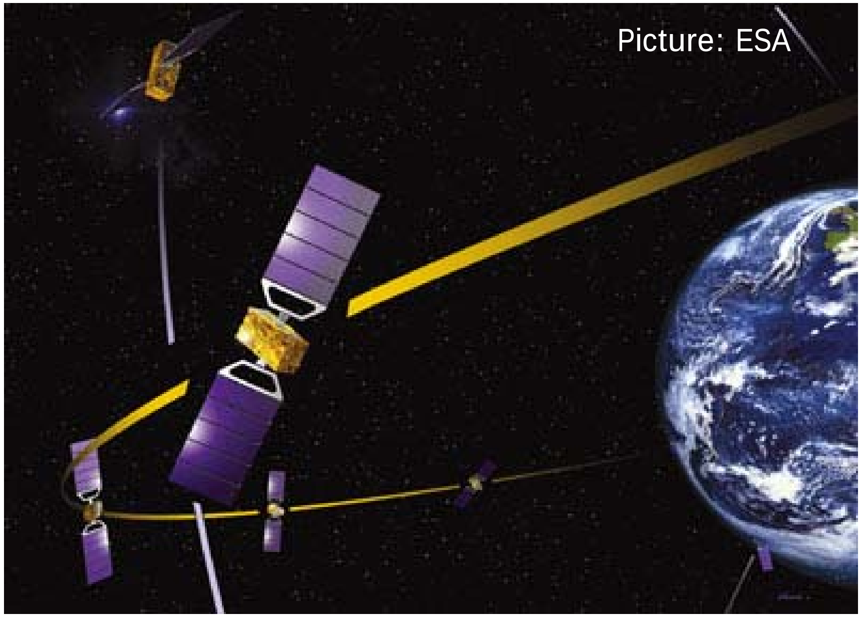
Satellite Navigation
error sources and position estimation


Picture: ESA

AE4E08

Sandra Verhagen

Course 2010 – 2011, lecture 5

Today’s topics

- Recap: GPS measurements and error sources
- Satellite clock and ephemeris
- Assignments VISUAL and RINEX
- Signal propagation errors: ionosphere and troposphere

- Book: Sections 5.2 – 5.3

Recap: Code and Carrier Phase measurements

$$\rho = r + I_\rho + T_\rho + c\left[\delta t_u - \delta t^s\right] + \varepsilon_\rho$$

$$\Phi = r + I_\phi + T_\phi + c\cdot\left(\delta t_u - \delta t^s\right) + \lambda \cdot A + \varepsilon_\Phi$$

Recap: error sources

- satellite:
  - orbit
  - clock
  - instrumental delays
- signal path
  - ionosphere
  - troposphere
  - multipath
- receiver
  - clock
  - instrumental delays
- other
  - spoofing
  - interference

Satellite clock and ephemeris

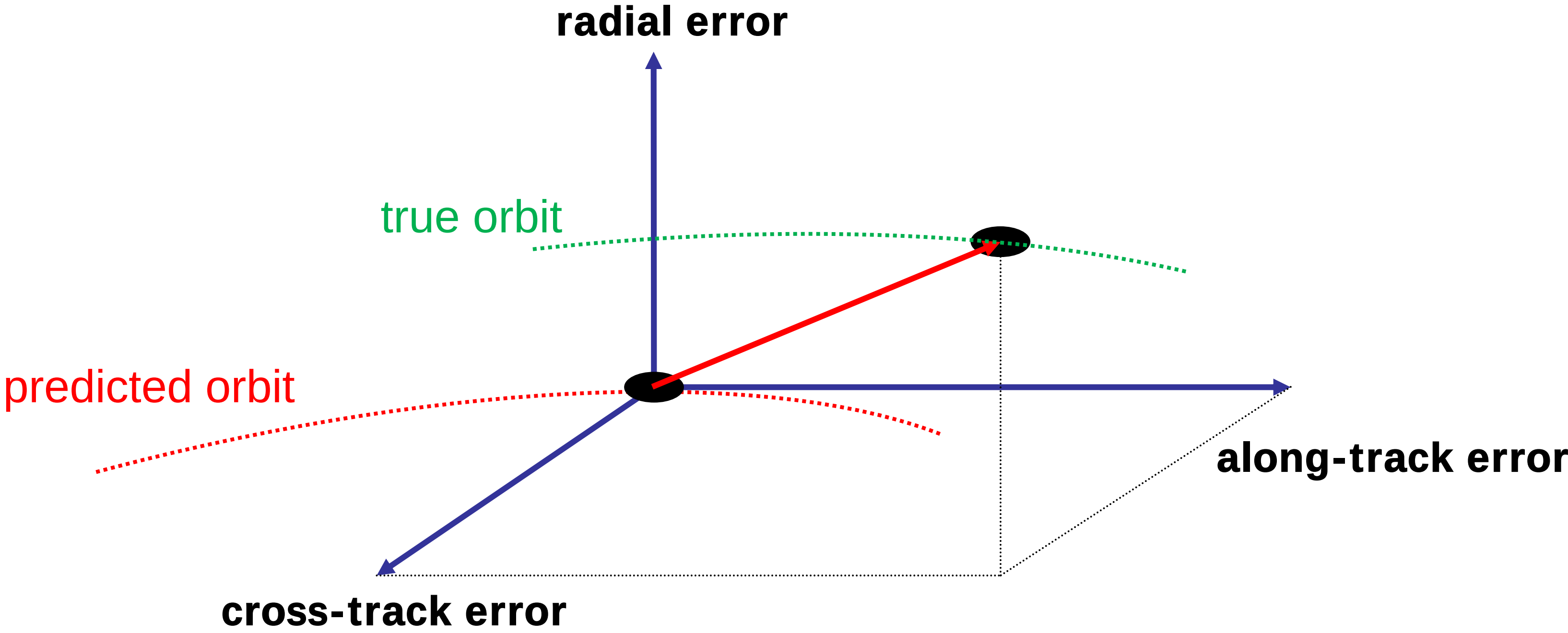
- determined by Control Segment based on measurements
- parameters in navigation message
- Kalman filter prediction
  - positions and velocities of satellites
  - phase bias, frequency bias, frequency drift rate clocks

  → prediction errors (grow with age of data)

Currently, ranging error < 3 meter rms

Satellite clock and ephemeris

- Ephemeris: **radial**, **along-track**, **cross-track** errors. Which is of the three is principal error source?



