Porous flow, general

Chapter 5

ct4310 Bed, Bank and Shoreline protection

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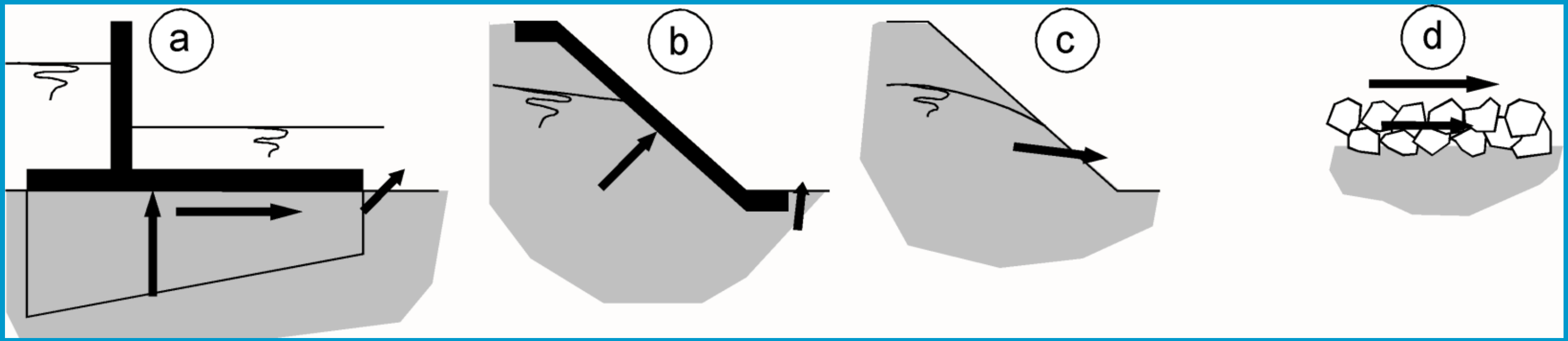
Introduction

- flow through granular medium (sand, pebbles)
- two aspects are relevant:
  - pressure
  - drag
- natural filters and geotextiles

Examples of loads due to porous flow



basic equations

Navier-Stokes

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{\partial^2 u}{\partial z^2}-\frac{\partial\overline{u'^2}}{\partial x}-\frac{\partial\overline{u'w'}}{\partial z}$$

Filter velocity

$$u_f=\frac{1}{A}\iint_A u\,dA=n\cdot u\qquad\left(n=\frac{V_P}{V_T}\right)$$

Combine terms

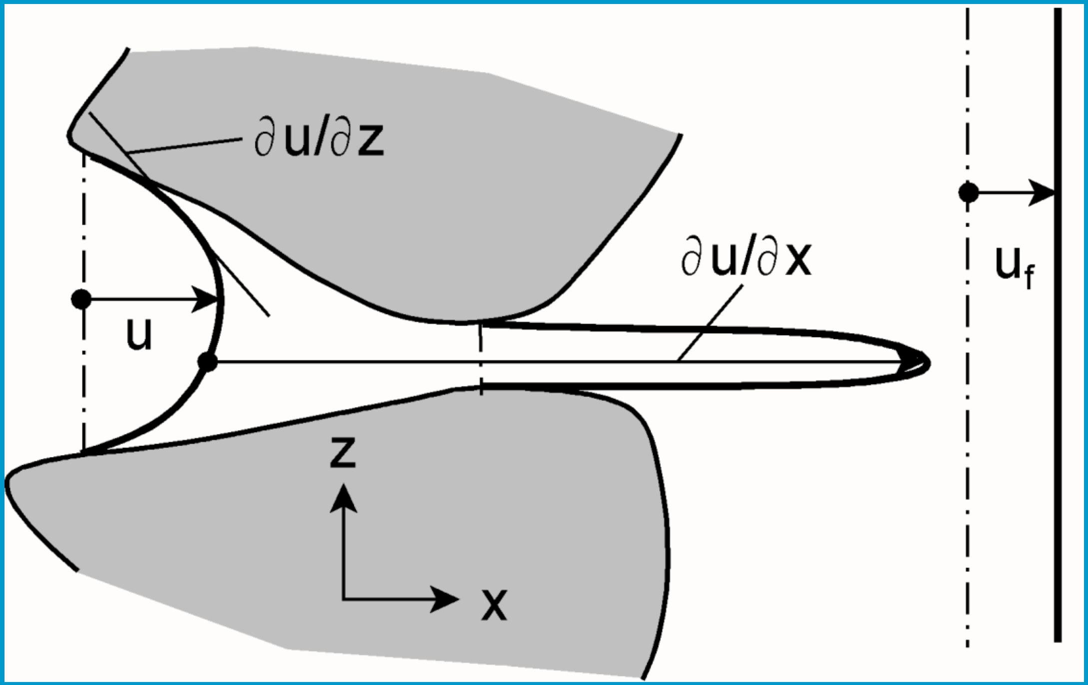
$$\frac{1}{\rho g}\frac{\partial p}{\partial x}=i=a u_f+b u_f\left|u_f\right|+c\frac{\partial u_f}{\partial t}\quad\text{with:}\quad a=\alpha\frac{(1-n)^2}{n^3}\frac{\upsilon}{g\,d_{(n)50}^2}$$

