System Identification & Parameter Estimation

Wb2301: SIPE lecture 4
Perturbation signal design

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Resume previous lectures: Discrete and Continuous Signals

- **Signal conversion: continue to discrete**
  AD conversion or sampling
  - Shannon (or Nyquist): \( f < f_s/2 \)
  - Dirac’s comb (\& limited T): \( \Delta f = 1/T \)

- **Signal conversion: discrete to continuous**
  DA conversion or reconstruction
  - zero order hold (introduces higher frequencies, above \( f_s \))
Contents Lecture 4

• Sources for error in (spectral) estimators:
  • Aliasing
  • Leakage
  • Low signal-to-noise ratio

• Optimal perturbation signals
  • Improve signal-to-noise ratio
  • Prevent aliasing
  • Prevent leakage

• Estimation with periodic signals
  • Spectral densities + variance
  • FRF + variance
Recording of signals

- In practical situations a **limited time** is recorded and digitized with a given **sample frequency**

- What errors can/do occur as a result
  - The sampling theorem (also Nyquist or Shannon)
    - Signal > f_s/2 => aliasing (frequency folding)
  - Resolution in frequency domain (Dirac’s comb)
    - $\Delta f = T^{-1}$
    - What if frequency does not ‘fit’ on $\Delta f$
Example aliasing

Matlab: Lec4_AliasingDemo.m

- 9 Hz sine
- $f_s = 10$ Hz
- $9 \text{ Hz} > f_s/2$, and 1 Hz is ‘seen’
- Frequencies above the Nyquist frequency are mirrored around $f_s/2$
How to prevent aliasing

- Sample high enough:
  - rule of thumb: sample frequency should be at least 2.5 times the cut-off frequency (preferably more)
- Apply anti-aliasing filter:
  - before digitizing!
  - analog filter!
Example anti-aliasing; EMG system

### Specifications

#### Main Amplifier Unit (Common)

<table>
<thead>
<tr>
<th>Specification</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Channels</td>
<td>4, 8 or 16 analog</td>
</tr>
<tr>
<td>Overall Amplification per Channel</td>
<td>0, 100, 1000, 10000, Custom (AUX setting)</td>
</tr>
<tr>
<td>Max. Output Voltage Range</td>
<td>± 5 Volts</td>
</tr>
<tr>
<td>Channel Frequency Response</td>
<td>20±5 Hz to 450±50 Hz, 80 dB/decade</td>
</tr>
</tbody>
</table>

- **EMG Sensors**: DE-2.1 (single differential) or DE-3.1 (double differential)
- **System Noise (R.T.I.)**: <1.2 μV (max) for the specified bandwidth
- **Channel Output Isolation**: 3750 V (max) @ 80 Hz for 60 sec.
- **Output Signal Connectors**: 4, 8, or 16 BNC connectors 68-pin 0.05" pitch high-density D-Sub male
- **Signal Quality Check**: Line Frequency Interference (50 or 60 Hz)
- **Channel Saturation Check**: ± 4.8 V threshold
- **Signal Quality Warnings**: Yellow LED, Selectable Audio Buzzer
- **Operating Temperature**: 15°C to 40°C

\[ f_s \gg 900 \text{ Hz} \]
Example: strain gauge amplifier

**Specifications**

**Voltage Specifications:**
- Electrical input: ±10mV, ±20mV, ±30mV, ±50mV, ±100mV
- Excitation voltage: 1~10Vdc (20mA max.)

**Voltage output:**
- Bipolar: ±5V, ±10V
- Unipolar: 0~5V, 0~10V
- Output impedance: <50Ω

**Current output:**
- Current: 0~20mA
- Current load resistor: 0~500Ω (Source)

**General**
- Three-way isolation: 1000 Vdc
- Accuracy: ±0.1% of full range
- Bandwidth: 600Hz (typical)@-3dB
- Operation temperature range: -25°C ~ 75°C
- Storage temperature range: -30°C ~ 85°C

**Supply Voltage**
- Input Range: 10~30Vdc
- Consumption: 1.44W (voltage output)
  1.74W (current output)
Example leakage

- Matlab: Lec4_LeakageDemo.m
- 1 second observation => resolution: 1Hz
- 5 Hz and 5.5 Hz sine
- Only frequencies with integer number of period are correctly observed!
- Other frequencies will ‘leak’ to neighboring frequencies
Effect of observation time

- Signal is infinite and a finite observation is made
  - Theoretical concept: signal is multiplied with a window
    - window(t) = 1 within observation time
    - window(t) = 0 elsewhere
  - What is effect of this window (‘windowing’)?
Window and it’s Fourier transform

- Note: multiplication in time-domain is convolution in frequency-domain (and vice versa)

- The sharp transitions in the (time) window introduces considerable side-lobes in frequency domain

- Matlab:
  Lec4_TheorieLeakage.m
  Lec4_TheorieLeakageHanning.m
Begin & end values are different!
How to reduce leakage