Satellite clock and ephemeris

Misra and Enge,
Section 4.2.4

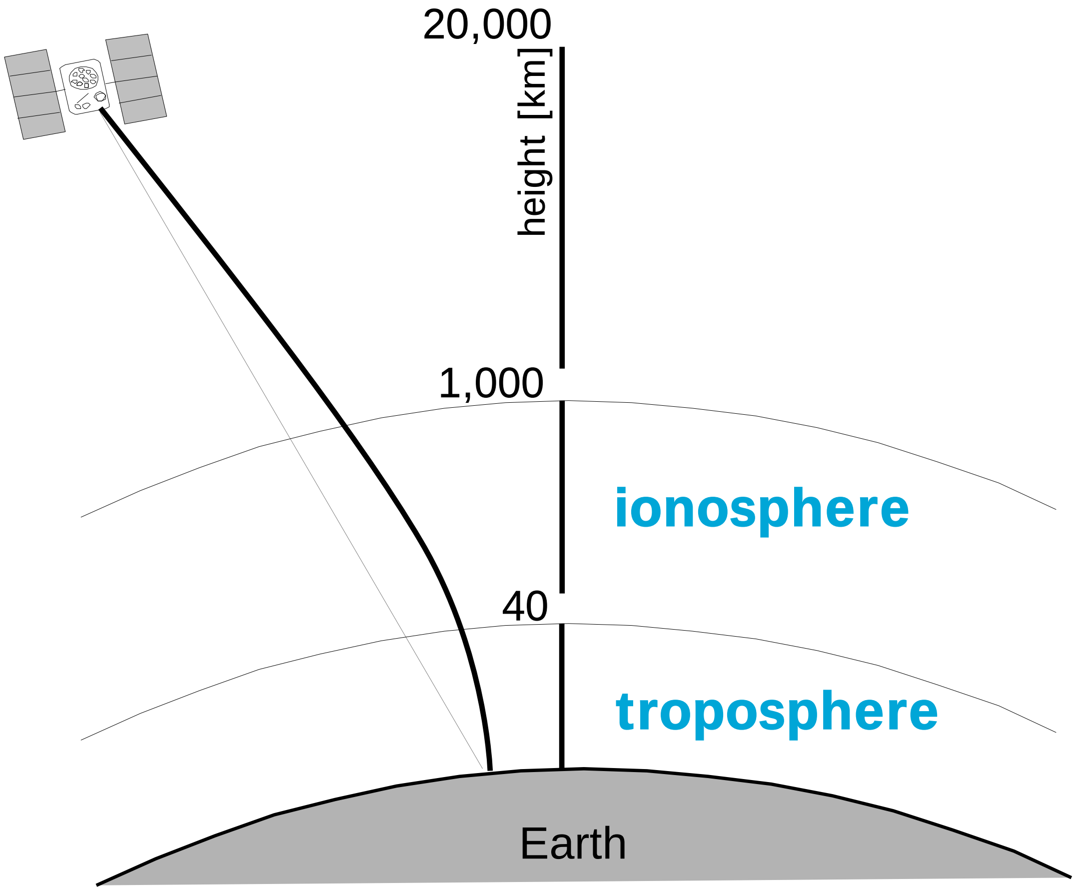
Satellite clock correction:

$$\delta t^s = t^s - t = a_{f0} + a_{f1}(t - t_{0c}) + a_{f2}(t - t_{0c})^2 + \Delta t_r$$

t_{0c}	reference epoch
a_{f0}	clock offset [s] ~1 µs – 1 ms
a_{f1}	fractional frequency offset [s/s] ~10^{-11} s/s
a_{f2}	fractional frequency drift [s/s²] ~0 s/s²
Δt_r	relativistic correction

broadcast with navigation message

Signal propagation errors



- Above ~1,000 km: vacuum
  → speed of light *c*
- ~ 50 – 1,000 km: **ionosphere**
- Below 40 km: **troposphere**

Atmosphere =
ionosphere + troposphere

Atmosphere changes **speed** and **direction** of propagation
→ **refraction**

Signal propagation errors

- Refractive index of a medium: $n = \dfrac{c}{v}$
- Refractive index changes along path
- Change in speed → change in travel time of signal

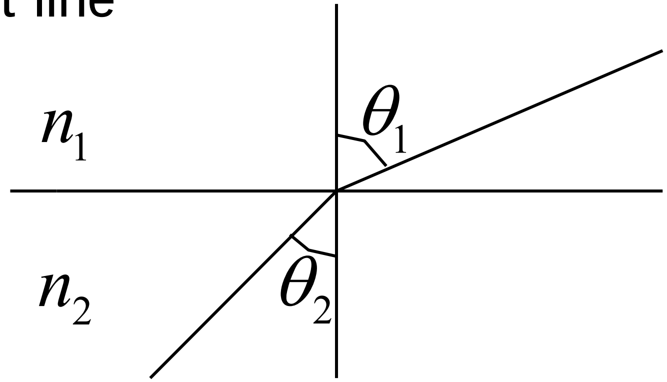
Signal propagation errors

- Refractive index of a medium: $n = \dfrac{c}{v}$

- Snell's law: changing refractive index results in bending of path → path longer than geometrical straight line

