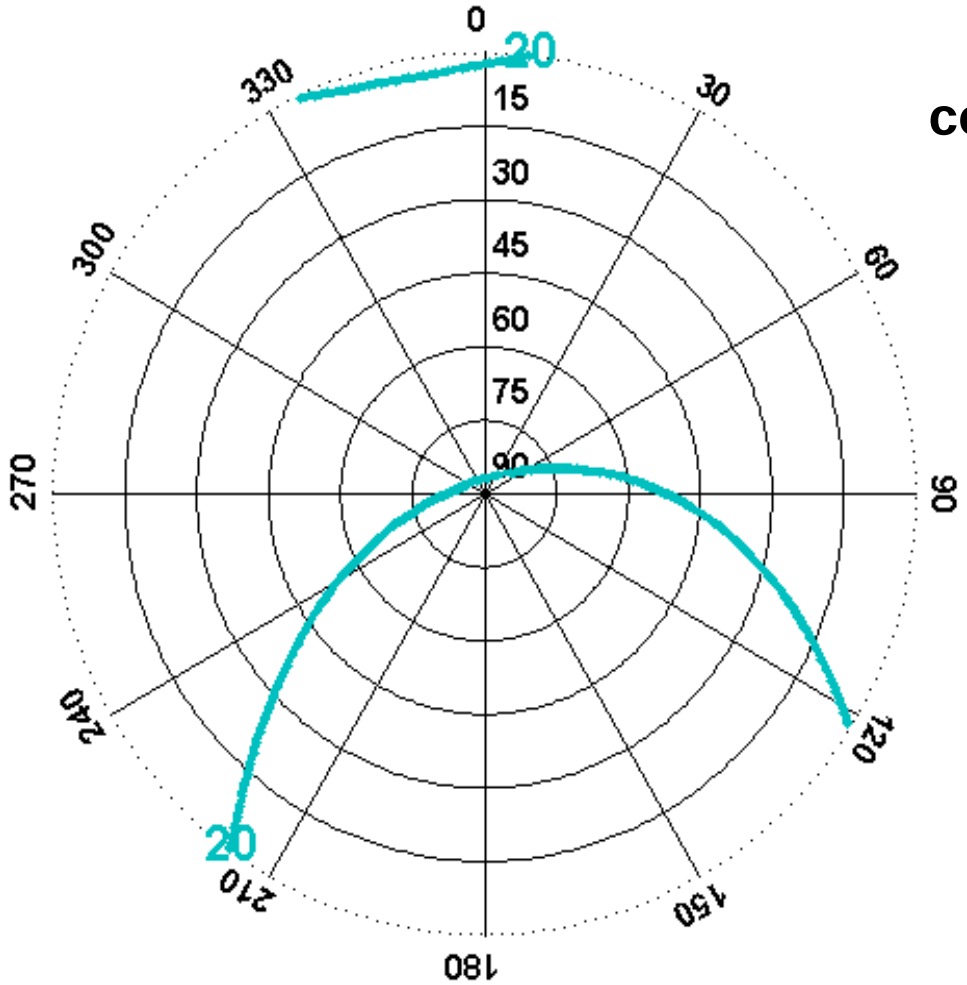
Rinex version 3

```
> 2009 06 01 19 24 20.0000000  0 16
G32  21416411.582 6 112543995.47606      -433.716 6        37.750
G14  17965923.281 7  94411564.82107      -926.294 7        46.000
G04  19499387.082 7 102469970.23607      -489.316 7        42.500
G05  16177386.182 8  85012737.46208      -503.029 8        48.000
G09  19680224.122 7 103420277.70507     -2994.644 7        43.000
G29  17588461.494 7  92427986.52407      3931.085 7        47.750
G30  15579821.979 8  81872514.87808      1440.192 8        48.250
G20  21201138.666 6 111412735.94806       220.982 6        41.000
G31  18633632.933 7  97920399.11307      3985.242 7        46.500
G12  16735629.190 8  87946324.57208      -650.783 8        48.000
G01  15869603.377 8  83395328.90308      1314.224 8        50.250    15869787.9
G02  18476058.079 7  97092340.45007      1970.052 7        46.500
E31  19662006.065 7 103324537.52107      2797.230 7        46.000    19661844.0
S20  34286369.717 6 180176083.47706      1054.396 6        41.250
S26  34483836.802 6 181213789.30506      1043.371 6        40.250
S24  34026955.157 7 178812854.29607       884.702 7        42.500
> 2009 06 01 19 24 30.0000000  0 16
G32  25614342.076 6 134604277.51506      -447.589 6        36.250
G14  22164792.959 7 116476773.11307      -941.472 7        46.250
G04  23697426.920 7 124530817.73607      -505.056 7        42.250
G05  20375449.952 8 107073712.86908      -516.946 8        48.250
G09  23883020.706 7 125506119.29207     -3005.558 7        43.250
G29  21778099.113 7 114444681.67207      3917.429 7        47.750
G30  19774194.116 8 103914088.18608      1425.642 8        48.250
G20  25397828.616 6 133466488.47906       205.658 6        39.250
G31  22823166.574 7 119936545.95607      3973.065 7        46.250
G12  20933973.121 8 110008770.85008      -663.817 8        48.250
G01  20064214.469 8 105438157.85208      1300.146 8        50.250    20064398.8
G02  22669424.181 7 119128628.34607      1954.341 7        46.500
E31  23853798.537 7 125352555.28107      2783.517 7        45.750    23853636.3
```