$$b=\beta\frac{(1-n)}{n^3}\frac{1}{g\,d_{(n)50}}$$

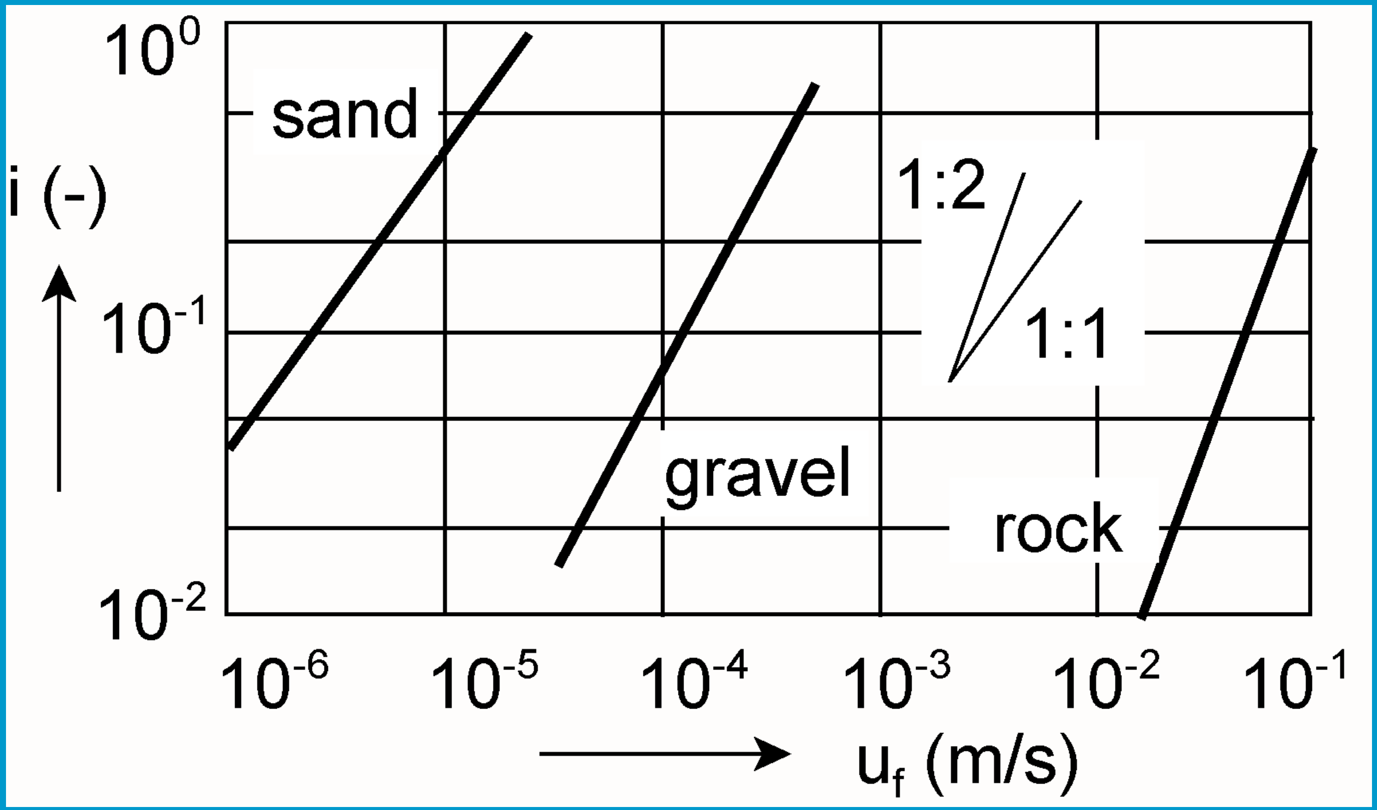
Forchheimer equation

=0 for stationary flow

velocities, gradients and averaging



relation between filter velocity and gradient for various materials



relation between velocity and pressure

$$u_f = k(i)^{\frac{1}{p}}$$

k permeability in m/s of porous material

for p=1 Darcy's law

p=2 Turbulent flow

values of k for various materials

Material	d_{50} (< 63.10^{-3} m) or d_{n50} (m)	Permeability, k (m/s)	Character of flow
Clay	< 2.10^{-6}	10^{-10} - 10^{-8}	laminar
Silt	2.10^{-6} - 63.10^{-6}	10^{-8} - 10^{-6}	laminar
Sand	63.10^{-6} - 2.10^{-3}	10^{-6} - 10^{-3}	laminar
Gravel	2.10^{-3} - 63.10^{-3}	10^{-3} - 10^{-1}	transition
Small rock	63.10^{-3} - 0.4	10^{-1} - 5.10^{-1}	turbulent
Large rock	0.4 - 1	5.10^{-1} - 1	turbulent

laminar flow

use pressure head instead of pressure

$$h = z + \frac{p}{\rho_w g}$$

Darcy relations become

$$u_f = -k_x \frac{\partial h}{\partial x} \qquad w_f = -k_z \frac{\partial h}{\partial z}$$

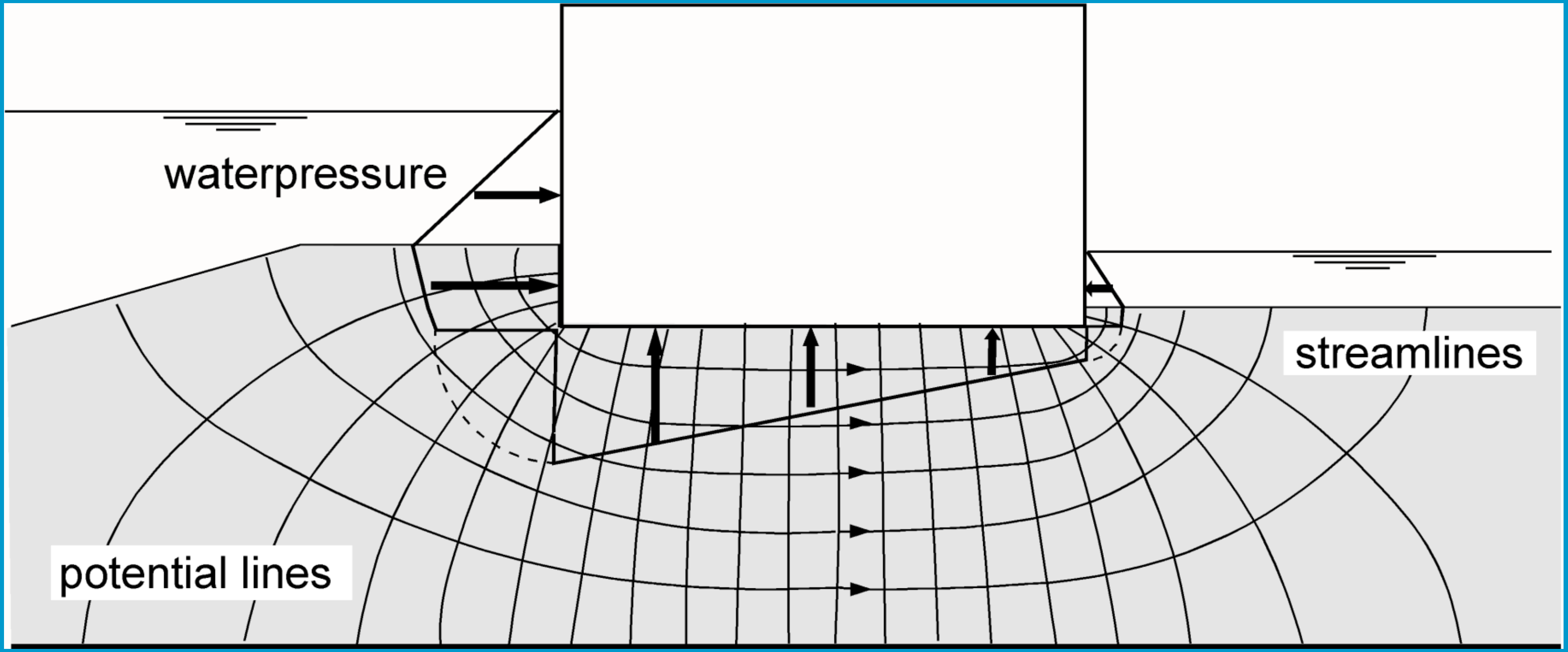
continuity equation is

$$\frac{\partial u_f}{\partial x} + \frac{\partial w_f}{\partial z} = 0$$

combining results in Laplace equation

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

groundwater flow under a caisson

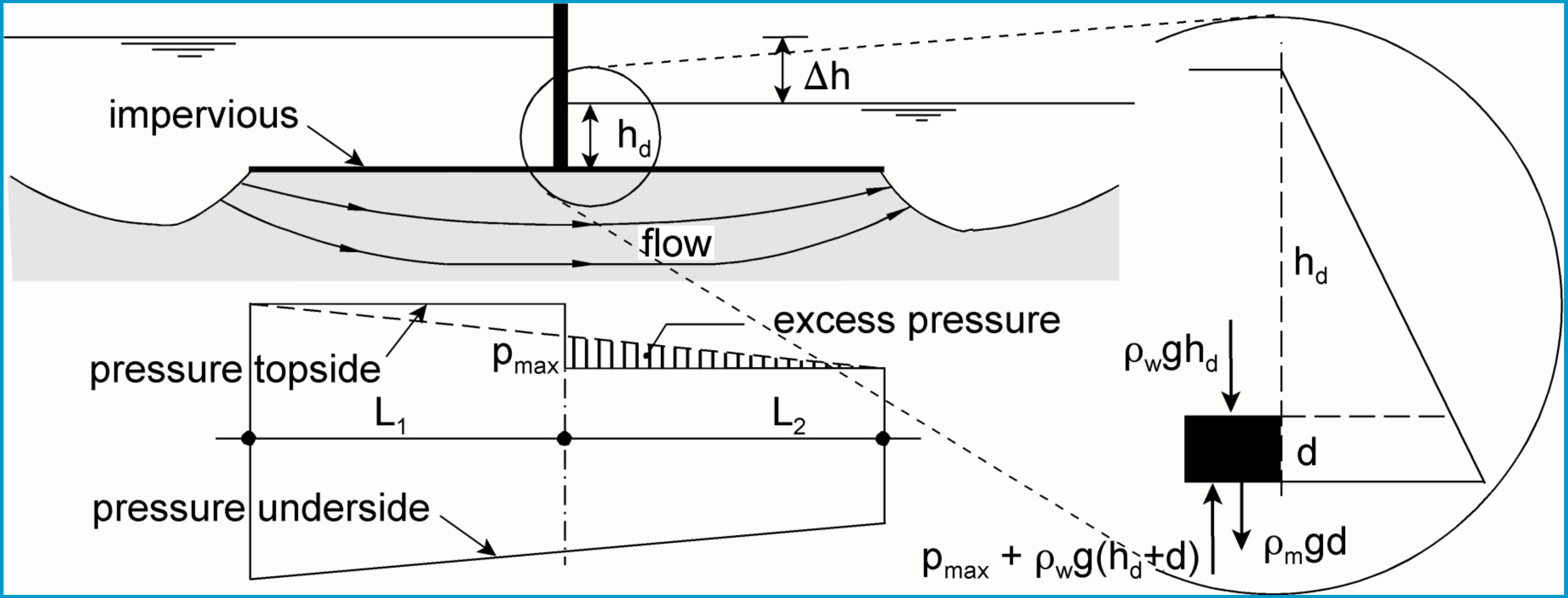


Flow force

$$F_f = \rho_w g i = \rho_w g \frac{\partial h}{\partial x}$$

This is the force, caused by the flow, acting on the grains.
Sometimes also called flow pressure.
However, dimension is N/m^3 !!

