Satellite Navigation

GPS measurements and error sources

AE4E08

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Course 2010 - 2011, lecture 4
Today’s topics

- Recap: GPS signal components
- Code Phase measurements → pseudoranges
- Carrier Phase measurements
- GPS measurements: example
- Outlook: error sources

- Book: Section 5.1
Recap: GPS signal components

- All signals and time information are coherently derived from the same clock with a frequency of $f_0 = 10.23$ MHz

<table>
<thead>
<tr>
<th>Signal components</th>
<th>Frequency</th>
<th>Wavelength / chiplength</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 carrier</td>
<td>1575.42 MHz ($154*f_0$)</td>
<td>19.05 cm</td>
</tr>
<tr>
<td>L2 carrier</td>
<td>1227.60 MHz ($120*f_0$)</td>
<td>24.45 cm</td>
</tr>
<tr>
<td>C/ A code on L1 with</td>
<td>1.023 Mbits/sec ($0.1*f_0$)</td>
<td>293 m</td>
</tr>
<tr>
<td>P code on L1 and L2</td>
<td>10.23 Mbits/sec ($f_0$)</td>
<td>29.3 m</td>
</tr>
<tr>
<td>Broadcast message</td>
<td>50 bits/sec</td>
<td></td>
</tr>
</tbody>
</table>
Recap: GPS signal components

From: Misra and Enge
Recap: GPS signal components

carrier $f(t)$

code and data $C(t) \oplus D(t)$

signal $S(t)$

phase shift $180^\circ$
**Code Phase measurements**

\[ \tau \quad ? \quad \text{transit time} \quad \rightarrow \quad 70 \text{ to } 90 \text{ ms} \]

\[ t \quad ? \quad \text{true GPS time at which code is received} \]

\[ t^s(t - \tau) \quad \text{emission time (imprinted on signal)} \]

\[ t_u(t) \quad \text{measured arrival time (clock reading)} \]

\[ \rho(t) = c \left[ t_u(t) - t^s(t - \tau) \right] \quad \text{pseudorange} \]
Code Phase measurements

\[ t_u(t) = t + \delta t_u(t) \]

receiver clock bias

\[ t_u(t) = t + \delta t_u(t) \]
**Code Phase measurements**

\[ t_u(t) = t + \delta t_u(t) \]

**receiver clock bias**

Receiver clocks: drift!

Deviation from GPS time limited to ±1 ms:

- continuous clock steering
- reset (clock jump!) when certain threshold is reached
Code Phase measurements

\[ t_u(t) = t + \delta t_u(t) \]

receiver clock bias

\[ t^s(t - \tau) = (t - \tau) + \delta t^s(t - \tau) \]

satellite clock bias

estimated by control segment
**Code Phase measurements**

\[ t_u(t) = t + \delta t_u(t) \]

\[ t^s(t - \tau) = (t - \tau) + \delta t^s(t - \tau) \]

\[ \rho(t) = c \left[ t_u(t) - t^s(t - \tau) \right] + \epsilon_{\rho}(t) \]

\[ = c \left[ t + \delta t_u(t) - (t - \tau) - \delta t^s(t - \tau) \right] + \epsilon_{\rho}(t) \]

\[ = c \tau + c \left[ \delta t_u(t) - \delta t^s(t - \tau) \right] + \epsilon_{\rho}(t) \]

Unmodeled effects and errors
\[ \rho(t) = c \tau + c \left[ \delta t_u(t) - \delta t^s(t - \tau) \right] + \varepsilon_\rho(t) \]

clock biases

noise + errors

distance traveled by signal

\[ \rho(t) \]
pseudorange

\[ \rho(t) = c\tau + c \left[ \delta t_u(t) - \delta t^s(t - \tau) \right] + \varepsilon_\rho(t) \]

- distance traveled by signal
- clock biases
- noise + errors

\[ c\tau = r(t, t - \tau) + I_\rho(t) + T_\rho(t) \]

- geometric range
- ionosphere and troposphere delays
pseudorange

\[ \rho = r + I_\rho + T_\rho + c \left[ \delta t_u - \delta t_s \right] + \varepsilon_\rho \]

pseudorange measurement = biased and noisy measurement of the geometric range \( r \)