G: GPS
E: Galileo
S: SBAS

date
time
no. observed GNSS satellites

C1 [m]
L1 [cyc]
D1 [Hz]
S1 [dB-Hz]

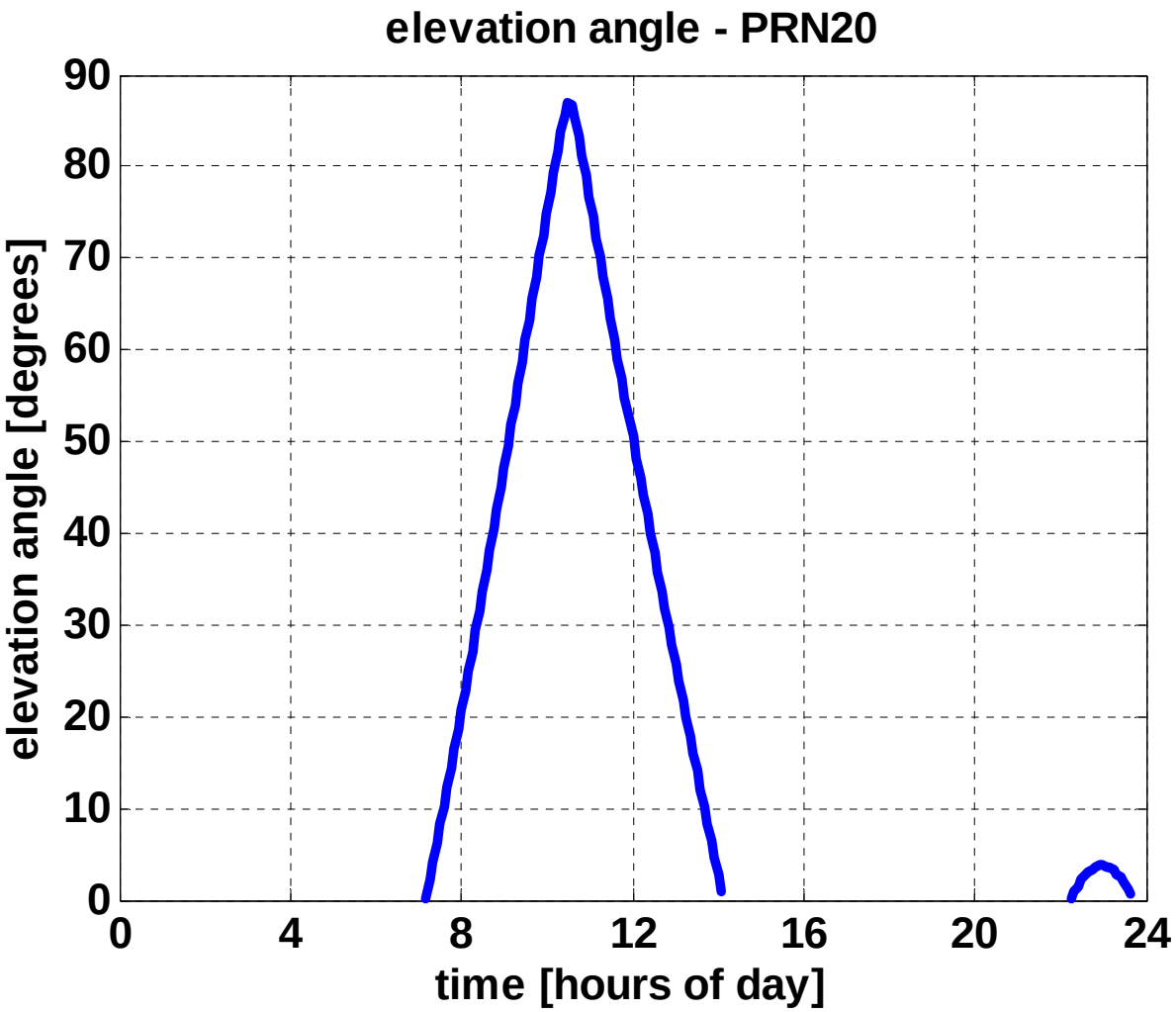


consider one GPS satellite

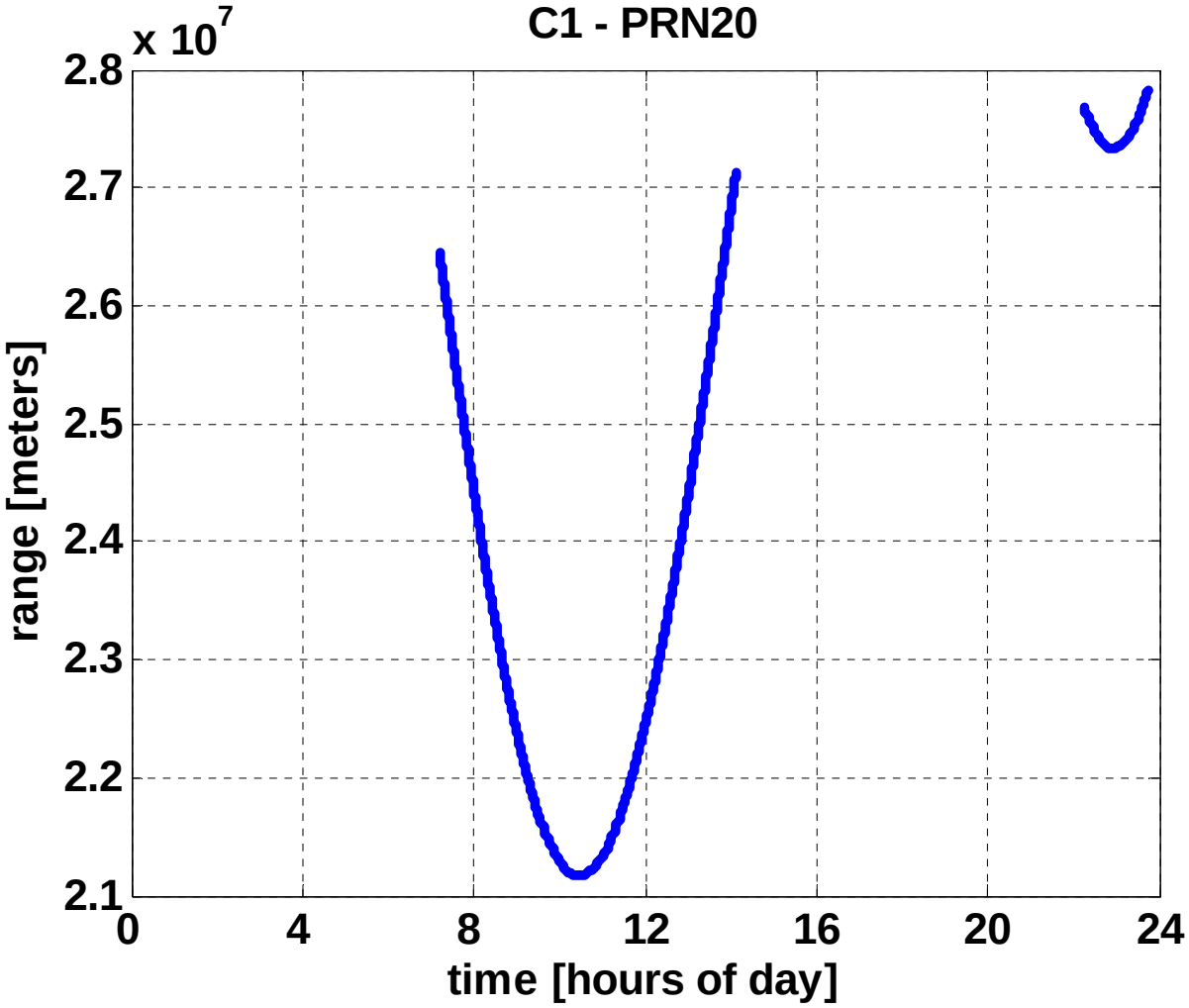
a = 26.560.000 m
r = 6.378.000 m

skyplot: local **azimuth** versus **elevation** of GPS satellite PRN 20
⇒ 'ground-track' as observed in Delft over 24 hr period
(Delft is at 52 degrees latitude North, the orbital plane has a 55 degrees inclination)