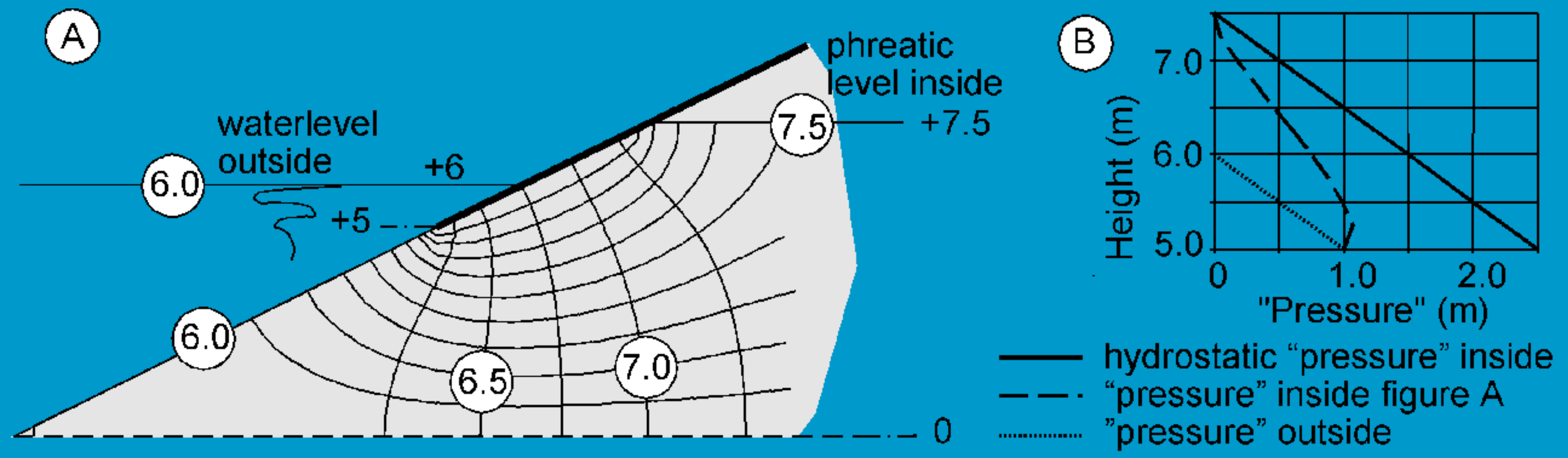
pressures in case of an impervious bed protection



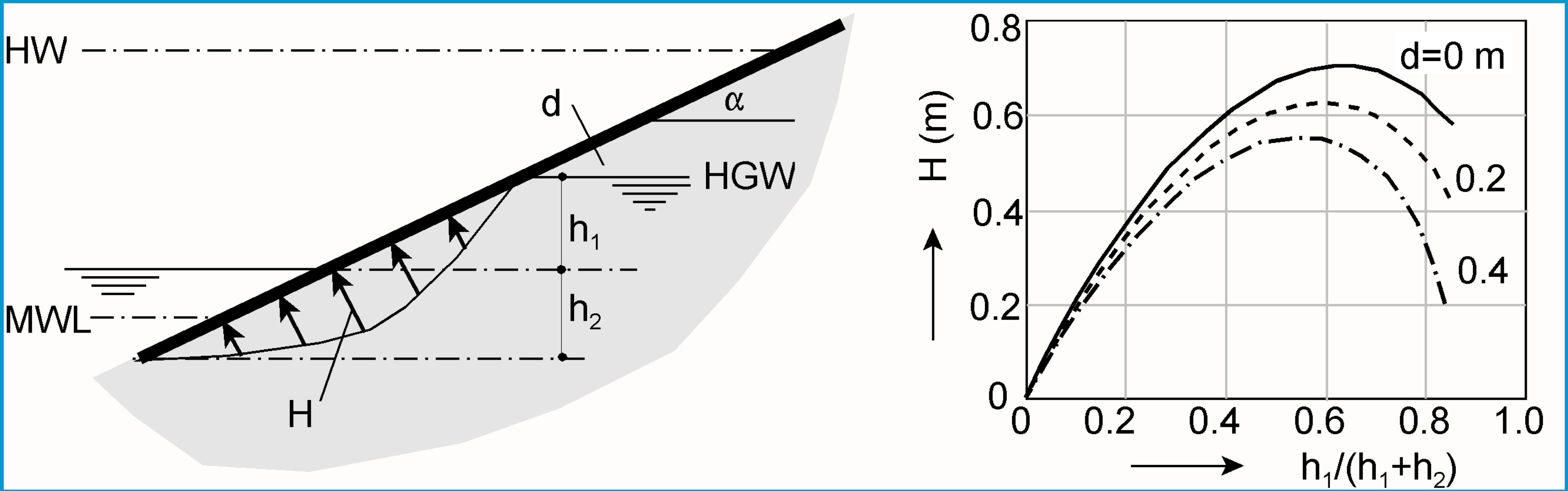
$$p_{\max} \approx \frac{L_2}{L_1 + L_2} \rho_w \, g \, \Delta h$$

