Satellite Navigation
error sources and position estimation

AE4E08

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Course 2010 - 2011, lecture 5
Today’s topics

• Recap: GPS measurements and error sources
• Satellite clock and ephemeris
• Assignments VISUAL and RINEX
• Signal propagation errors: ionosphere and troposphere

• Book: Sections 5.2 – 5.3
Recap: Code and Carrier Phase measurements

\[ \rho = r + I_\rho + T_\rho + c \left[ \delta t_u - \delta t^s \right] + \epsilon_\rho \]

\[ \Phi = r + I_\phi + T_\phi + c \cdot \left( \delta t_u - \delta t^s \right) + \lambda \cdot A + \epsilon_\Phi \]
Recap: error sources

- satellite:
  - orbit
  - clock
  - instrumental delays
- signal path
  - ionosphere
  - troposphere
  - multipath

- receiver
  - clock
  - instrumental delays
- other
  - spoofing
  - interference
Satellite clock and ephemeris

- determined by Control Segment based on measurements
- parameters in navigation message
- Kalman filter prediction
  - positions and velocities of satellites
  - phase bias, frequency bias, frequency drift rate clocks
  - prediction errors (grow with age of data)

Currently, ranging error < 3 meter rms
Satellite clock and ephemeris

- Ephemeris: radial, along-track, cross-track errors. Which is of the three is principal error source?
Satellite clock and ephemeris

Satellite clock correction:

\[
\delta t^s = t^s - t = a_{f0} + a_{f1}(t - t_{0c}) + a_{f2}(t - t_{0c})^2 + \Delta t_r
\]

- \( t_{0c} \): reference epoch
- \( a_{f0} \): clock offset [s] \( \sim 1 \ \mu s - 1 \ \text{ms} \)
- \( a_{f1} \): fractional frequency offset [s/s] \( \sim 10^{-11} \ \text{s/s} \)
- \( a_{f2} \): fractional frequency drift [s/s^2] \( \sim 0 \ \text{s/s}^2 \)
- \( \Delta t_r \): relativistic correction

Misra and Enge, Section 4.2.4
Signal propagation errors

- Above ~1,000 km: vacuum → speed of light $c$
- ~50 – 1,000 km: ionosphere
- Below 40 km: troposphere

Atmosphere = ionosphere + troposphere

Atmosphere changes speed and direction of propagation → refraction
Signal propagation errors

- Refractive index of a medium: $n = \frac{c}{v}$
- Refractive index changes along path
- Change in speed $\rightarrow$ change in travel time of signal
Signal propagation errors

• Refractive index of a medium: \( n = \frac{c}{v} \)

• Snell’s law: changing refractive index results in bending of path \( \rightarrow \) path longer than geometrical straight line
  \[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]
  \( \rightarrow \) effect very small

• Fermat’s principle of least time:
  transit time along curved path is shorter than for straight-line path
Signal propagation errors

- Refractive index of a medium: \( n = \frac{c}{v} \)

- Refractive index changes along path: \( n(l) \)

- Change in speed \( \rightarrow \) change in travel time \( \tau \) of signal

\[
\tau = \frac{1}{c} \int_{S}^{R} n(l) \, dl
\]
Signal propagation errors

- Refractive index of a medium:
  \[ n = \frac{c}{v} \]

- Refractive index changes along path: \( n(l) \)

- Change in speed \( \rightarrow \) change in travel time \( \tau \) of signal

\[
\tau = \frac{1}{c} \int_{S}^{R} n(l) dl
\]

- Excess delay:

\[
\Delta \tau = \frac{1}{c} \left[ \int_{S}^{R} n(l) dl - \int_{S}^{R} dl \right]
\]

\[
\Delta \rho = \int_{S}^{R} [n(l) - 1] dl
\]
Signal propagation errors: ionosphere

- **Dispersive** medium: refractive index depends on frequency of signal
- For GPS (L-band) signals: ionosphere is dispersive, troposphere is not
- In dispersive medium: different phase (carrier) and group (code) velocities, $v_p$ and $v_g$, resp.