Elevation

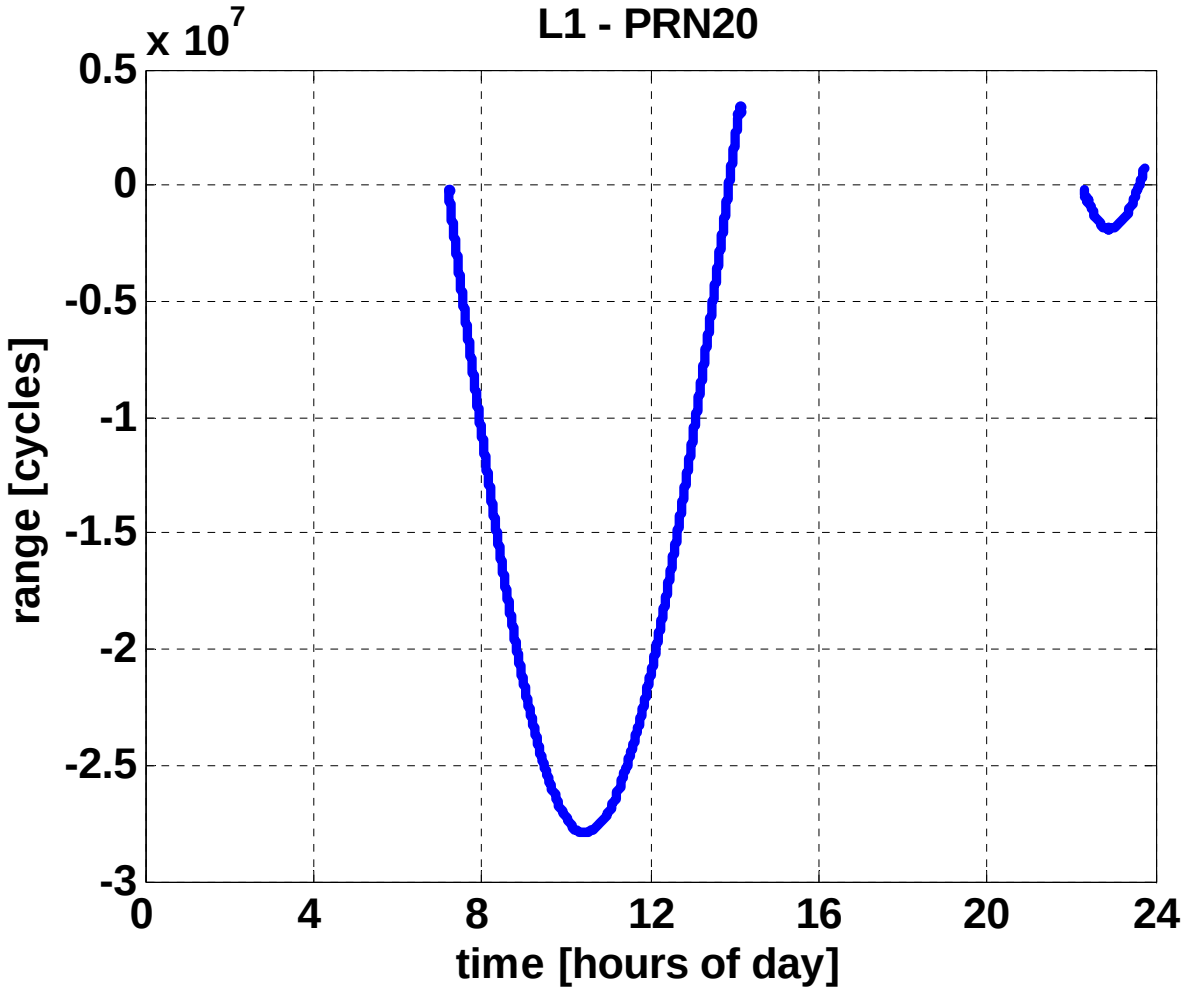

elevation angle - PRN20
elevation angle [degrees]
90
80
70
60
50
40
30
20
10
0
0
4
8
12
16
20
24
time [hours of day]

Pseudorange observation

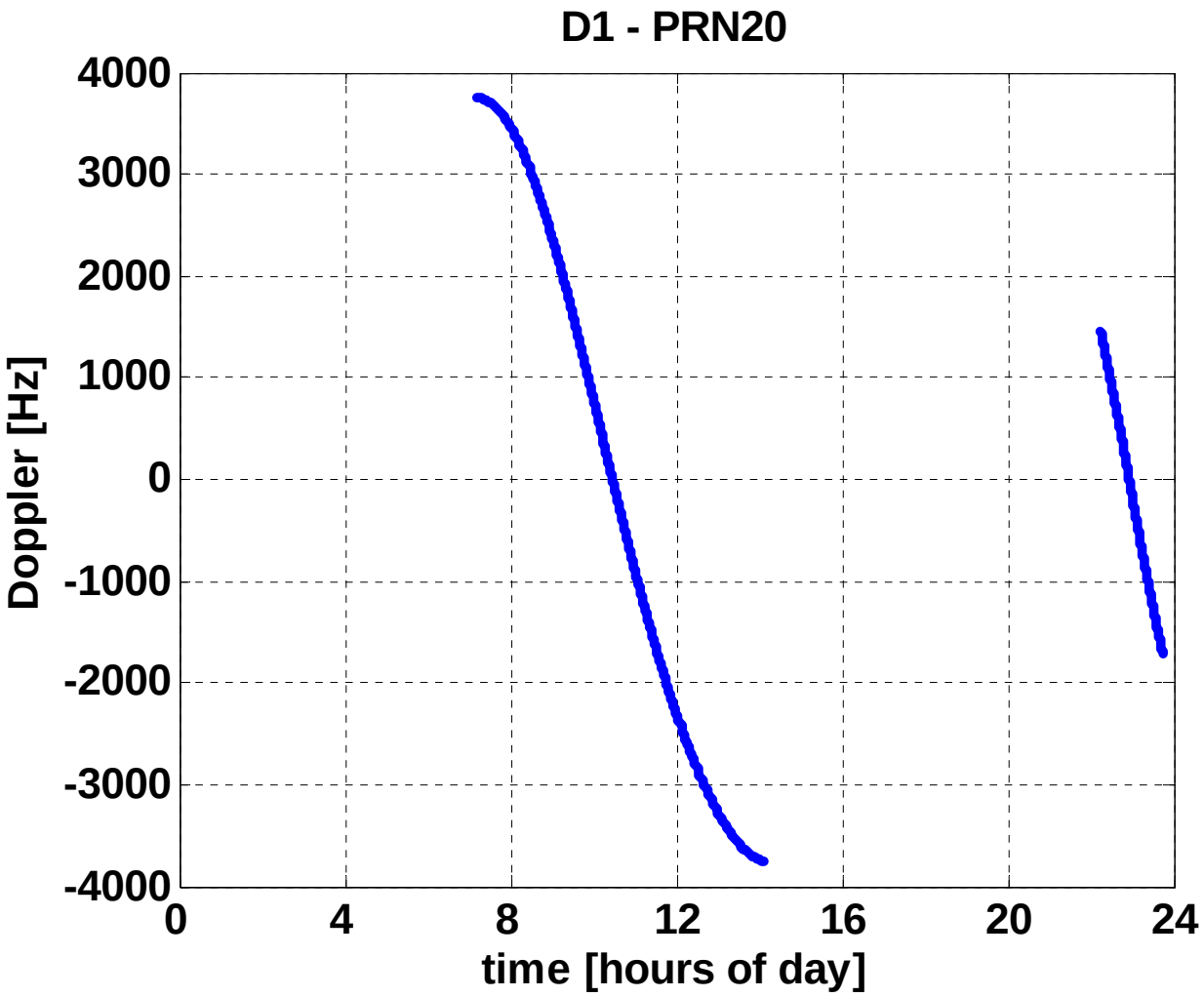


L1 carrier phase observation

carrier phase is ambiguous (just starts at zero here)