Not to be studied: part on “Constructing pseudorange measurements” in Section 5.1.1
pseudorange measurements: example
pseudorange measurements: example

Figure: Peter Buist
pseudorange measurements: example

Figure: Peter Buist
Carrier Phase measurements

Very precise!
Carrier Phase measurements

\[ \phi(t) = \phi(t_0) + f \cdot (t - t_0) \]

\[ f \cdot (t - t_0) \] = number of cycles since starting point of interval

Carrier phase: \[ \phi(t) = \phi(t_0) + f \cdot (t - t_0) \]
Carrier Phase measurements

Difference between phases of receiver-generated carrier signal and received carrier signal
Carrier Phase measurements

Carrier phase measurement:
- Difference between phases of receiver-generated carrier signal and received carrier signal
- Phase measurement + whole number of cycles traveled $\rightarrow$ range
- Change in phase continuously measured (incl. full cycles)
Carrier Phase measurements

\[ \phi(t) = \phi_u(t) - \phi^s(t - \tau) + N + \varepsilon_\phi \]

Recall: \( \phi(t) = \phi(t_0) + f \cdot (t - t_0) \)

\[ \phi_u(t) = \phi_u(t_0) + f \cdot (t - t_0) + f \cdot (\delta t_u(t) - \delta t_u(t_0)) \]

clock biases

\[ \phi^s(t - \tau) = \phi^s(t_0) + f \cdot (t - \tau - t_0) + f \cdot (\delta t_u(t - \tau) - \delta t_u^s(t_0)) \]
**Carrier Phase measurements**

\[ \phi(t) = \phi_u(t) - \phi^s(t - \tau) + N + \varepsilon_{\phi} \]

\[ \phi_u(t) = \phi_u(t_0) + f \cdot (t - t_0) + f \cdot (\delta t_u(t) - \delta t_u(t_0)) \]

\[ \phi^s(t - \tau) = \phi^s(t_0) + f \cdot (t - \tau - t_0) + f \cdot (\delta t^s(t - \tau) - \delta t^s(t_0)) \]

\[ \phi(t) = f \cdot \tau + f \cdot (\delta t_u(t) - \delta t^s(t - \tau)) \]

\[ + \phi_u(t_0) - \phi^s(t_0) \]

\[ - f \cdot (\delta t_u(t_0) - \delta t^s(t_0)) \]

\[ + N + \varepsilon_{\phi} \]

<table>
<thead>
<tr>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>- clock biases</td>
</tr>
<tr>
<td>- initial phases</td>
</tr>
<tr>
<td>- clock biases at $t_0$</td>
</tr>
<tr>
<td>- integer ambiguity (constant)</td>
</tr>
<tr>
<td>- noise and errors</td>
</tr>
</tbody>
</table>
Carrier Phase measurements

\[ \phi(t) = f \cdot \tau + f \cdot \left( \delta t_u(t) - \delta t_s(t - \tau) \right) + A + \epsilon_\phi \]

\[ \lambda \cdot \phi(t) = c \cdot \tau + c \cdot \left( \delta t_u(t) - \delta t_s(t - \tau) \right) + \lambda \cdot A + \lambda \cdot \epsilon_\phi \]

\[ f = \frac{c}{\lambda} \]
Carrier Phase measurements

\[ \lambda \cdot \phi(t) = c \cdot \tau + c \cdot (\delta t_u(t) - \delta t^s(t - \tau)) + \lambda \cdot A + \lambda \cdot \varepsilon_\phi \]

\[ \Phi = r + I_\phi + T_\phi + c \cdot (\delta t_u - \delta t^s) + \lambda \cdot A + \varepsilon_\Phi \]

Ambiguities must be resolved to take advantage of high precision phase measurements
GPS measurements: example

Dataset  June 1\textsuperscript{st}, 2005 -  GPS week 1325
00:00-23:59 (GPS-time)
10 seconds interval
Trimble 4700 receiver
dual-frequency GPS (L1 & L2)
choke-ring antenna
at GNSS-observatory in Delft