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

→ effect very small

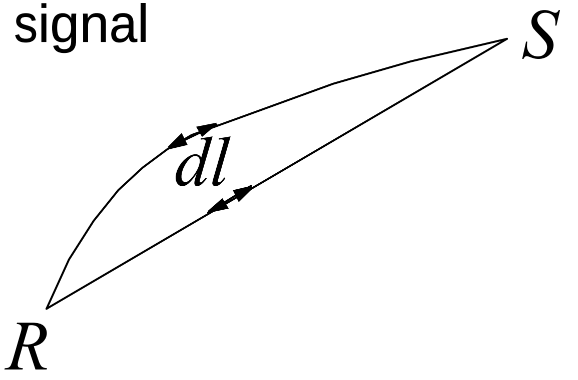


- Fermat's principle of least time:
  transit time along curved path is shorter than for straight-line path

Signal propagation errors

- Refractive index of a medium: $n = \frac{c}{v}$
- Refractive index changes along path: $n(l)$
- Change in speed → change in travel time τ of signal

$$\tau = \frac{1}{c} \int_S^R n(l)\,dl$$



Signal propagation errors

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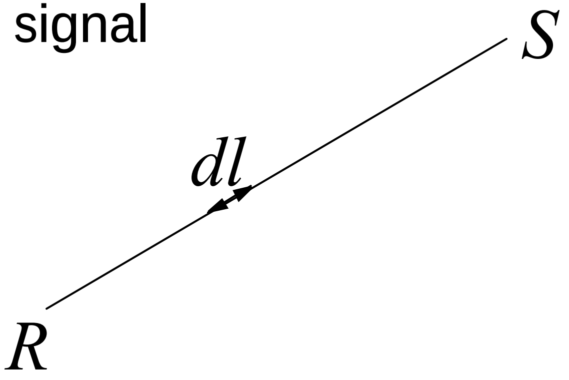


- Excess delay: $\Delta\tau = \dfrac{1}{c}\left[\displaystyle\int_S^R n(l)\,dl - \int_S^R dl\right] \Rightarrow \Delta\rho = \displaystyle\int_S^R \left[n(l) - 1\right] dl$

Signal propagation errors: ionosphere

- **Dispersive** medium: refractive index depends on **frequency** of signal
- For GPS (L-band) signals: **ionosphere is dispersive**, troposphere is not
- In dispersive medium: different phase (carrier) and group (code) velocities, v_p and v_g , resp.

modulated carrier wave → superposition of a **group** of waves of different frequencies

$$n_p = \frac{c}{v_p}$$

$$n_g = \frac{c}{v_g} = n_p + f\frac{dn_p}{df}$$

Signal propagation errors: ionosphere

- ionosphere contains free electrons and ions
- ionization caused by sun's radiation → state depends on solar activity
- temporal variations:
  - during the day, peak at 2 PM local time
  - day to day due to solar activity, geomagnetic disturbances
  - seasonal
  - 11-year solar cycle
  - local short term effects due to traveling ionospheric disturbances

Signal propagation errors: ionosphere

- propagation speed depends on **total electron content** (**TEC**) = number of electrons in tube of 1 m^2 from receiver to satellite

$$\text{TEC} = \int_S^R n_e(l)\,dl \qquad [\text{TECU}]$$

- with $n_e(l)$ the electron density along the path
- 1 TECU (TEC Unit) = 10^{16} electrons / m^2
- **VTEC** = TEC in vertical direction [in book TECV]

Signal propagation errors: ionosphere

Global map of TEC (computed from global network of GPS receivers)