- No window = rectangular window
- A rectangular window has substantial side-lobs

=> premultiply signals with a window before FFT
- Hanning window (cosine window: \( w=0.5 \left( 1+\cos \pi t/T \right) \))

Note:
- 1. This method is also called tapering
- 2. Do not to confuse with windowing of the covariance (which is used to reduce the variance of the spectral estimator based on FFT of covariance, see Lecture 3)
SIPE, lecture 4

-1 0 1

0

-1

1

0

-1

1
time domain

0

-10 -5 0 5 10

0

-5

0

5

10

frequency domain

5 Hz

5.5 Hz
Remarks windowing

- Shown examples are extremes: methods are developed for ‘noise’ signals, and not a single sine

- Numerous windows exist
  With windowing the begin & end effects are tempered, Hanning window is the most used window

- Premultiplying with window reduces side-lobes but introduces other artifact
  => there is always a trade-off between advantages and disadvantages!
How to prevent leakage

• Can we make ‘leakage’ free signals? (and consequently do not have to apply windowing to reduce leakage!)

• White noise contains all frequencies, so leakage will always occur (at least in theory)
Signal-to-Noise ratio

- SNR: ratio between signal power and noise power
- Definition:

\[ SNR = \frac{P_{signal}}{P_{noise}} = \left( \frac{A_{signal}}{A_{noise}} \right)^2 \]

\[ SNR(dB) = 10^{10 \log \left( \frac{P_{signal}}{P_{noise}} \right)} = 20^{10 \log \left( \frac{A_{signal}}{A_{noise}} \right)} \]
Signal-to-Noise Ratio (SNR)

- Improve SNR by increasing signal (u), or decreasing noise (n)
- Increase signal: turn up the volume!
- Decrease noise: average (in time or frequency)
Ideal perturbation

Properties
- Persistently exciting
- Introducing no bias and variance

Additional
- Long enough to gain sufficient frequency resolution
- Short enough to limit measurement time
  (Humans: fatigue, attention, etc)
- In case of human subjects: unpredictable
White noise vs. colored noise

- White noise
  - Leakage and aliasing, bad SNR

- Colored noise (=filtered white noise)
  - Improved SNR, no aliasing, leakage not solved.
  - Note that discrete noise is always ‘colored’ as result from sample frequency
Example colored noise: boost the signal

- Matlab: Lec4_ColoredNoise.m

- For most systems the input amplitude is restricted!

- If white and colored noise have the same variance (or the same maximum amplitude)
- The power in colored noise is concentrated in a limited number of frequencies
- => the power per frequency will be higher (Parseval!)
- => better SNR (within the bandwidth)
Improving the estimate 1

Bias (structural errors):

- Main causes
  - finite observation of stochastic input (leakage!)
  - frequency averaging
- Cure
  - application of 'leakage free' deterministic perturbation signals
  - moderate frequency averaging
  - apply method that do not need frequency averaging
Improving the estimate 2

Variance (random errors):
• Main causes
  • noise
• Cure
  • improve SNR
  • averaging (time or frequency domain)
Multisine signals

• With a limited observation time only the frequencies with an integer number of period can be ‘seen’ by spectral estimators.

• Idea: construct signal with all frequencies with integer number of period

• => Multisine signals

• Advantages:
  • no leakage, no aliasing
  • better SNR compared to white noise
Example multisine signal

- Matlab: Lec4_crest_example.m

- If white noise and multisine have same variance

- The power in multisine is concentrated in a limited number of frequencies
  - => the power per frequency is higher in multisine (Parseval!)
  - => better SNR
  - => no leakage, no aliasing
Cresting

- An normal uncrested multisine signal (as white noise) has many outliers
- Probability density function is Gaussian (matlab)

- 'Trick' of cresting: reduces the variance of the multisine signal, by removing outliers: cresting
- Probability density function is altered!

- Crest factor = $\frac{\max|x(t)|}{\sigma_x}$

- Even more improved SNR
More advanced multisine tricks

• Basically two possibilities to improve multisine

• Reduce number of frequencies
  • Power per frequency can be increased, better SNR
  • Example: linear frequency spacing
  • Example: (quasi-)logarithmic frequency spacing
    • => Bode diagram has logarithmic frequency axis!