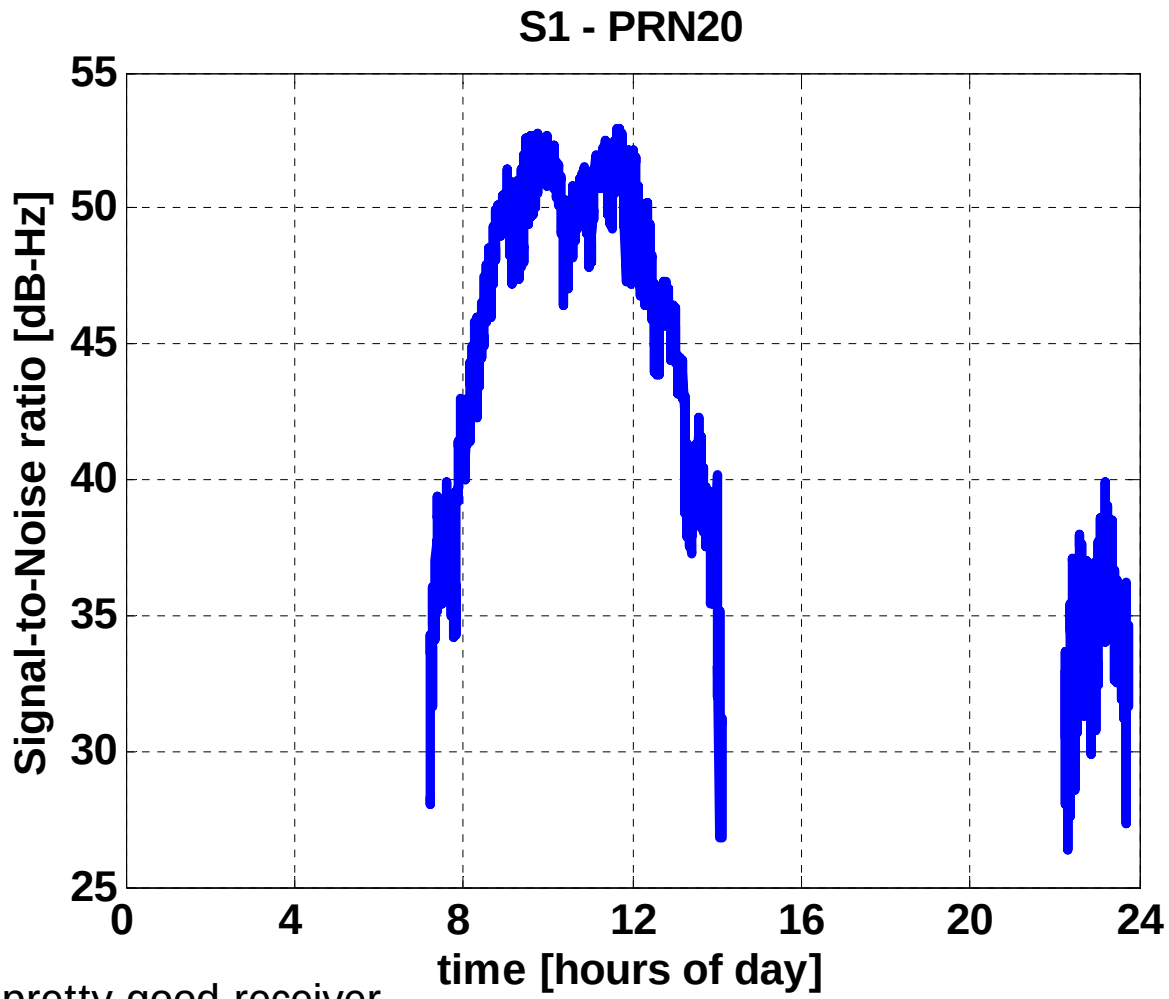


L1 Doppler frequency observation



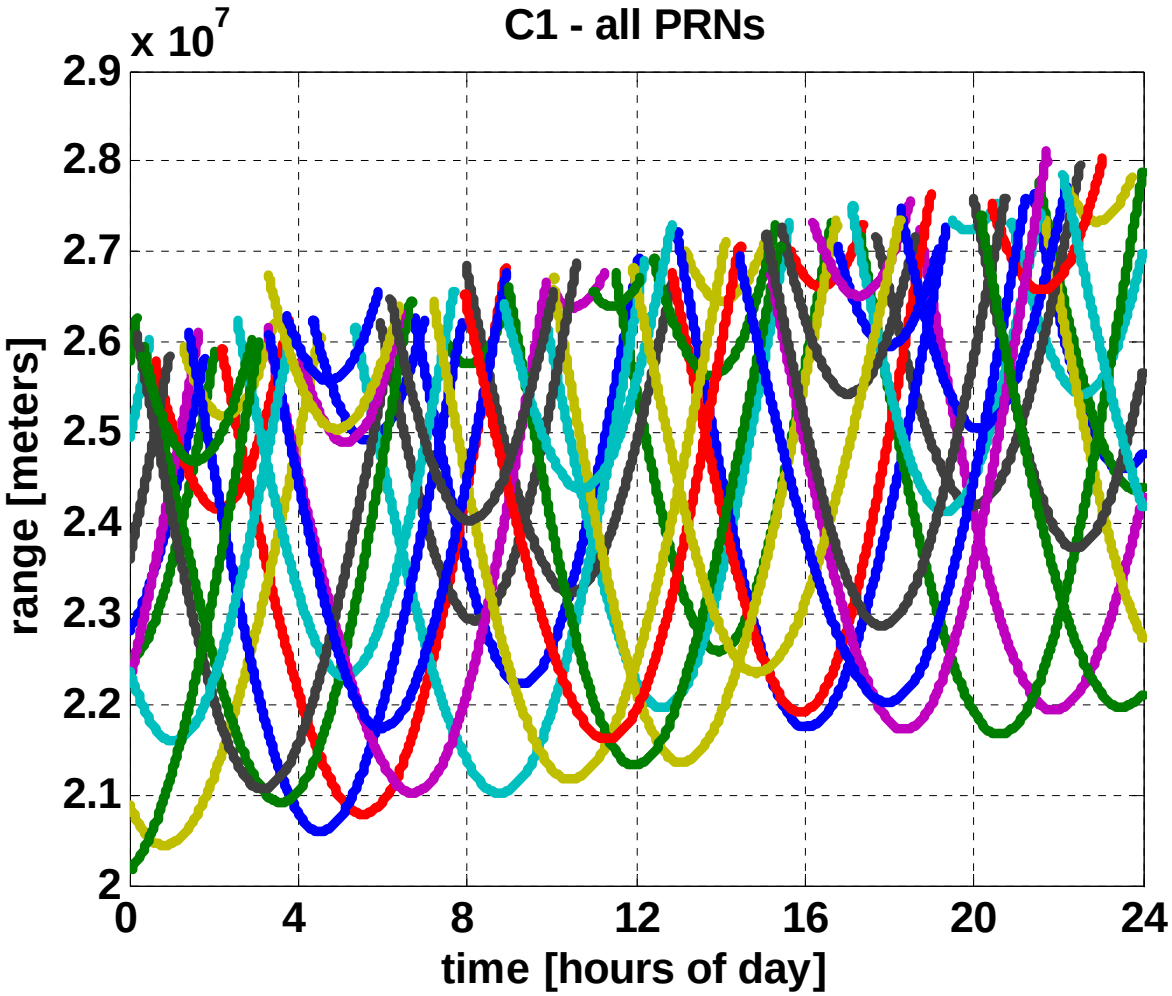
Signal strength