$$\left(\rho_m - \rho_w\right) g \, d \geq p_{\max}$$

flow net and pressures under an impervious layer on a slope

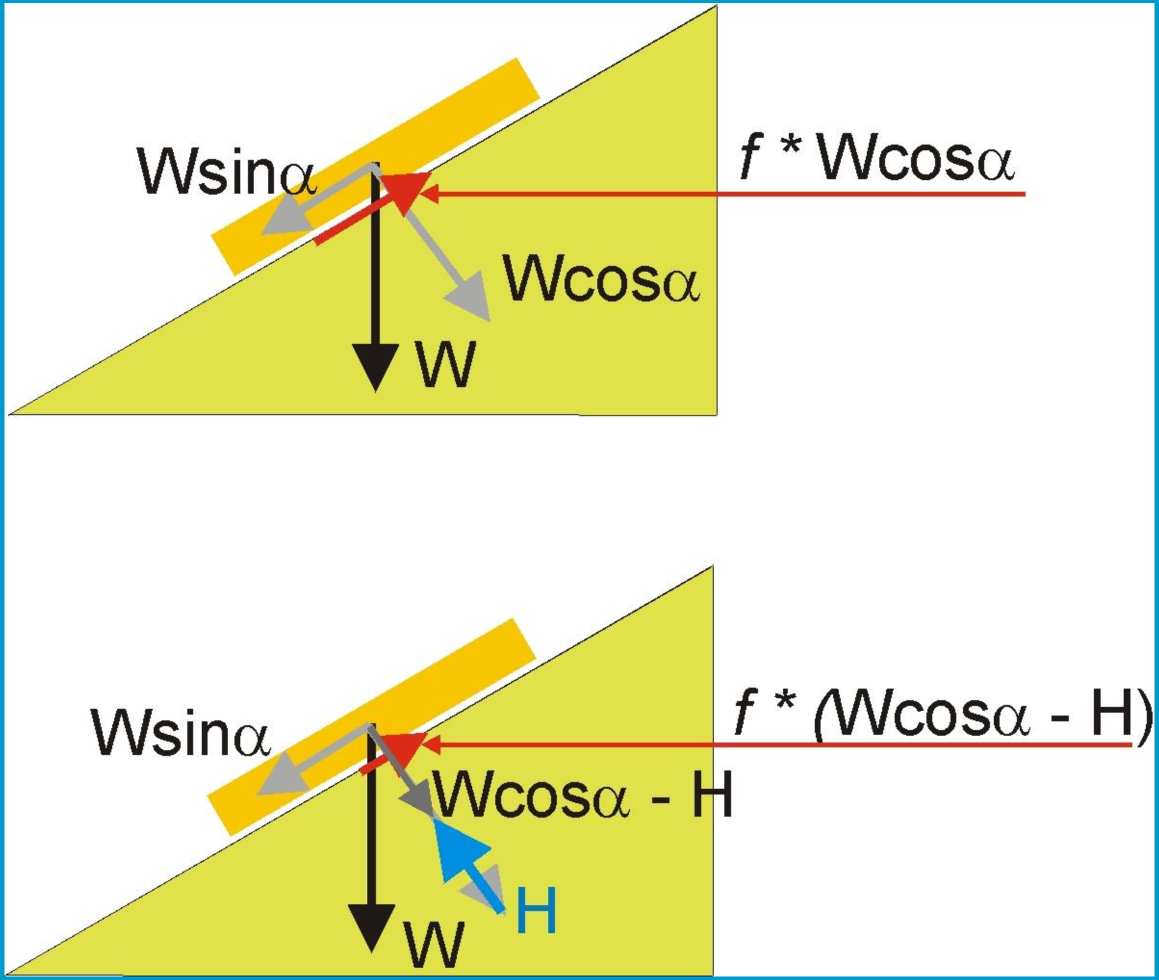


pressures under impervious slope protection

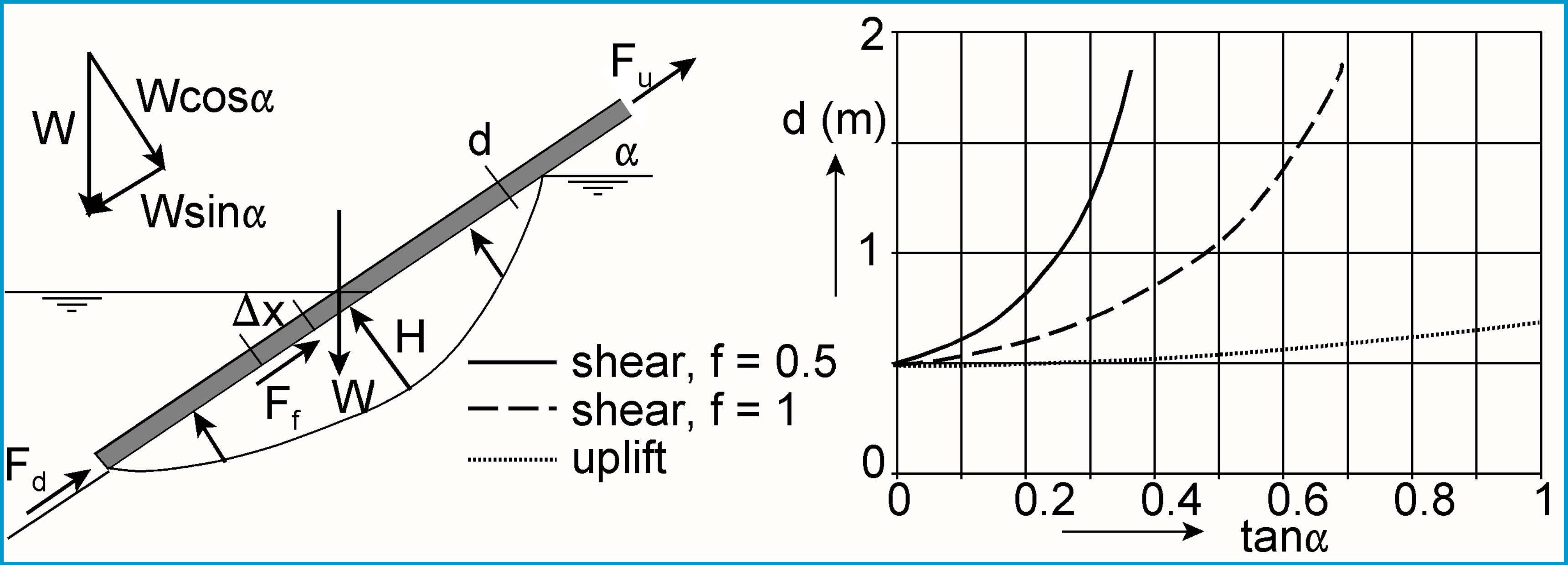


$$H = \frac{h_1}{\pi} \arccos\left[2\left(\frac{h_1 + d\cos\alpha}{h_1 + h_2} \right)^{\frac{\pi}{\arctan(\cot\alpha)+\pi/2}} - 1 \right]$$

simple stability of a block



stability of impervious layer on slope



shear and uplift

above water ρ_w is zero

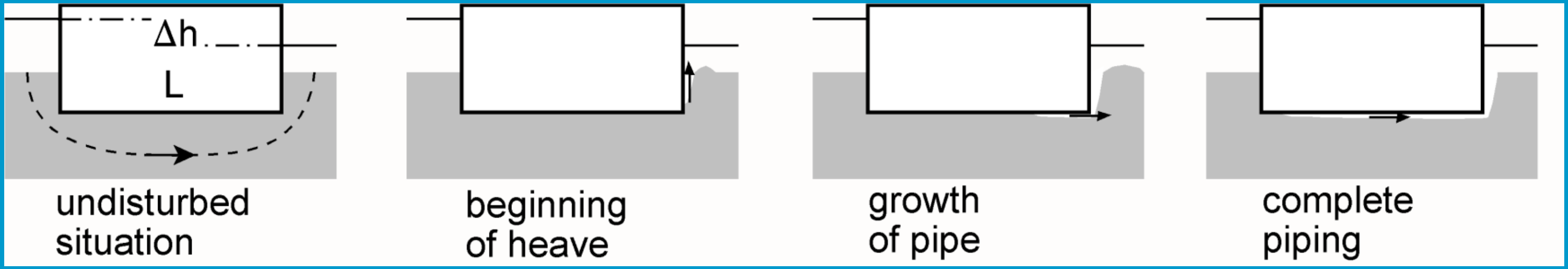
$$f\left[\left(\rho_m-\rho_w\right)g\,d\,\Delta x\cos\alpha - H\,\rho_w g\,\Delta x\right]$$

$$\geq \left(\rho_m-\rho_w\right)g\,d\,\Delta x\sin\alpha$$

$$\frac{H}{\Delta d}=\frac{f\cos\alpha-\sin\alpha}{f}$$

$$\left(\rho_m-\rho_w\right)g\,d\,\Delta x\cos\alpha \geq H\,\rho_w g\Delta x \quad \rightarrow \quad \frac{H}{\Delta d}=\cos\alpha$$

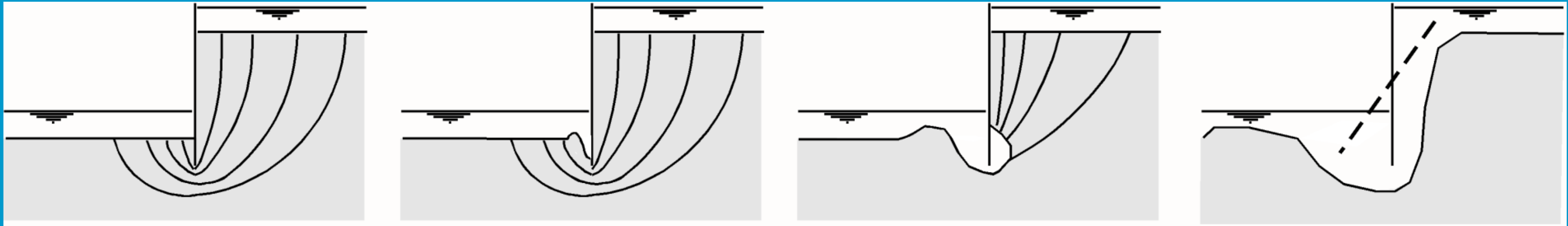
heave and piping under a structure



$$\rho_w \, g \, i \approx (1-n)\left(\rho_g - \rho_w\right) g$$

$$h_u - h_t \leq \frac{1}{\gamma} d \; \frac{\rho_s - \rho_w}{\rho_w}$$

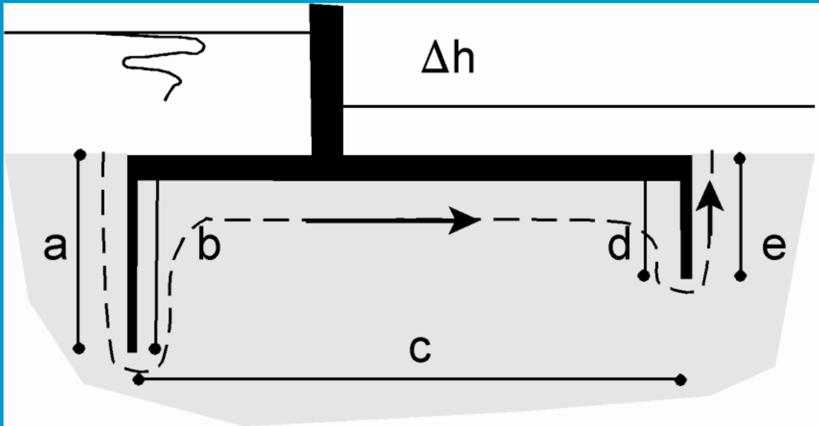
piping behind a sheet piling



Phase 1 Undisturbed situation	Phase 2 Beginning of heave	Phase 3 Progress of pipe formation	Phase 4 Complete piping and collapse