RINEX: Receiver Independent Exchange format

data (delf1520) provided by H. van der Marel
plots by Q. Le, photo by R. Kremers
2.10  OBSERVATION DATA    G (GPS)    RINEX VERSION / TYPE

teqc  2002Mar14     Automatic GPS proces20050608 13:15:24UTC
Linux 2.0.36|Pentium II|gcc|Linux|486/DX+
BIT 2 OF LLI FLAGS DATA COLLECTED UNDER A/S CONDITION
DELT-16
13502M004
H. VAN DER MAREL    AGRS.NL (KAD,MD,TUD)
96517    TRIMBLE 4700    N1.30/S0.00
93258    TRM29659.00    UNAV
3924687.7080   301132.7690  5001910.7700
0.0000        0.0000  0.0000
1     1
7    L1  L2  C1  P2  D1  S1  S2
18653
10.0000

AGRS.NL - Active GPS Reference System for the Netherlands
E-mail: H.vanderMarel@geo.tudelft.nl

The coordinates in the RINEX header are given in the system ETRS89 and were based on the ITRF96 solution.
SNR is mapped to RINEX snr flag value [1-9]
L1: 3 -> 1; 8 -> 5; 40 -> 9
L2: 1 -> 1; 5 -> 5; 60 -> 9
2005 6 1 0 0 0.0000000 09G 5G 1G 4G 2G14G30G 6G25G 9
-16228703.44749 -12554107.65847 22235768.1704 22235762.2514 -2897.9544
49.0004         37.2004
-15372176.02649 -11906719.62447 22815508.4054 22815503.6974 -979.5314
47.0004         31.7004
<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>Observed GPS Satellites</th>
</tr>
</thead>
<tbody>
<tr>
<td>05/06/10</td>
<td>00:00:00</td>
<td>3G 5G 1G 4G 2G 14G 30G 6G 25G 9G</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>L1 [cyc]</th>
<th>L2 [cyc]</th>
<th>C1 [m]</th>
<th>P2 [m]</th>
<th>D1 [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2403869.02749</td>
<td>-1832558.68946</td>
<td>24942446.2624</td>
<td>24942444.1874</td>
<td>-2885.6404</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S1 [dB-Hz]</th>
<th>S2 [dB-Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10805888.10549</td>
<td>-8401681.84846</td>
</tr>
</tbody>
</table>

| 27874983.73749 | 21695668.02248 | 20156621.4464 | 20156615.6674 | -732.5684 |
| 53.7004 | 45.9004 |

| -24032176.01649 | -18578191.10848 | 20873900.3734 | 20873895.5104 | 1508.0864 |
| 52.2004 | 42.7004 |

<p>| -18815259.15049 | -14652081.30047 | 22360956.9864 | 22360950.5734 | 2220.6834 |</p>
<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>No. Observed GNSS Satellites</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009-06-01</td>
<td>19:24:20</td>
<td>16</td>
</tr>
<tr>
<td>2009-06-01</td>
<td>19:24:30</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rinex version 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 [cyc]</td>
</tr>
<tr>
<td>C1 [m]</td>
</tr>
<tr>
<td>D1 [Hz]</td>
</tr>
<tr>
<td>S1 [dB-Hz]</td>
</tr>
</tbody>
</table>