actually Carrier-to-Noise density ratio



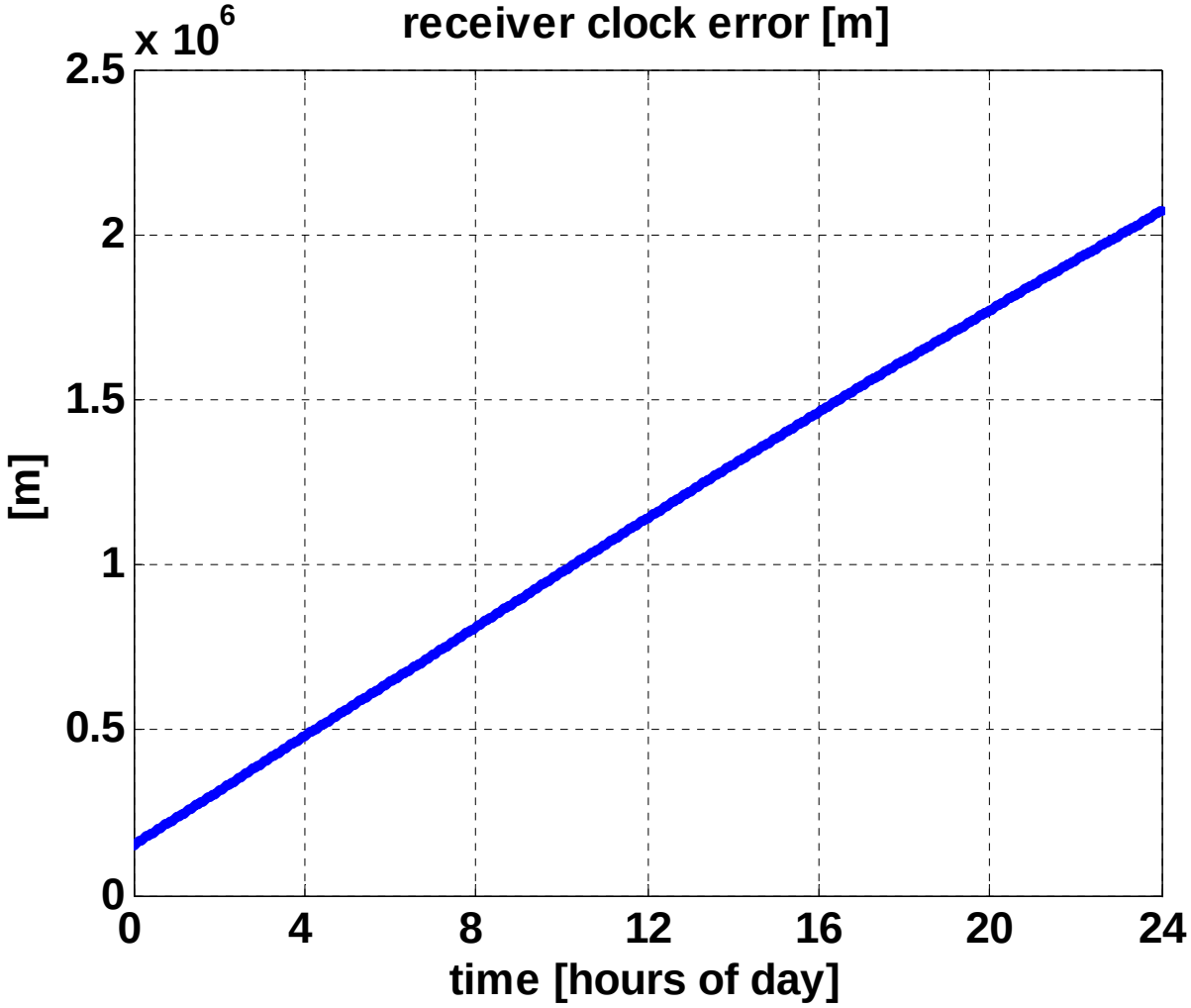
down to 25 dB-Hz, pretty good receiver ...