• Shape of the gain per frequency
  • shape input signal such that output signal becomes flat
    • => ideal case for white noise disturbance on the output
  • shape input signal such that output signal has same shape as output noise
Matlab demo

- Lec4_PeriodicDemo.m
Reducing variance

- Average in time
  - Losing frequency resolution
  - Improves SNR
- Average adjacent frequencies
  - Can introduce bias
- Average in frequency domain
  - Multiple realizations or chop in multiple segments. Calculate spectral densities for each segment and average spectral densities over the segments ('Welch' method)
  - Losing frequency resolution
  - Improves SNR
Summary

• Sources for error in (spectral) estimators:
  • Aliasing \( f > \frac{f_s}{2} \)
  • Leakage \( f \neq k\Delta f \) with integer \( k \)
  • Low signal-to-noise ratio (SNR)

• Multisine signals
  • No aliasing, no leakage
  • Signal can be ‘shaped’ in frequency domain
  • Cresting further improves the SNR
Continuous and transient perturbations

- Transient perturbations
  - Impulses
  - Steps
  - Ramps

- Continuous perturbations
  - Random
    - White noise
    - Colored noise
  - Periodic
    - Sinusoids
    - Multisines
    - Binary noise (not discussed: switches randomly between two values)
FRF measurements with multiple periods of a periodic signal

- Periodic signal:
  - N samples per period
  - M periods

- Noise \((n_u, n_y)\) on measured input \(u(t)\) and output \(y(t)\)
FRF measurements with multiple periods of a periodic signal

- Sample mean (in freq domain)

\[ U(f) = \frac{1}{M} \sum_{l=1}^{M} U^{[l]}(f) \]

\[ S_{UU}(f) = \frac{1}{N} U^*(f) U(f) = \frac{1}{N} |U(f)|^2 \]

- Sample (co-)variance (in freq domain)

\[ \sigma_u^2(f) = \frac{1}{N(M-1)} \sum_{l=1}^{M} \left| U^{[l]}(f) - U(f) \right|^2 \]

\[ \sigma_{uy}^2(f) = \frac{1}{N(M-1)} \sum_{l=1}^{M} \left( U^{[l]}(f) - U(f) \right)^* \left( Y^{[l]}(f) - Y(f) \right) \]
FRF measurements with multiple periods of a periodic signal

- Periodic signal
  - N samples per period
  - M periods

- Frequency response function (FRF)
  \[ H(f) = \frac{Y(f)}{U(f)} \]

- Variance FRF
  \[ \sigma_H^2(f) = \frac{1}{M} |H(f)|^2 \left( \frac{\sigma_Y^2(f)}{S_{YY}(f)} + \frac{\sigma_U^2(f)}{S_{UU}(f)} - 2 \text{re} \left( \frac{\sigma_{UY}^2(f)}{S_{UY}(f)} \right) \right) \]
FRF measurements with multiple periods of a periodic signal

- No noise on input:

- Variance FRF

\[ \sigma_H^2(f) = \frac{1}{M} |H(f)|^2 \left( \frac{\sigma_Y^2(f)}{S_{YY}(f)} \right) \]
Example multiple periods

- Lec4_AveragePeriods.m

- Assume signal $u$ which is contaminated with noise $n$
  $$u(t) = x(t) + n(t)$$

- With multiple periods of periodic signal:
  $$X(f) \approx U(f) = \frac{1}{M} \sum_{l=1}^{M} U^{[l]}(f)$$
  $$S_{nn}(f) \approx \sigma_u^2(f) = \frac{1}{N(M-1)} \sum_{l=1}^{M} \left| U^{[l]}(f) - U(f) \right|^2$$

  - Sample variance (in freq domain) is (approximately) equal to the auto-spectral density of the noise!
Relevant Book Chapters

- Pintelon and Schoukens, System identification, a frequency domain approach

- Lecture 4:
  - Leakage and windowing/tapering: SP 2.2.2 & 2.2.3
  - Multisine signals: SP 2.3-2.6, 4.1-4.3

- Note that S&P use different scaling for the DFT (and order for $S_{uu}$)
  - This course (and most used, o.a. W&K, Matlab):
    \[ U(f) = \sum_{t=1}^{N} u(t)e^{-j2\pi \frac{ft}{N}}; \quad S_{uu} = \frac{1}{N} U^*(f)U(f) \]
  - S&P:
    \[ U(f) = \frac{1}{\sqrt{N}} \sum_{t=1}^{N} u(t)e^{-j2\pi \frac{ft}{N}}; \quad S_{uu} = U(f)U^*(f) \]
Readings

• Book Westwick & Kearney
  • Chapter 1, all (lecture 1)
  • Chapter 2, sec. 2.1 – 2.3.4 (lecture 1+2)
  • Chapter 3, sec. 3.1 – 3.2 (lecture 2)
  • Chapter 5, sec. 5.1 – 5.3 (lecture 3)

• Book Pintelon & Schoukens
  • Chapter 1, sec 1.1 – 1.4 (optional, lecture 1)
  • Chapter 2, all (lecture 4)
  • Chapter 4, all (lecture 4)

• Articles
  • de Vlugt et al. (lecture 5)
Next week: lecture 5

- Up to now:
  - Single input – single output (SISO) systems
    - Frequency response function (FRF)
    - Coherence

- Next week:
  - Open-loop and closed-loop
  - Multi input – multi output (MIMO) systems
    - MIMO frequency response function
    - Multiple coherence: is an output linearly related with the inputs
    - Partial coherence: is one output linearly related with one input
    - Open-loop and closed-loop