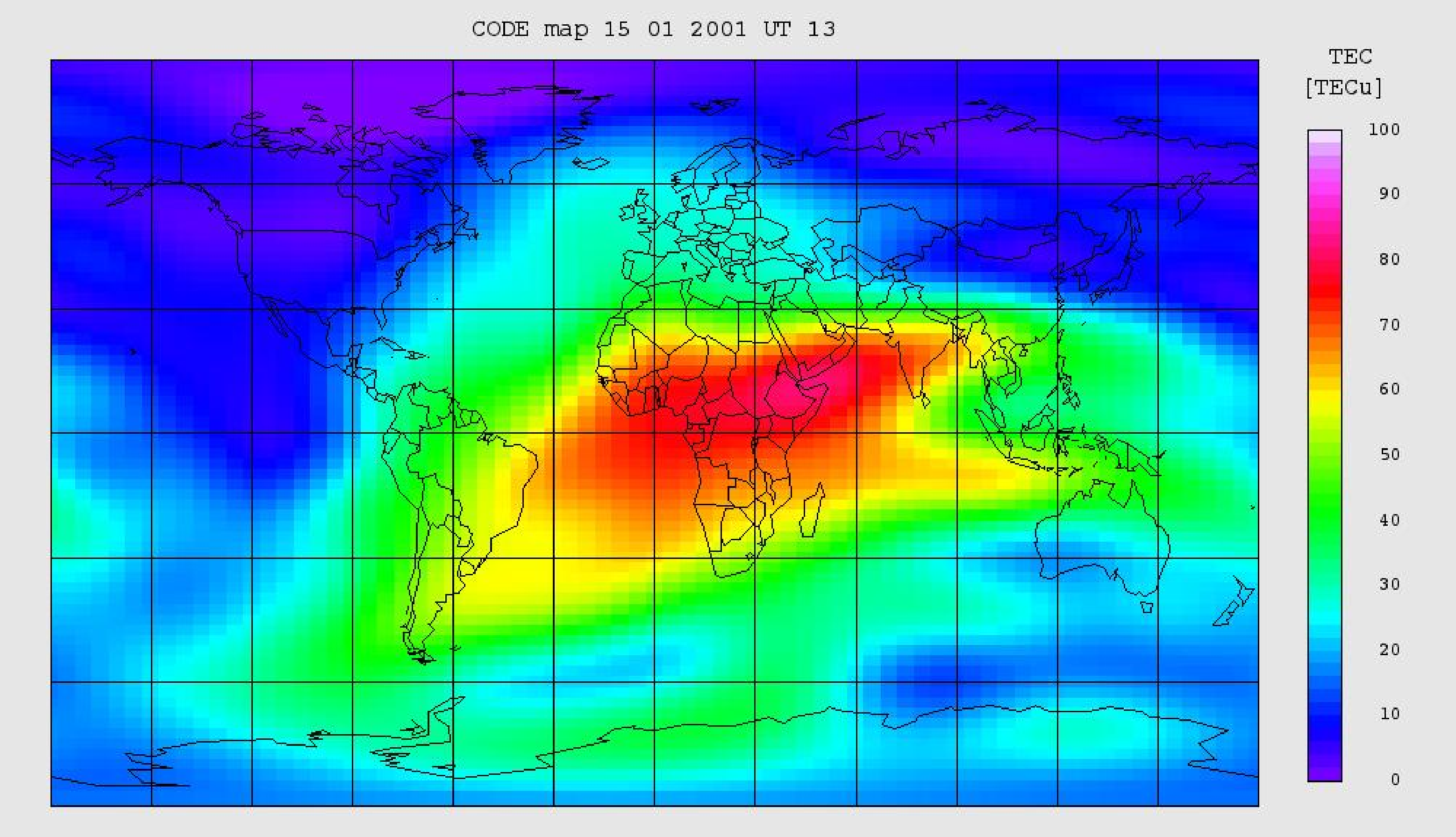
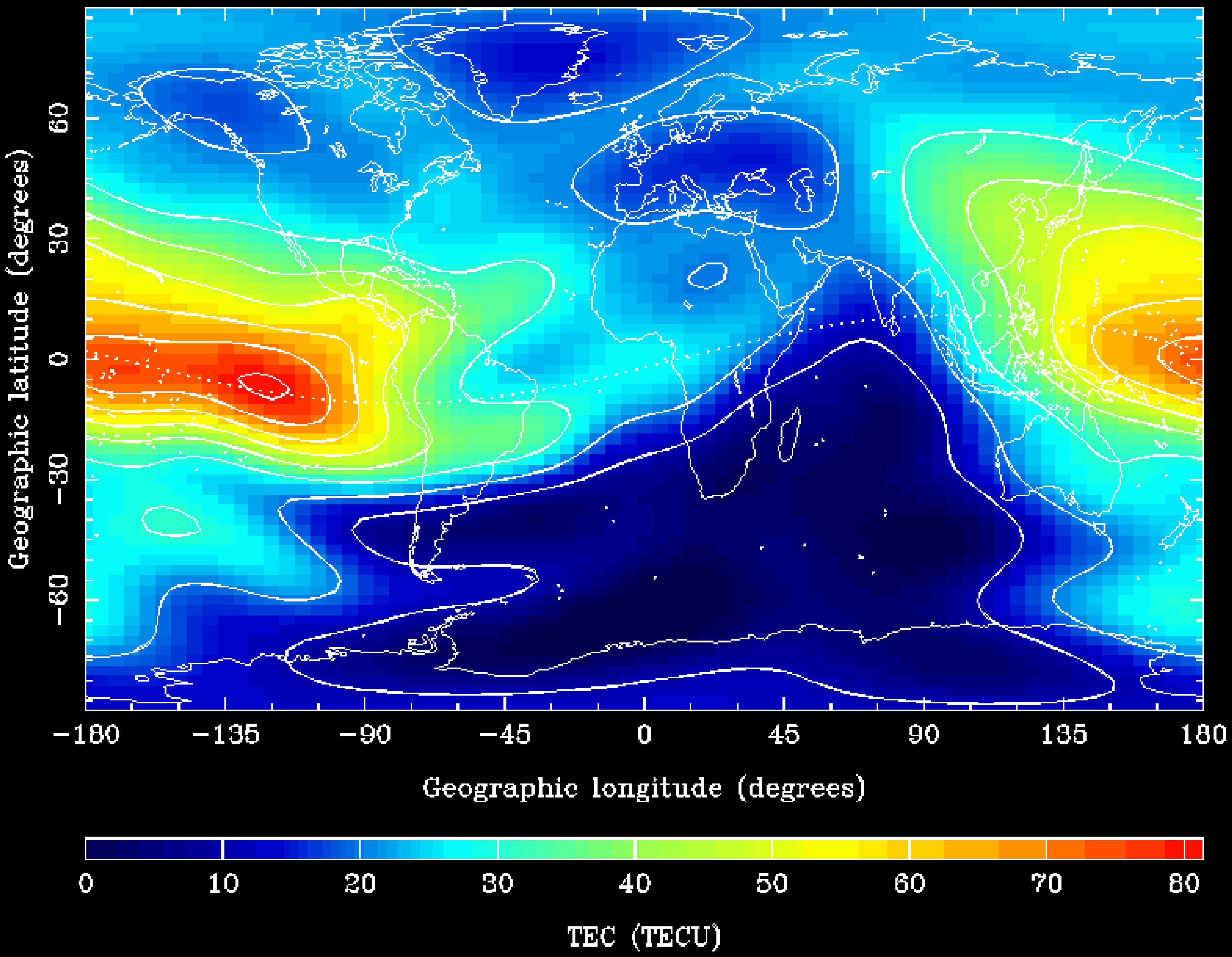