Bligh and Lane

$$\frac{\Delta h_c}{L} \leq \frac{1}{C_{creep}}$$

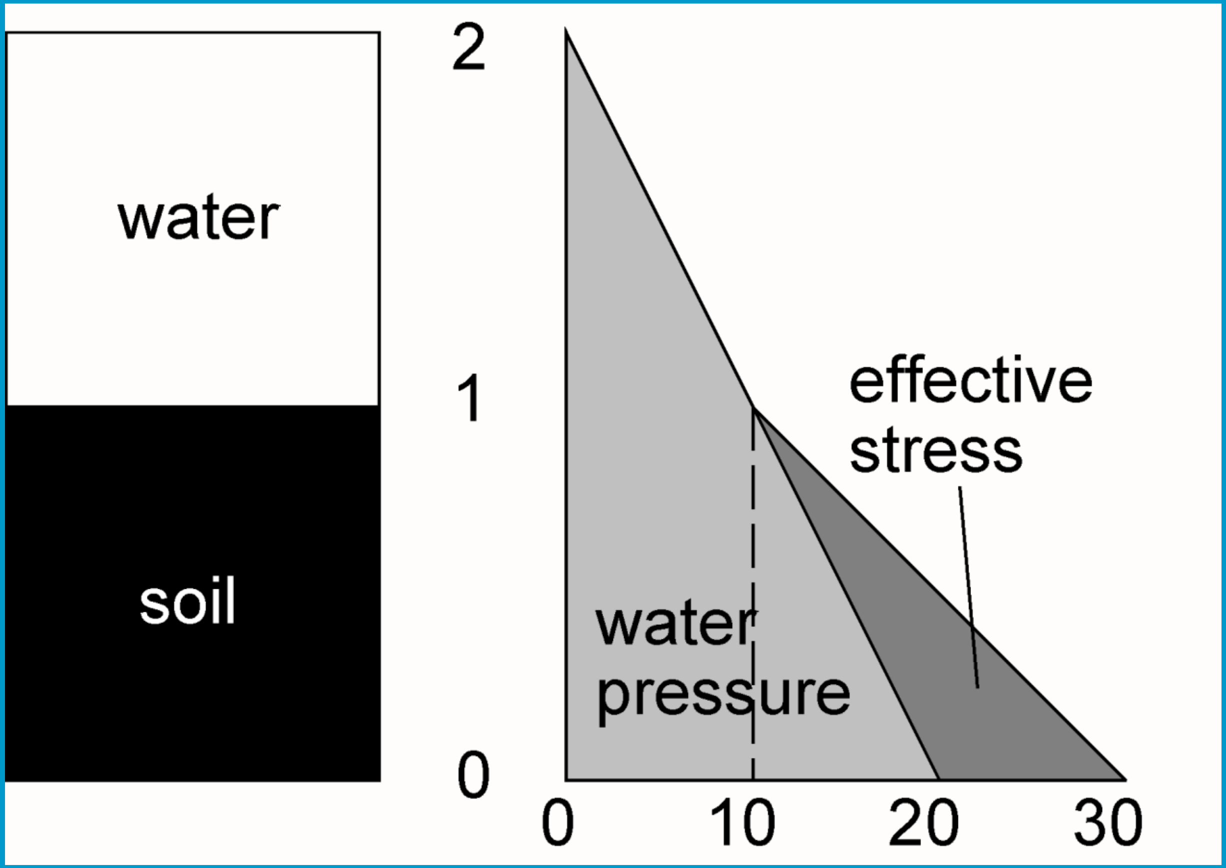


Bligh: $L = a + b + c + d + e$ for $c > 2b$

$L = a + c + e$ for $c < 2b$

Lane: $L = L_v + 1/3L_h = a + b + d + e + c/3$

water and grains (static)



water and grains (dynamic)

stress at bottom:

wet sand: 1*2000*g=20 kN/m^2

water: 2*1000*g = 20 kN/m^2

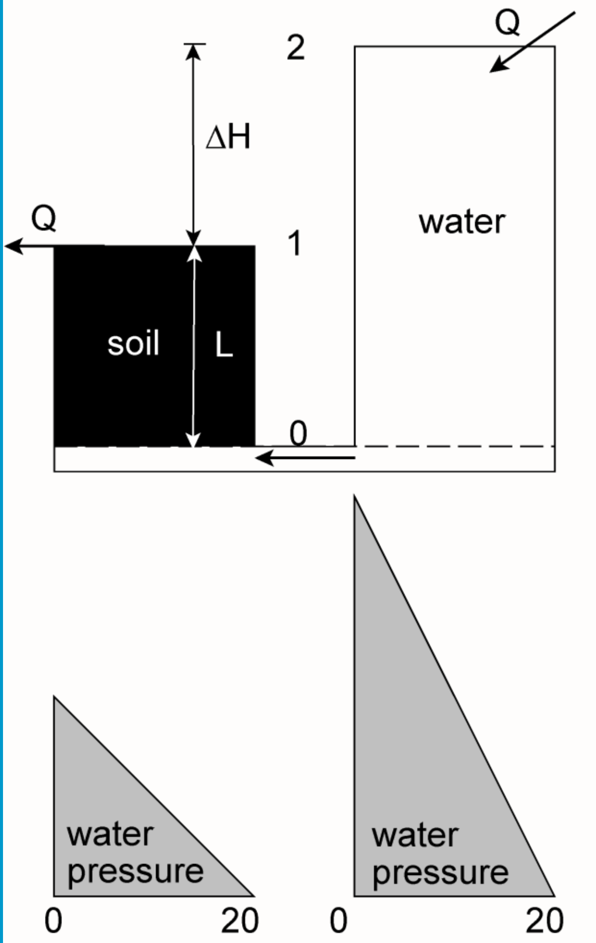
effective grain stress:

$\sigma' = \sigma - p$ = 20-20 = 0 kN/m^2

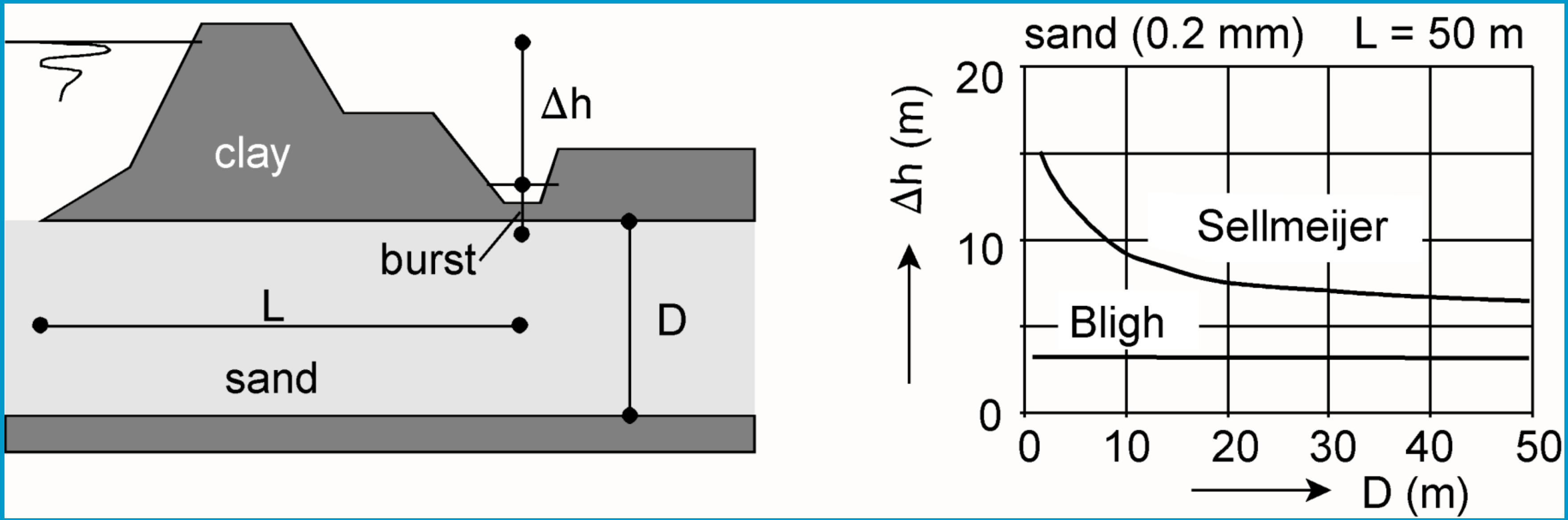
eq. 5.9: $F_f = \rho_w g i = \rho_w g \dfrac{\partial h}{\partial x}$