\[
\begin{align*}
n_p &= \frac{c}{v_p} \\
n_g &= \frac{c}{v_g} = n_p + f \frac{dn_p}{df}
\end{align*}
\]
Signal propagation errors: ionosphere

- ionosphere contains free electrons and ions
- ionization caused by sun’s radiation → state depends on solar activity
- temporal variations:
  - during the day, peak at 2 PM local time
  - day to day due to solar activity, geomagnetic disturbances
  - seasonal
  - 11-year solar cycle
  - local short term effects due to traveling ionospheric disturbances
Signal propagation errors: ionosphere

- propagation speed depends on **total electron content (TEC)** = number of electrons in tube of 1 m² from receiver to satellite

\[
\text{TEC} = \int_{S}^{R} n_e(l) dl \quad [\text{TECU}]
\]

- with \( n_e(l) \) the electron density along the path

- 1 TECU (TEC Unit) = \( 10^{16} \) electrons / m²

- **VTEC** = TEC in vertical direction [in book TECV]
Signal propagation errors: ionosphere

Global map of TEC (computed from global network of GPS receivers)

Figure from S.M. Radicella – ARPL; Data Astronomical Institute University of Berne
Signal propagation errors: ionosphere

- highest ionospheric delay within ±20° of magnetic equator
- solar flares
  - magnetic storms
  - large and quickly varying electron densities, esp. polar regions
  - rapid fluctuations in phase and amplitude of GPS signals, called scintillation and fading, resp.
  - may cause losses of lock
Signal propagation errors: ionosphere

**Phase advance**

\[ n_p = \frac{c}{v_p} \approx 1 - \frac{40.3 n_e}{f^2} \]

\[ \Delta \tau_p = \frac{1}{c} \int_S^n (n_p(l) - 1) dl \]

\[ = -\frac{1}{c} \int_S^n 40.3 n_e(l) \frac{dl}{f^2} = -\frac{40.3 \cdot \text{TEC}}{cf^2} \] [s]

\[ I_\phi = c \Delta \tau_p = -\frac{40.3 \cdot \text{TEC}}{f^2} \] [m]

\[ \Phi = r + I_\phi + T_\phi + c \cdot (\delta t_u - \delta t^s) + \lambda \cdot A + \epsilon_\Phi \]
Signal propagation errors: ionosphere

group delay

\[ n_g = n_p + f \frac{dn_p}{df} = 1 + \frac{40.3n_e}{f^2} \]

\[ I_\rho = c\Delta \tau_g = \frac{40.3 \cdot \text{TEC}}{f^2} \quad [\text{m}] \]

\[ \rho = r + I_\rho + T_\rho + c\left[ \delta t_u - \delta t^s \right] + \varepsilon_\rho \]

\[ I_\rho = -I_\phi = I \]
Signal propagation errors: ionosphere

- IP: ionospheric pierce point
- $h_I$: mean ionosphere height

\[
\sin \zeta' = \frac{R_E}{R_E + h_I} \sin \zeta
\]

\[
I(\zeta) = \frac{1}{\cos \zeta'} I_z
\]
Signal propagation errors: ionosphere

\[
\frac{1}{\cos \zeta'}
\]

obliquity factor

zenith angle $\zeta$
Signal propagation errors: ionosphere

zenith delay mid-latitudes:
• 1-3 m at night
• 5-15 m mid-afternoon

peak solar cycle near equator:
• max. ~36 m
Signal propagation errors: ionosphere

\[ I_{L1} = \frac{40.3 \cdot \text{TEC}}{f_{L1}^2} \]

\[ I_{L2} = \frac{40.3 \cdot \text{TEC}}{f_{L2}^2} = \frac{f_{L1}^2}{f_{L2}^2} I_{L1} \]

\[ \rho_{Li} = r + I_{Li} + T + c \left[ \delta t_u - \delta t_s \right] + \varepsilon_{\rho_{Li}} \]