Figure from S.M. Radicella – ARPL; Data Astronomical Institute University of Berne

CODE'S GLOBAL IONOSPHERE INFO FOR DAY 177, 2000 – 00:15 UT



Signal propagation errors: ionosphere

- highest ionospheric delay within ±20^o of magnetic equator
- solar flares
  - → magnetic storms
  - → large and quickly varying electron densities, esp. polar regions
  - → rapid fluctuations in **phase** and **amplitude** of GPS signals, called **scintillation** and **fading**, resp.
  - → may cause losses of lock

Signal propagation errors: ionosphere

phase **advance**

$$n_p = \frac{c}{v_p} \approx 1 - \frac{40.3 n_e}{f^2}$$

$$\text{TEC} = \int_S^R n_e(l)\,dl$$

$$\Delta\tau_p = \frac{1}{c}\int_S^R \left(n_p(l) - 1\right) dl$$

$$= -\frac{1}{c}\int_S^R \frac{40.3 n_e(l)}{f^2}\,dl = -\frac{40.3 \cdot \text{TEC}}{c f^2}$$ [s]

$$I_\phi = c\Delta\tau_p = -\frac{40.3 \cdot \text{TEC}}{f^2}$$ [m]

$$\Phi = r + I_\phi + T_\phi + c\cdot\left(\delta t_u - \delta t^s\right) + \lambda \cdot A + \varepsilon_\Phi$$

Signal propagation errors: ionosphere

group **delay**

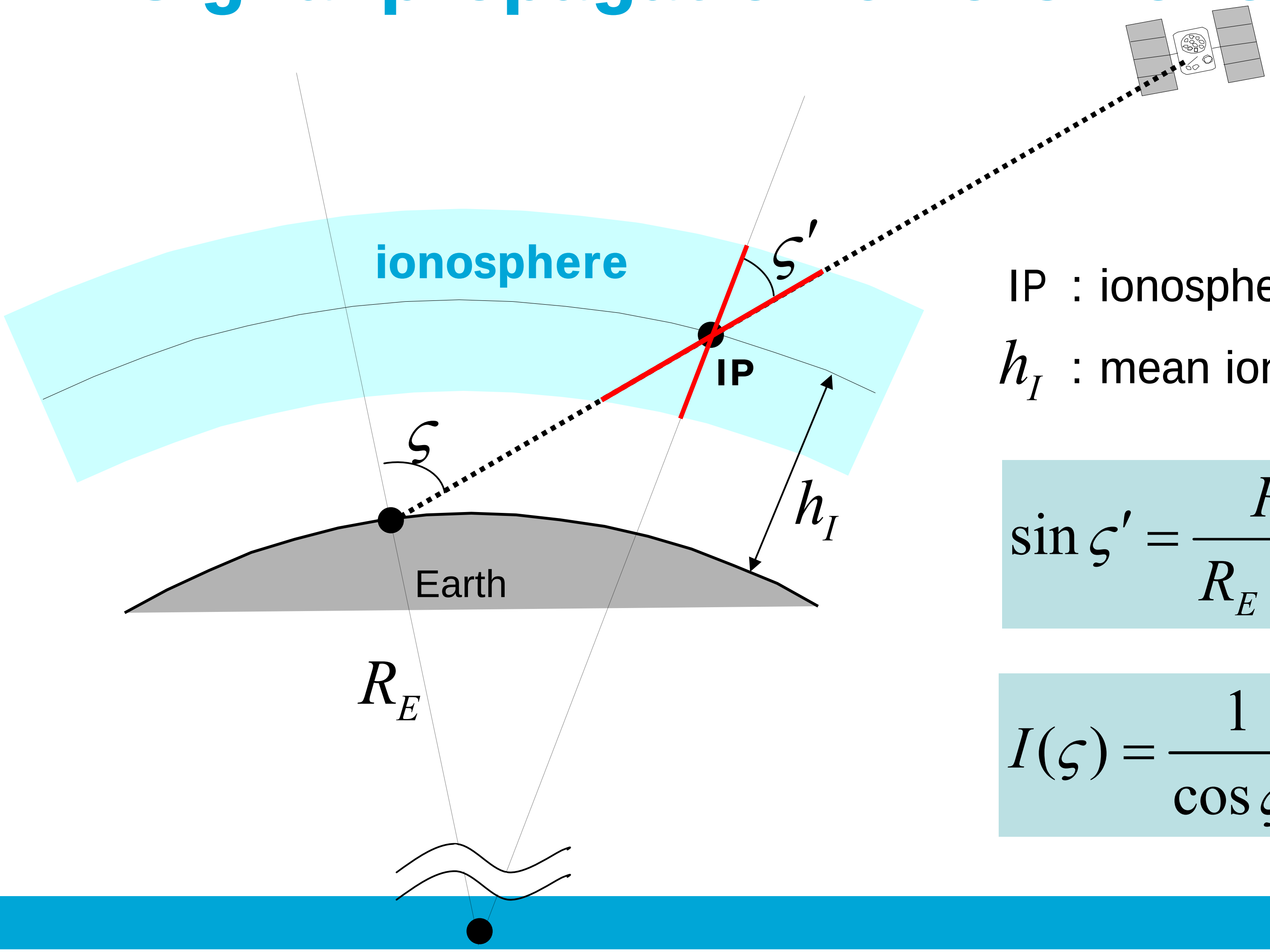
$$n_g = n_p + f\frac{dn_p}{df} = 1 + \frac{40.3 n_e}{f^2}$$