Pseudorange observations



they get longer as time proceeds ...?

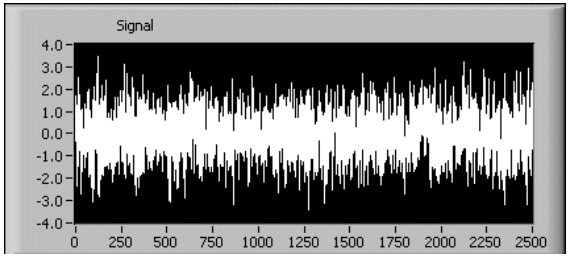
Receiver clock error



oscillator in receiver has stability of about 10^{-7} s/s

Noise and bias

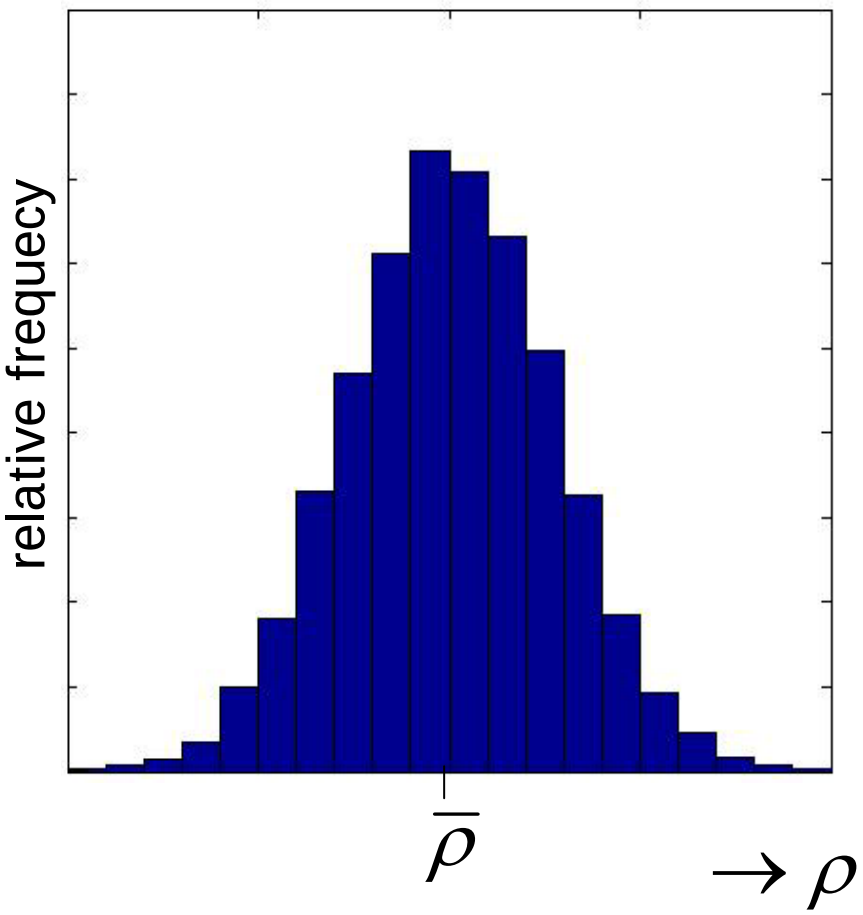
- Noise:
  quickly varying, averages out to zeros