= 1000*10*1/1 = 10 kN/m^3

which compensates effective weight of grains under water



Sellmeijer



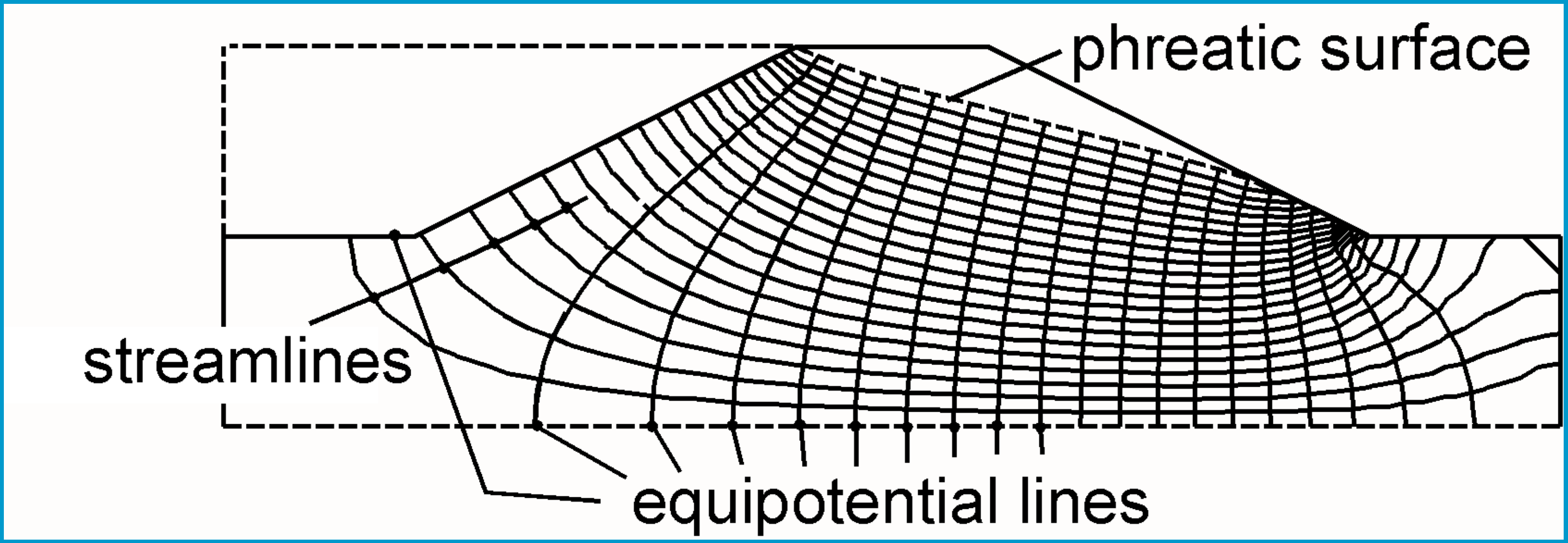
$$\frac{\Delta h_c}{L} \leq 0.87\,\alpha\,c\left(\frac{\rho_g - \rho_w}{\rho_w}\right)(0.68 - 0.10\ln c)$$

$$\text{with:}\quad \alpha = \left(\frac{D}{L}\right)^{\frac{0.28}{\left(\frac{D}{L}\right)^{2.8} - 1}} \quad \text{and:}\quad c = 0.25 d_{70}\left(\frac{g}{\upsilon k L}\right)^{\frac{1}{3}}$$

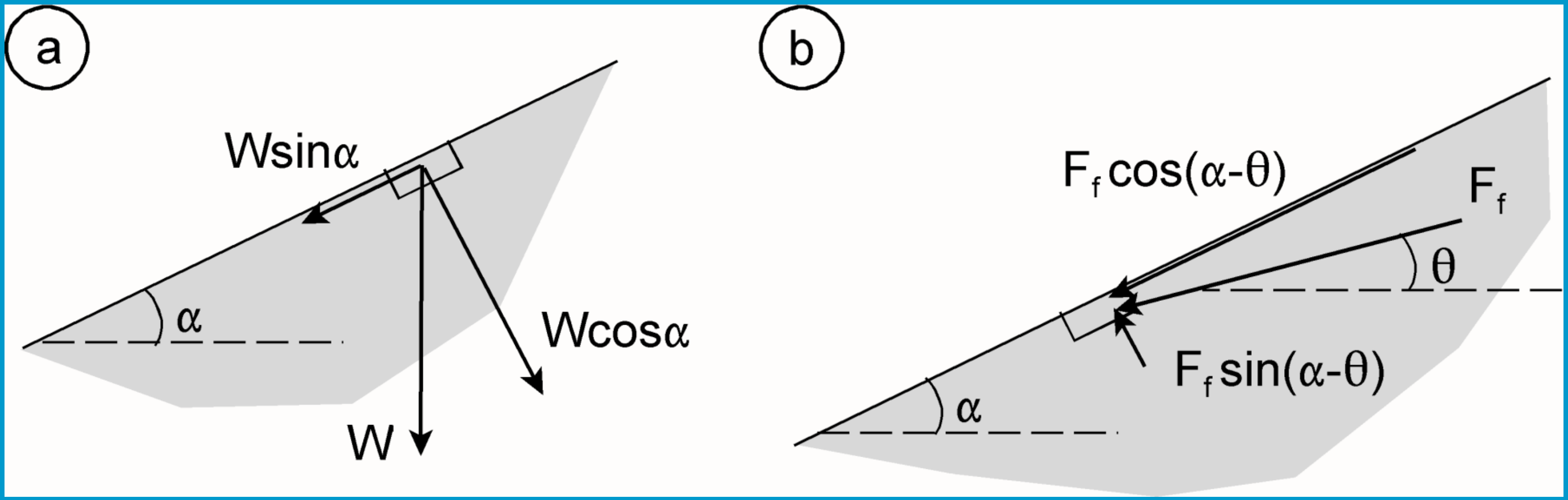
More details in ct5314 -dikes

TU Delft

porous flow in a dike



forces on a slope with porous flow

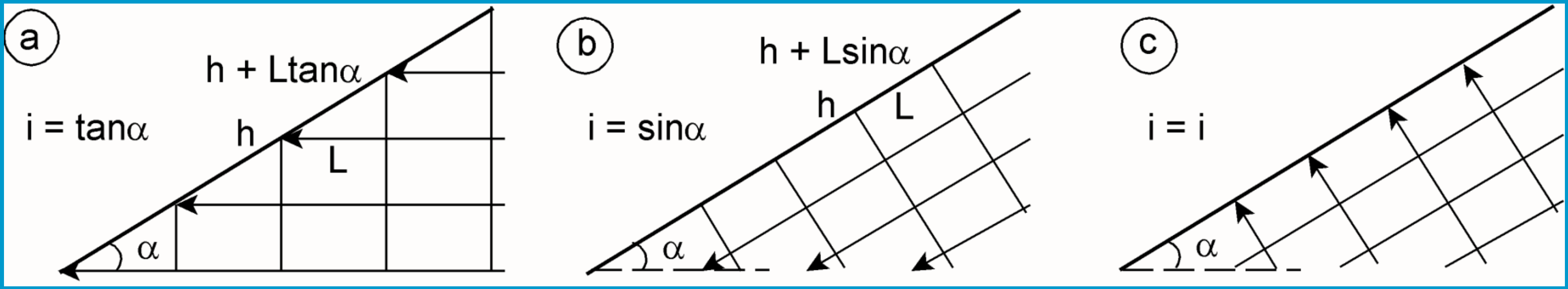


$$\tan\phi \geq \left[\frac{\sin\alpha + i\cos(\alpha-\theta)}{\cos\alpha - i\sin(\alpha-\theta)}\right]$$

for porous flow (*i*=0) this gives:

$$\varphi \geq \alpha$$

flow gradients and micro-stability



seepage parallel to the slope

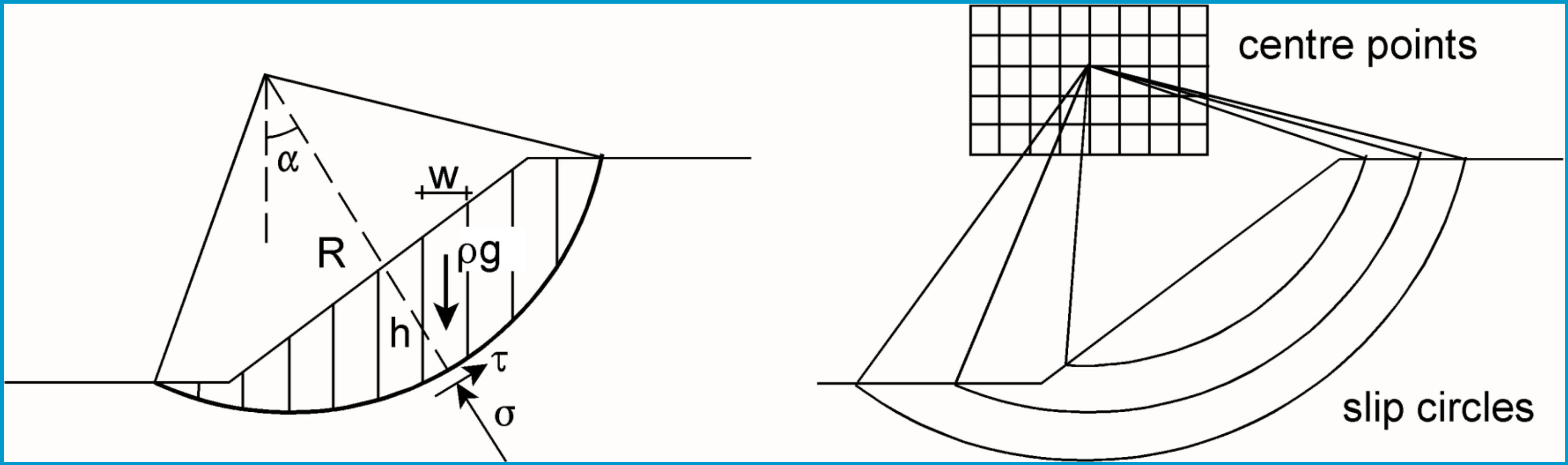
$$\tan\phi \geq \frac{\sin\alpha + \sin\alpha}{\cos\alpha} \quad \rightarrow \quad \tan\phi \geq 2\tan\alpha$$

seepage perpendicular to the slope

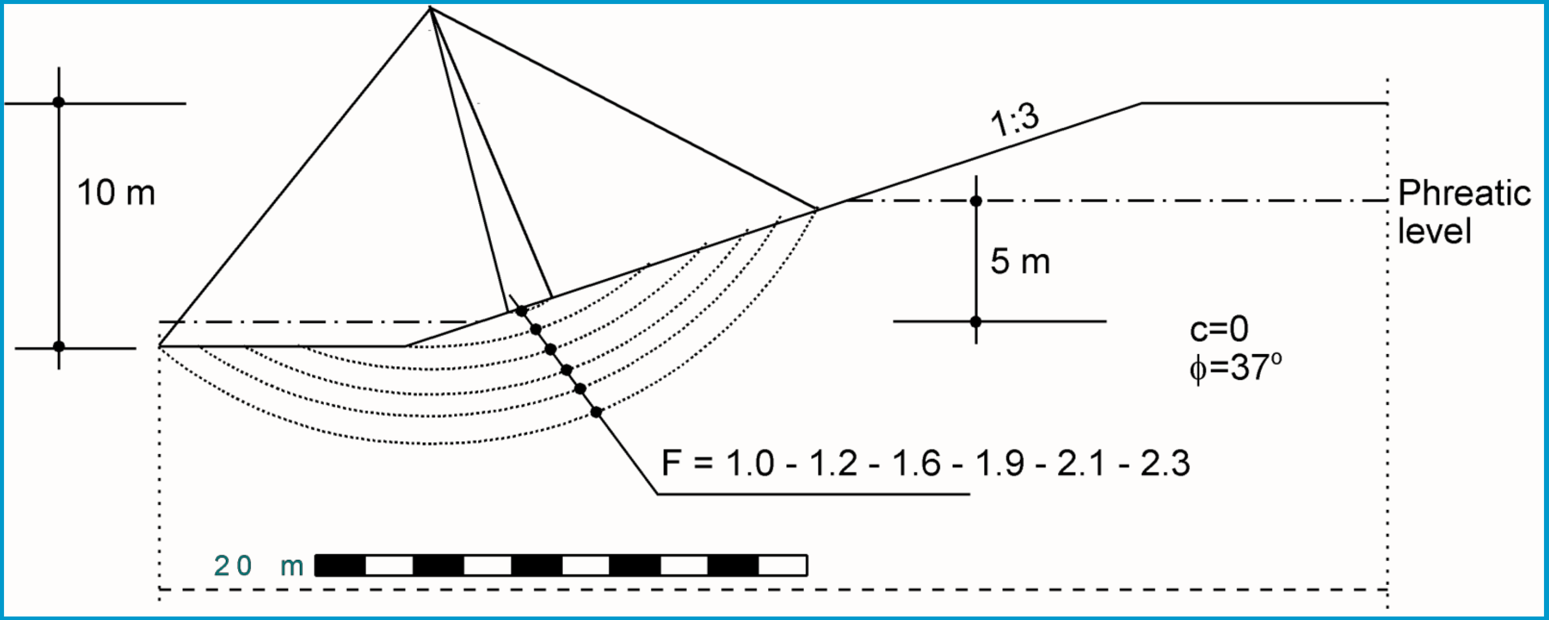
$$\tan\phi \geq \frac{\sin\alpha}{\cos\alpha - i}$$