$$I_\rho = c\Delta\tau_g = \frac{40.3 \cdot \mathrm{TEC}}{f^2}$$ [m]

$$\rho = r + I_\rho + T_\rho + c\left[\delta t_u - \delta t^s\right] + \varepsilon_\rho$$

$$I_\rho = -I_\phi = I$$

Signal propagation errors: ionosphere



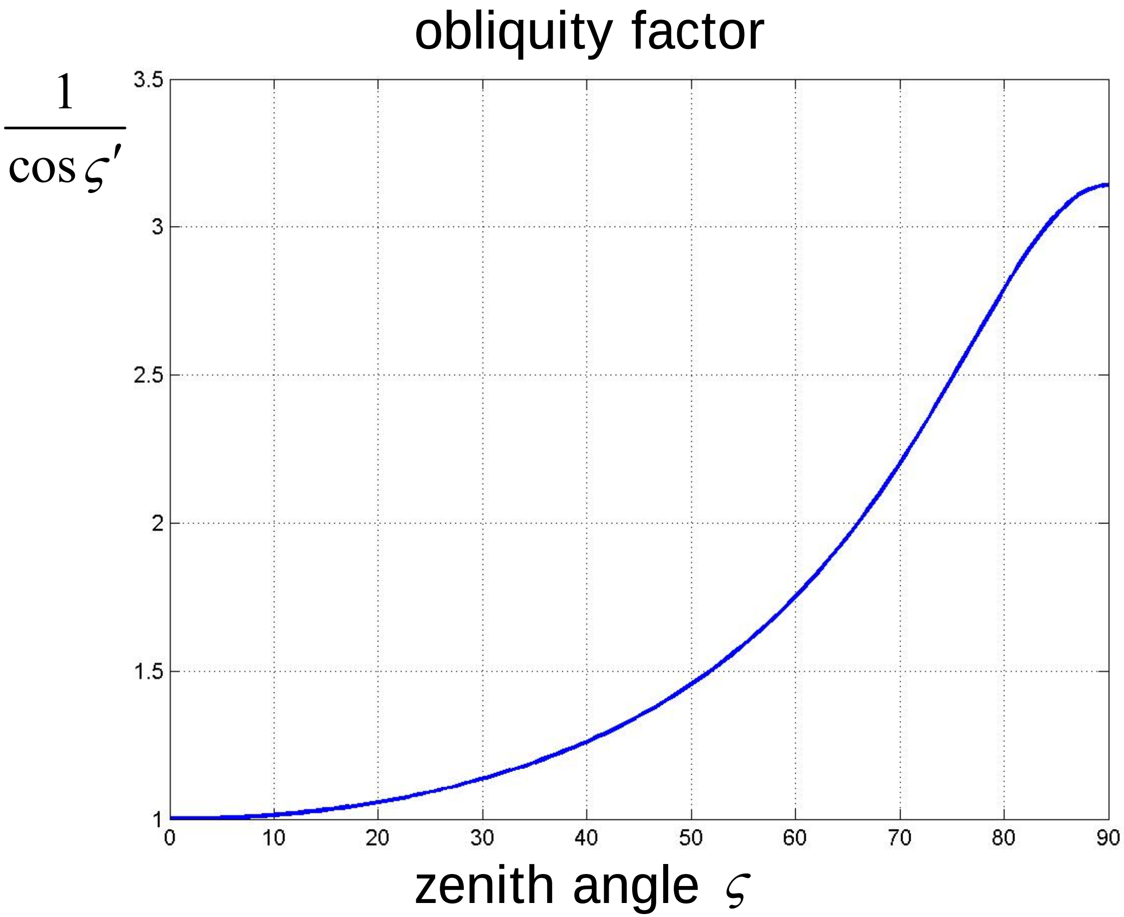
IP : ionospheric pierce point

h_I : mean ionosphere height

$$\sin\varsigma' = \frac{R_E}{R_E + h_I}\sin\varsigma$$

$$I(\varsigma) = \frac{1}{\cos\varsigma'} I_z$$

Signal propagation errors: ionosphere



Signal propagation errors: ionosphere

zenith delay mid-latitudes:

- 1-3 m at night
- 5-15 m mid-afternoon

peak solar cycle near equator:

- max. ~36 m

Signal propagation errors: ionosphere

$$I_{L1} = \frac{40.3 \cdot \text{TEC}}{f_{L1}^2}$$

$$I_{L2} = \frac{40.3 \cdot \text{TEC}}{f_{L2}^2} = \frac{f_{L1}^2}{f_{L2}^2} I_{L1}$$

$$\rho_{Li} = r + I_{Li} + T + c\left[\delta t_u - \delta t^s\right] + \varepsilon_{\rho_{Li}}$$

ionosphere-free combination:

$$a\rho_{L1} - b\rho_{L2} = \underbrace{r + T + c\left[\delta t_u - \delta t^s\right]}_{= \rho_{IF}} + \varepsilon_{\rho_{IC}}$$

