Porous flow, general

Chapter 5

cT4310 Bed, Bank and Shoreline protection

H.J. Verhagen

June 3, 2012
Introduction

• flow through granular medium (sand, pebbles)
• two aspects are relevant:
  • pressure
  • drag
• natural filters and geotextiles
Examples of loads due to porous flow
basic equations

Navier-Stokes

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2} - \frac{\partial u'^2}{\partial x} - \frac{\partial u'w'}{\partial z}
\]

Filter velocity

\[
u_f = \frac{1}{A} \int \int_A u \, dA = n \cdot u \quad \left( n = \frac{V_P}{V_T} \right)
\]

Combine terms

\[
\frac{1}{\rho g} \frac{\partial p}{\partial x} = i = a u_f + b u_f \left| u_f \right| + c \frac{\partial u_f}{\partial t}
\]

Forchheimer equation

=0 for stationary flow

with:

\[
a = \alpha \frac{(1-n)^2}{n^3} \frac{\nu}{g d_{(n)50}^2}
\]

\[
b = \beta \frac{(1-n)}{n^3} \frac{1}{g d_{(n)50}}
\]
velocities, gradients and averaging
relation between filter velocity and gradient for various materials
relation between velocity and pressure

\[ u_f = k \left( \frac{1}{p} \right) \]

\( k \) permeability in m/s of porous material

for \( p=1 \) Darcy’s law

\( p=2 \) Turbulent flow
values of $k$ for various materials

<table>
<thead>
<tr>
<th>Material</th>
<th>$d_{50}$ ($&lt; 63.10^{-3}$ m) or $d_{n50}$ (m)</th>
<th>Permeability, $k$ (m/s)</th>
<th>Character of flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>$&lt; 2.10^{-6}$</td>
<td>$10^{-10}$ - $10^{-8}$</td>
<td>laminar</td>
</tr>
<tr>
<td>Silt</td>
<td>$2.10^{-6}$ - $63.10^{-6}$</td>
<td>$10^{-8}$ - $10^{-6}$</td>
<td>laminar</td>
</tr>
<tr>
<td>Sand</td>
<td>$63.10^{-6}$ - $2.10^{-3}$</td>
<td>$10^{-6}$ - $10^{-3}$</td>
<td>laminar</td>
</tr>
<tr>
<td>Gravel</td>
<td>$2.10^{-3}$ - $63.10^{-3}$</td>
<td>$10^{-3}$ - $10^{-1}$</td>
<td>transition</td>
</tr>
<tr>
<td>Small rock</td>
<td>$63.10^{-3}$ - 0.4</td>
<td>$10^{-1}$ - $5.10^{-1}$</td>
<td>turbulent</td>
</tr>
<tr>
<td>Large rock</td>
<td>0.4 - 1</td>
<td>$5.10^{-1}$ - 1</td>
<td>turbulent</td>
</tr>
</tbody>
</table>
laminar flow

use pressure head instead of pressure

Darcy relations become

$$u_f = -k_x \frac{\partial h}{\partial x} \quad w_f = -k_z \frac{\partial h}{\partial z}$$

continuity equation is

$$\frac{\partial u_f}{\partial x} + \frac{\partial w_f}{\partial z} = 0$$

combining results in Laplace equation

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$
groundwater flow under a caisson

waterpressure

potential lines

streamlines
Flow force

\[ F_f = \rho_w g i = \rho_w g \frac{\partial h}{\partial x} \]

This is the force, caused by the flow, acting on the grains.
Sometimes also called flow pressure.
However, dimension is N/m\(^3\) !!
pressures in case of an impervious bed protection

\[ p_{\text{max}} \approx \frac{L_2}{L_1 + L_2} \rho_w g \Delta h \]

\[ (\rho_m - \rho_w) g d \geq p_{\text{max}} \]
flow net and pressures under an impervious layer on a slope
pressures under impervious slope protection

\[ H = \frac{h_1}{\pi} \arccos \left[ 2 \left( \frac{h_1 + d \cos \alpha}{h_1 + h_2} \right) \frac{\pi}{\arctan(\cot \alpha) + \frac{\pi}{2}} - 1 \right] \]
simple stability of a block

\[ W \sin \alpha \]
\[ f \times W \cos \alpha \]