| G32 | 21416411.582 | 6 | 112543995.47606 | -433.716 | 6 | 37.750 |
| G14 | 17965923.281 | 7 | 94411564.82107  | -926.294 | 7 | 46.000 |
| G04 | 19499387.082 | 7 | 102469970.23607 | -489.316 | 7 | 42.500 |
| G05 | 16177386.182 | 8 | 85012737.46208  | -503.029 | 8 | 48.000 |
| G09 | 19680224.122 | 7 | 103420277.70507 | -2994.644| 7 | 43.000 |
| G29 | 17588461.494 | 7 | 92427986.52407  | 3931.085 | 7 | 47.750 |
| G30 | 15579821.979 | 8 | 81872514.87808  | 1440.192 | 8 | 48.250 |
| G20 | 21201138.666 | 6 | 111412735.94806 | 220.982  | 6 | 41.000 |
| G31 | 18633632.933 | 7 | 97920399.11307  | 3985.242 | 7 | 46.500 |
| G32 | 17588461.494 | 7 | 92427986.52407  | 3931.085 | 7 | 47.750 |
| G14 | 17965923.281 | 7 | 94411564.82107  | -926.294 | 7 | 46.000 |
| G04 | 19499387.082 | 7 | 102469970.23607 | -489.316 | 7 | 42.500 |
| G05 | 16177386.182 | 8 | 85012737.46208  | -503.029 | 8 | 48.000 |
| G09 | 19680224.122 | 7 | 103420277.70507 | -2994.644| 7 | 43.000 |
| G29 | 17588461.494 | 7 | 92427986.52407  | 3931.085 | 7 | 47.750 |
| G30 | 15579821.979 | 8 | 81872514.87808  | 1440.192 | 8 | 48.250 |
| G20 | 21201138.666 | 6 | 111412735.94806 | 220.982  | 6 | 41.000 |
| G31 | 18633632.933 | 7 | 97920399.11307  | 3985.242 | 7 | 46.500 |
| G12 | 20933973.121 | 8 | 110008770.85008 | -663.817 | 8 | 48.250 |

Satellite Navigation (AES408) - Lecture 4

TUDelft
skyplot: local **azimuth** versus **elevation** of GPS satellite PRN 20
⇒ ‘ground-track’ as observed in Delft over 24 hr period
(Delft is at 52 degrees latitude North, the orbital plane has a 55 degrees inclination)
Elevation

elevation angle - PRN20

time [hours of day]
elevation angle [degrees]
L1 carrier phase observation

carrier phase is ambiguous (just starts at zero here)

L1 - PRN20

range [cycles]

time [hours of day]
L1 Doppler frequency observation

D1 - PRN20

Doppler [Hz]

time [hours of day]

-4000 -3000 -2000 -1000 0 1000 2000 3000 4000

0 4 8 12 16 20 24
Signal strength
actually Carrier-to-Noise density ratio
S1 - PRN20

down to 25 dB-Hz, pretty good receiver …
Pseudorange observations

they get longer as time proceeds …?
Receiver clock error

oscillator in receiver has stability of about $10^{-7}$ s/s
Noise and bias

- **Noise**: quickly varying, averages out to zeros
- **Bias**: systematic / persistent over longer time, or outlier in observation
Normal distribution

- assume satellite and receiver are not moving
- range measurement repeated 10,000 times
- fluctuations due to measurement noise
Normal distribution

- assume satellite and receiver are not moving
- range measurement repeated 10,000 times
- fluctuations due to measurement noise
- generally, normal distribution assumed
A random variable $\bar{x}$ has a normal or Gaussian distribution with parameters $\bar{x}$ and $\sigma_x$.

$\bar{x}$ : mean
$\sigma_x$ : standard deviation
$\sigma_x^2$ : variance

Notation: $\bar{x} \sim N(\bar{x}, \sigma_x^2)$
Standard deviation and RMS error

- Standard deviation: measure for fluctuations

- Empirical standard deviation: \( \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}} \) with \( \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \)

- RMS error: \( \sqrt{\frac{\sum_{i=1}^{n} (x - x_i)^2}{n}} \)

- \( P(|x - \bar{x}| \leq \sigma_x) = 68.3\% \)
- \( P(|x - \bar{x}| \leq 2\sigma_x) = 95.4\% \)
Error sources

- satellite:
  - orbit
  - clock
  - instrumental delays
- signal path
  - ionosphere
  - troposphere
  - multipath
- receiver
  - clock
  - instrumental delays
- other
  - spoofing
  - interference
Summary and outlook

• GPS: history and overview (Chapter 1, Sections 2.1, 2.2, 4.4)
• GPS signals (Section 2.3)
• Future GNSS (Chapter 3)
• GPS receivers (paper Braasch and Van Dierendonck)
• GPS measurements (Section 5.1)

Homework (optional): on blackboard

Next: error sources and PVT estimation