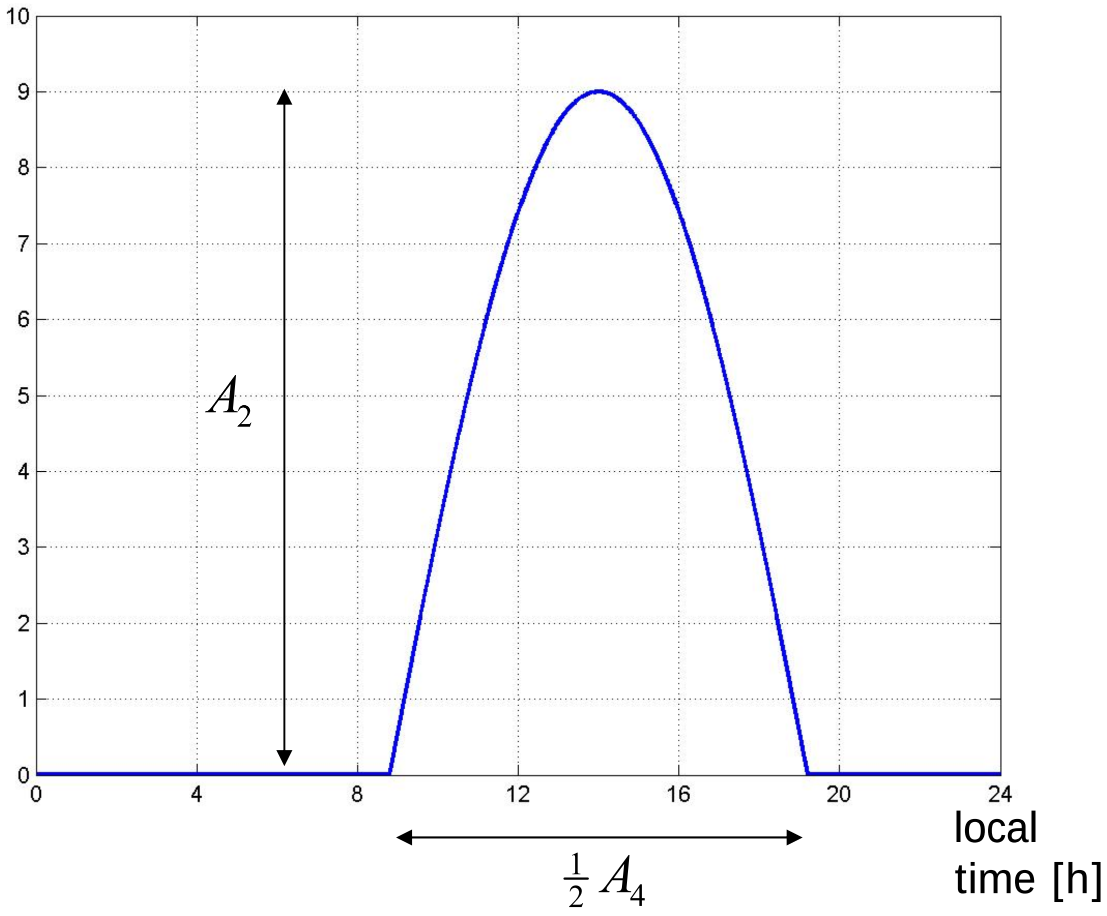
bias removed;
noise increased

Signal propagation errors: ionosphere

Klobuchar model

A_2 and A_4 broadcasted with navigation message

~50% reduction
RMS range error



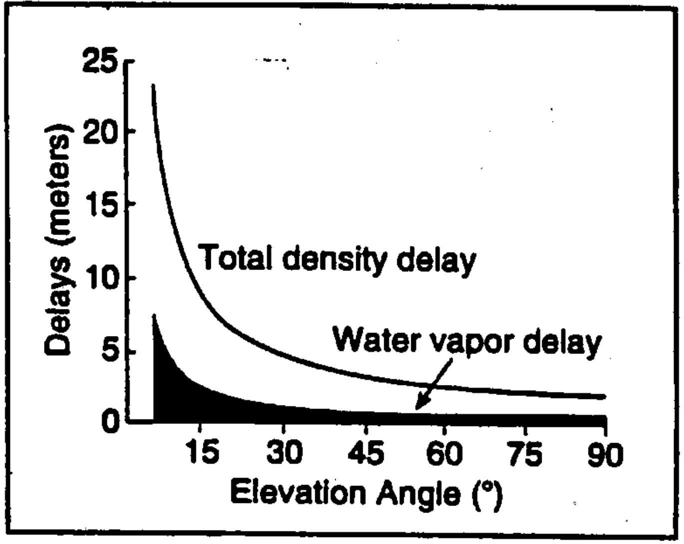
Signal propagation errors: ionosphere

- **NeQuick** model (proposed for Galileo)
  - 3-D electron density model
  - One location dependent input parameter (Az)
  - Az is given for Galileo in broadcast message
  - Slant-TEC is compute by numerical integration along line-of-sight
- Compute corrections from IGS **Global Ionosphere Maps** (GIM)
  - 2-D grid of VTEC (2.5^o latitude x 5^o longitude @ 2 hours)
  - Interpolate VTEC to ionospheric point at time of observation
  - Map VTEC to slant direction using mapping function