overall assumption: there is no cohesion

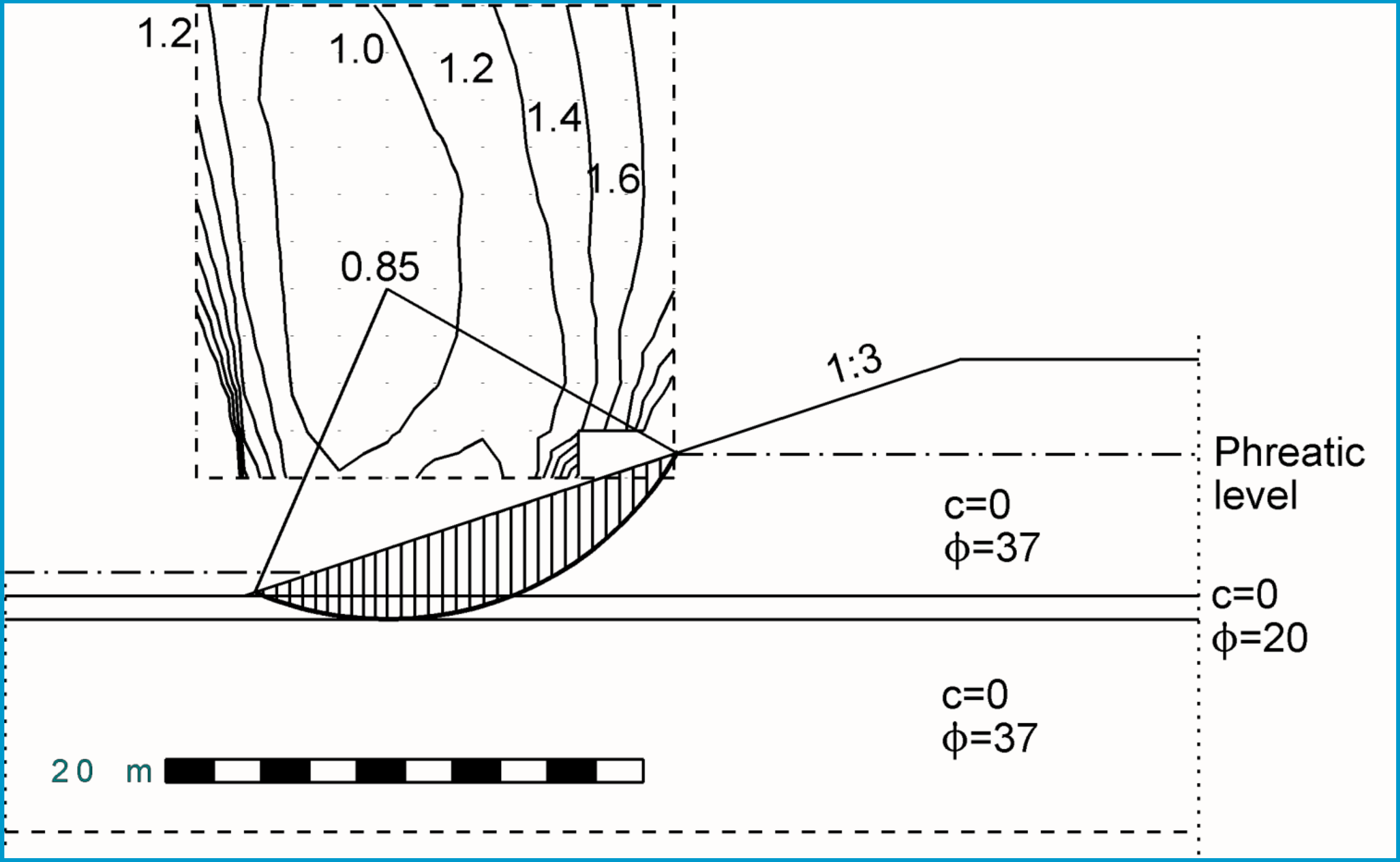
macro stability of slopes



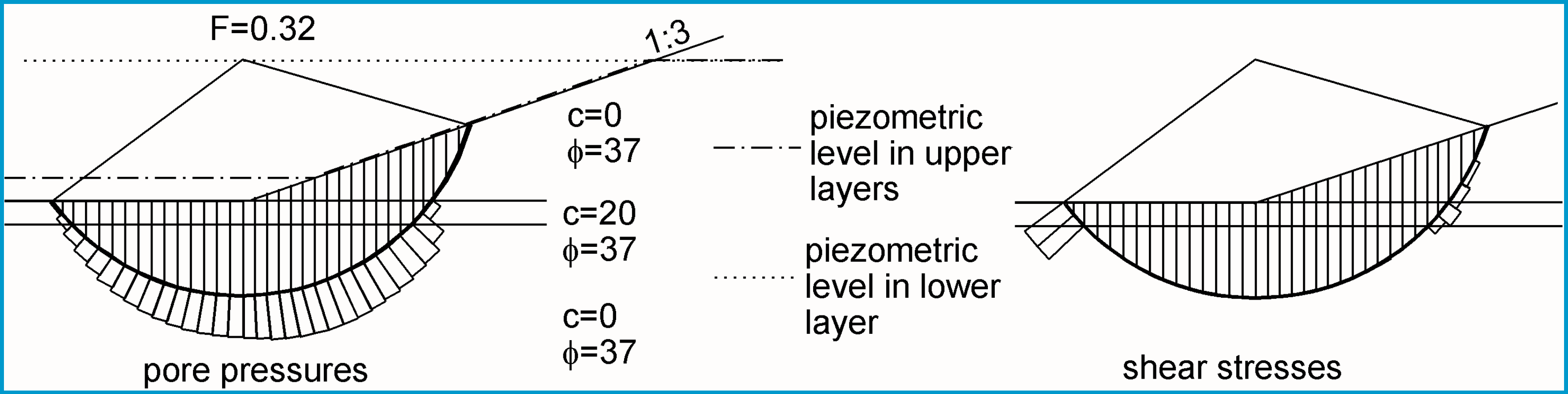
F-values for various slip circles with one centre point



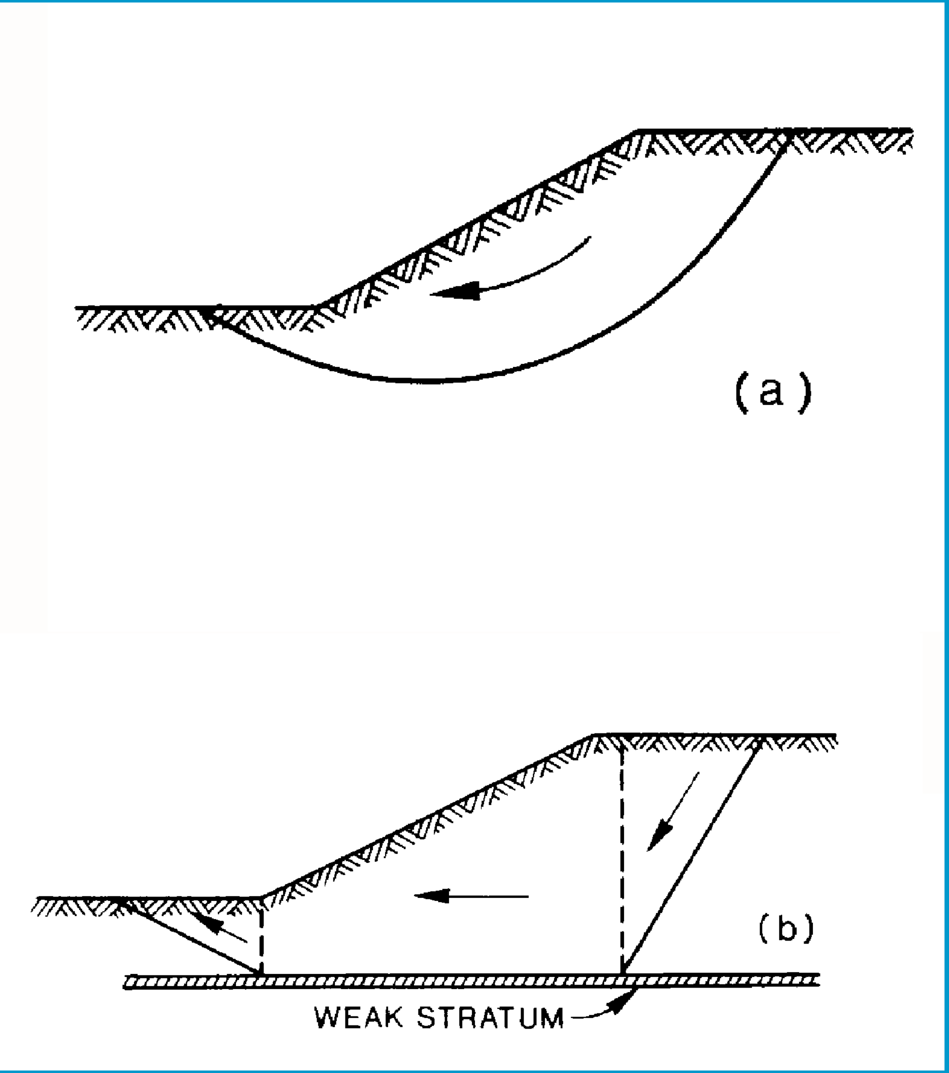
critical slip circles with bad soil layer



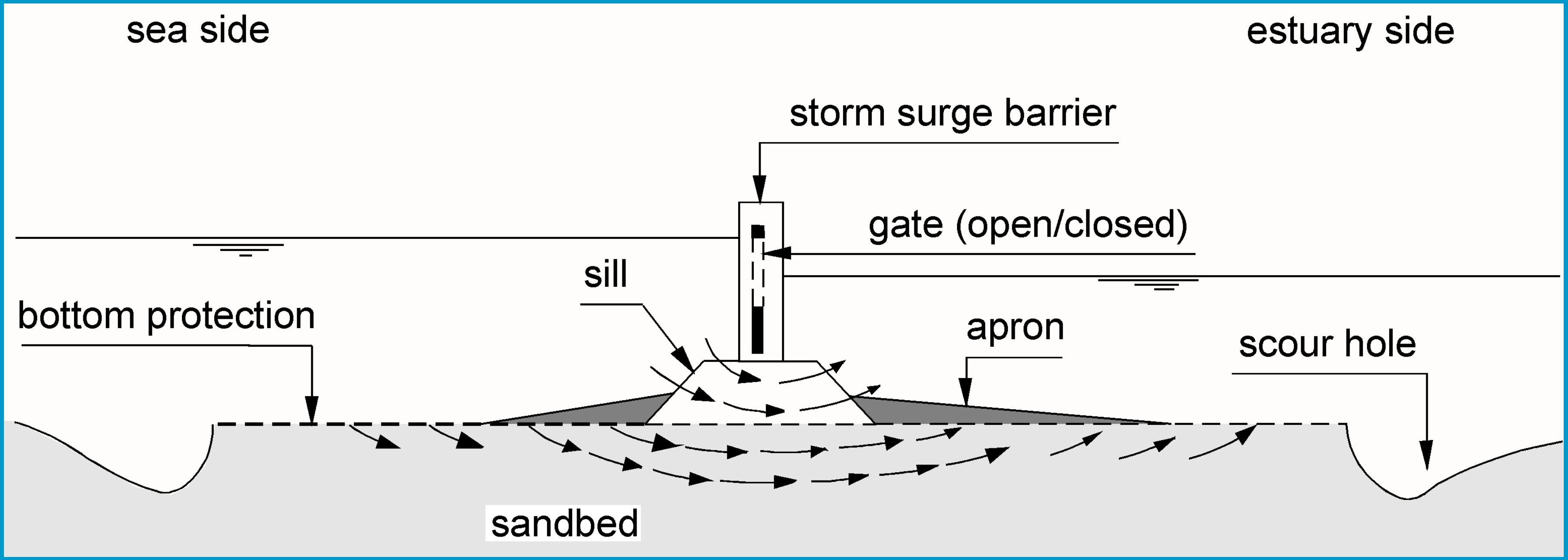
slip circle with high pore pressures



a “cut-off” slip circle



flow under gate



various options of load reduction at barrier