- Bias:
  systematic / persistent over longer time,
  or outlier in observation

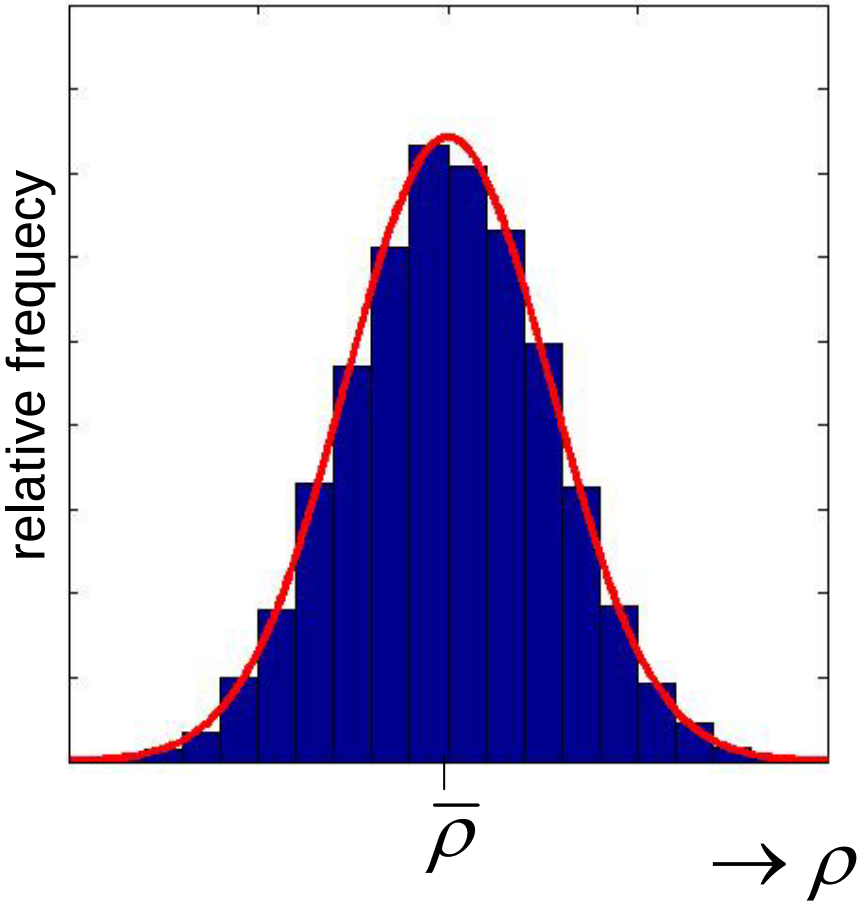
Normal distribution

- assume satellite and receiver are not moving
- range measurement repeated 10,000 times
- fluctuations due to measurement noise



Normal distribution

- assume satellite and receiver are not moving
- range measurement repeated 10,000 times
- fluctuations due to measurement noise
- generally, normal distribution assumed



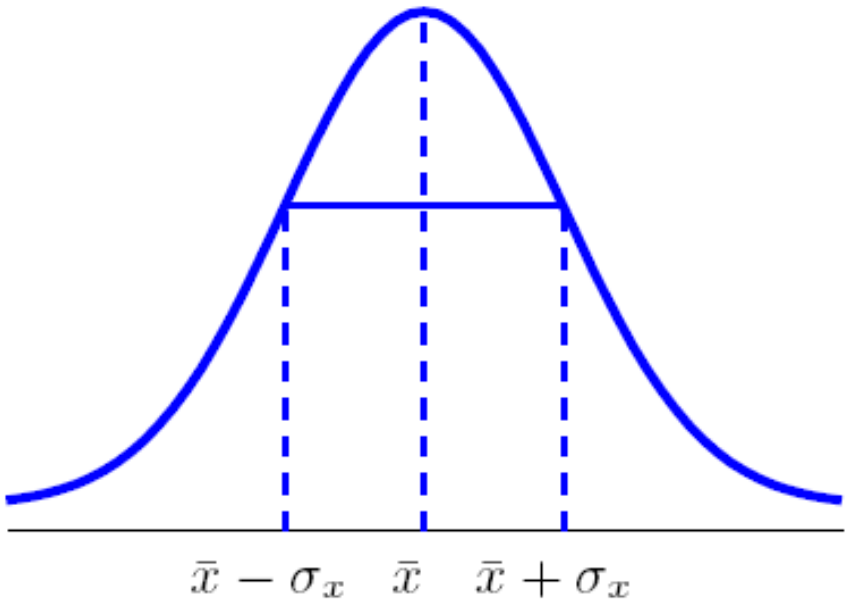
Normal distribution

A random variable $\underline{x}$ has a **normal** or **Gaussian** distribution with parameters $\bar{x}$ and σ_x

$\bar{x}$: mean

σ_x : standard deviation

σ_x^2 : variance



Notation: $\boxed{\underline{x} \sim \mathrm{N}(\bar{x}, \sigma_x^2)}$

Standard deviation and RMS error

- Standard deviation: measure for fluctuations
- Empirical standard deviation: $\sqrt{\dfrac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}}$ with $\bar{x}=\dfrac{1}{n}\sum_{i=1}^{n}x_i$

$$P(|\underline{x}-\bar{x}|\le\sigma_x)=68.3\%$$
$$P(|\underline{x}-\bar{x}|\le 2\sigma_x)=95.4\%$$

- RMS error: $\sqrt{\dfrac{\sum_{i=1}^{n}(x-x_i)^2}{n}}$

Error sources

- satellite:
  - orbit
  - clock
  - instrumental delays
- signal path
  - ionosphere
  - troposphere
  - multipath
- receiver
  - clock
  - instrumental delays
- other
  - spoofing
  - interference

Summary and outlook

- GPS: history and overview (Chapter 1, Sections 2.1, 2.2, 4.4)
- GPS signals (Section 2.3)
- Future GNSS (Chapter 3)
- GPS receivers (paper Braasch and Van Dierendonck)
- GPS measurements (Section 5.1)

Homework (optional): on blackboard

Next: error sources and PVT estimation