Ionosphere-free combination:

\[ a \rho_{L1} - b \rho_{L2} = r + T + c \left[ \delta t_u - \delta t_s \right] + \varepsilon_{\rho_{IC}} \]

\[ = \rho_{IF} \]
Signal propagation errors: ionosphere

Klobuchar model

$A_2$ and $A_4$
broadcasted with
navigation message

~50% reduction
RMS range error
Signal propagation errors: ionosphere

- **NeQuick** model (proposed for Galileo)
  - 3-D electron density model
  - One location dependent input parameter (Az)
  - Az is given for Galileo in broadcast message
  - Slant-TEC is computed by numerical integration along line-of-sight
- Compute corrections from IGS **Global Ionosphere Maps** (GIM)
  - 2-D grid of VTEC (2.5° latitude x 5° longitude @ 2 hours)
  - Interpolate VTEC to ionospheric point at time of observation
  - Map VTEC to slant direction using mapping function
Signal propagation errors: troposphere

- 9 km (poles) – 16 km (equator)
- Dry gases and water vapor
- Recall: non-dispersive, i.e. refraction does not depend on frequency
- Propagation speed lower than in free space: apparent range is longer (~2.5 – 25 m)
- Same phase and group velocities

\[ T\rho_{L1} = T\rho_{L2} = T\phi_{L1} = T\phi_{L2} = T \]
Signal propagation errors: troposphere

- Refractivity $N = (n - 1) \times 10^6$

\[
N = N_d + N_w
\]

\[
T = 10^{-6} \int N(l)dl = 10^{-6} \int [N_d(l) + N_w(l)]dl = T_d + T_w
\]

\[
N_d \approx 77.64 \frac{P}{T}
\]

\[
N_w \approx 3.73 \times 10^5 \frac{e}{T^2}
\]

$P$ : total pressure [mbar]
$T$ : temperature [K]
$e$ : partial pressure water vapor [mbar]

if known $\rightarrow$ refractivity known
Signal propagation errors: troposphere

$$T = m_d(\text{el}) \cdot T_{z,d} + m_w(\text{el}) \cdot T_{z,w}$$

- tropospheric delay
- computed hydrostatic delay
- mapping functions
- Unknown tropospheric zenith wet delay
Signal propagation errors: troposphere

- **Saastamoinen model:**
  zenith dry and wet delays calculated from temperature, pressure and humidity (measurements or standard atmosphere), height and latitude
- **Hopfield model:**
  dry and wet refractivities calculated

- Dry delay in zenith direction 2.3 – 2.6 m at sea level
  - can be predicted with accuracy of few mm’s
- Wet delay depends on water vapor profile along path, 0 – 80 cm
  - accuracy of models few cm’s
- If no actual meteorological observations available (standard atmosphere applied): total zenith delay error 5 – 10 cm
# Signal propagation errors: summary

<table>
<thead>
<tr>
<th></th>
<th>ionosphere</th>
<th>troposphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>height</td>
<td>50 – 1000 km</td>
<td>0 – 16 km</td>
</tr>
<tr>
<td>variability</td>
<td>diurnal, seasonal, solar cycle (11 yr), solar flares</td>
<td>low</td>
</tr>
<tr>
<td>zenith delay</td>
<td>meters – tens of meters</td>
<td>2.3 – 2.6 m (sea level)</td>
</tr>
<tr>
<td>obliquity factor</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30°</td>
<td>1.8</td>
<td>2</td>
</tr>
<tr>
<td>15°</td>
<td>2.5</td>
<td>4</td>
</tr>
<tr>
<td>3°</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>modeling error (zenith)</td>
<td>1 - &gt;10 m</td>
<td>5 – 10 cm (no met. data)</td>
</tr>
<tr>
<td>dispersive</td>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>

All values are approximate, depending on location and circumstances.
Signal propagation errors

Homework exercise:

- make plots of the different mapping functions (page 173 Misra and Enge) as function of the elevation angle (ranging from 0 – 90°)
- compare them with each other AND with the obliquity factor of the ionosphere delay (slide 22)
- try to explain the differences
- more details: see assignment on blackboard
Summary and outlook

• GPS measurements and error sources

Next:
Position, Velocity and Time (PVT) estimation