\[ W \cos \alpha \]

\[ W \]

\[ f \times (W \cos \alpha - H) \]

\[ W \cos \alpha - H \]

\[ W \]

\[ H \]
stability of impervious layer on slope
shear and uplift

above water $\rho_w$ is zero

\[
\begin{align*}
    f \left[ (\rho_m - \rho_w) g d \Delta x \cos \alpha - H \rho_w g \Delta x \right] \\
    \geq (\rho_m - \rho_w) g d \Delta x \sin \alpha
\end{align*}
\]

\[
\frac{H}{\Delta d} = \frac{f \cos \alpha - \sin \alpha}{f}
\]

\[
(\rho_m - \rho_w) g d \Delta x \cos \alpha \geq H \rho_w g \Delta x \quad \rightarrow \quad \frac{H}{\Delta d} = \cos \alpha
\]
heave and piping under a structure

\[ \rho_w g i \approx (1-n)(\rho_g - \rho_w)g \]

\[ h_u - h_t \leq \frac{1}{\gamma d} \frac{\rho_s - \rho_w}{\rho_w} \]
piping behind a sheet piling

Phase 1
Undisturbed situation

Phase 2
Beginning of heave

Phase 3
Progress of pipe formation

Phase 4
Complete piping and collapse
Bligh and Lane

\[ \frac{\Delta h_c}{L} \leq \frac{1}{C_{\text{creep}}} \]

Bligh:  \[ L = a + b + c + d + e \quad \text{for } c > 2b \]
\[ L = a + c + e \quad \text{for } c < 2b \]

Lane:  \[ L = L_v + 1/3L_h = a + b + d + e + c/3 \]
water and grains (static)
water and grains (dynamic)

stress at bottom:
- wet sand: 1*2000*g = 20 kN/m²
- water: 2*1000*g = 20 kN/m²

effective grain stress:
\[ \sigma' = \sigma - p = 20 - 20 = 0 \text{ kN/m}^2 \]

eq. 5.9:
\[ F_f = \rho_w g i = \rho_w g \frac{\partial h}{\partial x} \]
\[ = 1000 * 10 * 1 / 1 = 10 \text{ kN/m}^3 \]

which compensates effective weight of grains under water
\[
\frac{\Delta h_c}{L} \leq 0.87 \alpha c \left( \frac{\rho_g - \rho_w}{\rho_w} \right) (0.68 - 0.10 \ln c)
\]

with:
\[
\alpha = \left( \frac{D}{L} \right) \left( \frac{D}{L} \right)^{-1} = 0.28 \left( \frac{D}{L} \right)^{-2.8}
\]
and:
\[
c = 0.25 d_{70} \left( \frac{g}{\nu kL} \right)^{\frac{1}{3}}
\]
porous flow in a dike
forces on a slope with porous flow

\[
\tan \phi \geq \left[ \frac{\sin \alpha + i \cos(\alpha - \theta)}{\cos \alpha - i \sin(\alpha - \theta)} \right]
\]

for porous flow \((i = 0)\)
this gives:
\[
\varphi \geq \alpha
\]
flow gradients and micro-stability

seepage parallel to the slope

$$\tan \phi \geq \frac{\sin \alpha + \sin \alpha}{\cos \alpha} \quad \rightarrow \quad \tan \phi \geq 2 \tan \alpha$$

seepage perpendicular to the slope

$$\tan \phi \geq \frac{\sin \alpha}{\cos \alpha - i}$$

overall assumption: there is no cohesion
macro stability of slopes
F-values for various slip circles with one centre point

F = 1.0 - 1.2 - 1.6 - 1.9 - 2.1 - 2.3

c=0  
ϕ=37°

10 m

5 m

1:3

Phreatic level

20 m
critical slip circles with bad soil layer
slip circle with high pore pressures

\[ F = 0.32 \]

\[ \begin{align*}
&c = 0 \\
&\phi = 37 \\
&c = 20 \\
&\phi = 37 \\
&c = 0 \\
&\phi = 37 \\
\end{align*} \]

piezometric level in upper layers

piezometric level in lower layer

shear stresses
a “cut-off” slip circle
flow under gate
various options of load reduction at barrier

Piezometric head compared with downstream waterlevel

Percentage of total flow compared with case A