Signal propagation errors: troposphere

- 9 km (poles) – 16 km (equator)
- Dry gases and water vapor
- Recall: non-dispersive, i.e. refraction does not depend on frequency
- Propagation speed lower than in free space:
  apparent range is longer (~2.5 – 25 m)
- Same phase and group velocities

$$T_{\rho_{L1}} = T_{\rho_{L2}} = T_{\phi_{L1}} = T_{\phi_{L2}} = T$$



Signal propagation errors: troposphere

- Refractivity $N = (n-1) \times 10^6$

$$N = N_d + N_w$$

$$T = 10^{-6} \int N(l)dl = 10^{-6} \int \left[N_d(l) + N_w(l) \right] dl = T_d + T_w$$

$$N_d \approx 77.64 \frac{P}{T}$$

$$N_w \approx 3.73 \cdot 10^5 \frac{e}{T^2}$$

P : total pressure [mbar]
T : temperature [K]
e : partial pressure water vapor [mbar]

if known → refractivity known

Signal propagation errors: troposphere

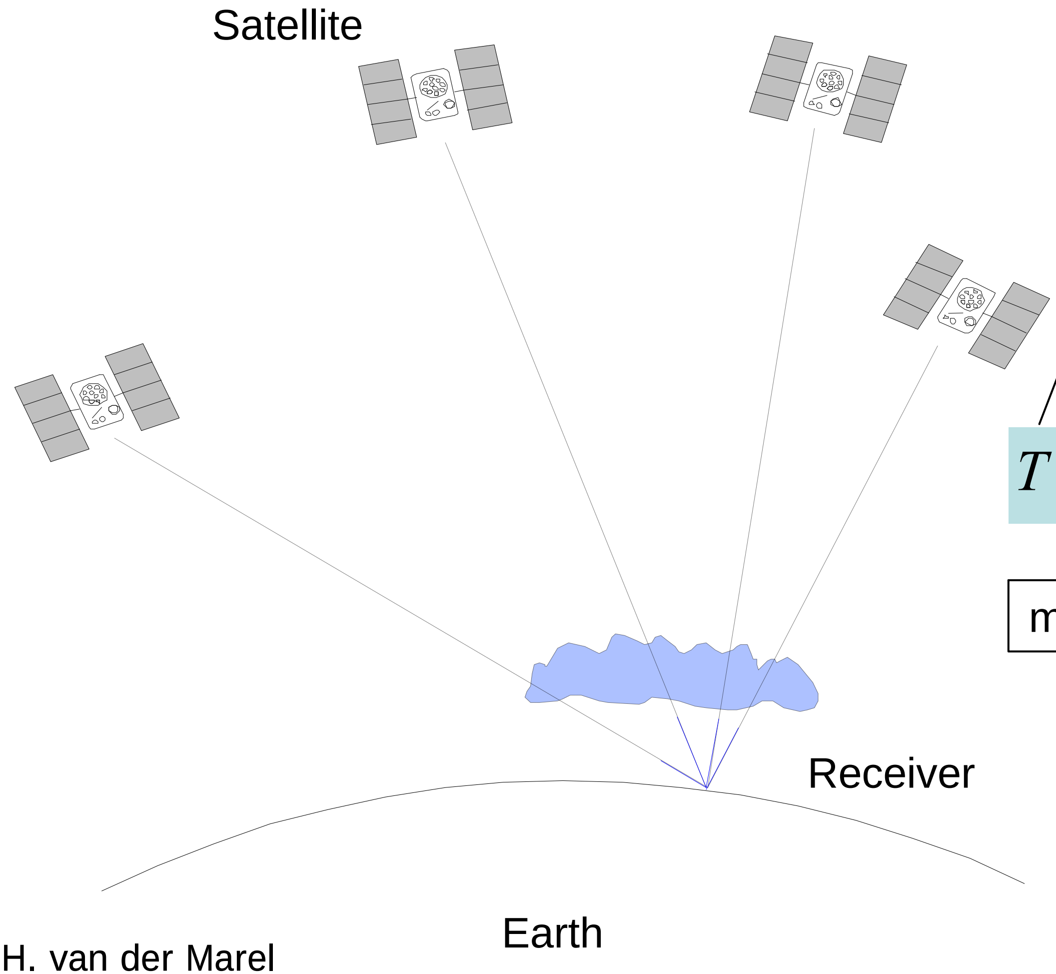


Figure: H. van der Marel

$$T = m_d(el) * T_{z,d} + m_w(el) * T_{z,w}$$

tropospheric delay

computed hydrostatic delay

mapping functions

Unknown tropospheric zenith wet delay

Signal propagation errors: troposphere

- **Saastamoinen model**:
  zenith dry and wet delays calculated from temperature, pressure and humidity (measurements or standard atmosphere), height and latitude
- **Hopfield model**:
  dry and wet refractivities calculated

- Dry delay in zenith direction 2.3 – 2.6 m at sea level
  → can be predicted with accuracy of few mm's
- Wet delay depends on water vapor profile along path, 0 – 80 cm
  → accuracy of models few cm's
- If no actual meteorological observations available (standard atmosphere applied): total zenith delay error 5 – 10 cm

Signal propagation errors: summary

	ionosphere	**troposphere**
height	50 – 1000 km	0 – 16 km
variability	diurnal, seasonal, solar cycle (11 yr), solar flares	low
zenith delay	meters – tens of meters	2.3 – 2.6 m (sea level)
obliquity factor 30°	1.8	2
15°	2.5	4
3°	3	10
modeling error (zenith)	1 - >10 m	5 – 10 cm (no met. data)
dispersive	yes	no

all values are approximate, depending on location and circumstances

Signal propagation errors

Homework exercise:

- make plots of the different mapping functions (page 173 Misra and Enge) as function of the elevation angle (ranging from 0 – 90^o)
- compare them with each other AND with the obliquity factor of the ionosphere delay (slide 22)
- try to explain the differences
- more details: see assignment on blackboard

Summary and outlook

- GPS measurements and error sources

Next:

Position, Velocity and Time (PVT) estimation