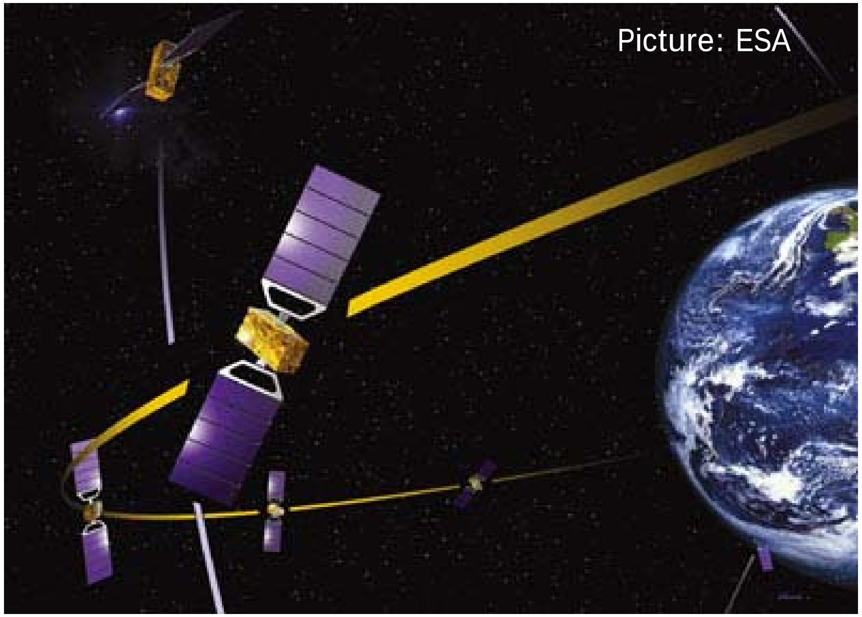
Satellite Navigation
PVT estimation and integrity



AE4E08

Sandra Verhagen

Course 2010 – 2011, lecture 7

Today's topics

- Position, Velocity and Time estimation
- Performance measures
- RAIM → exam material, but not in book!

- Study Sections 6.1, 6.2.1
- Read Sections 6.2.2, 6.3

Recap: Code and Carrier Phase measurements

$$\rho_{Li} = r + \frac{f_1^2}{f_i^2} I_{L1} + T + c\left[\delta t_u - \delta t^s\right] + \varepsilon_{\rho_{Li}}$$

$$\Phi_{Li} = r - \frac{f_1^2}{f_i^2} I_{L1} + T + c\left[\delta t_u - \delta t^s\right] + \lambda A_{Li} + \varepsilon_{\Phi_{Li}}$$

Linearization

$$\mathbf{y} = \rho^{(k)} = \left\| \mathbf{x}^{(k)} - \mathbf{x} \right\| + b + \tilde{\varepsilon}_{\rho}^{(k)}$$

$$\mathbf{y} = \mathbf{H}(\mathbf{v}_0) + \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{v}} \left(\mathbf{v} - \mathbf{v}_0 \right) + \ldots + \boldsymbol{\varepsilon}$$

$$H(\mathbf{v}_0) = \rho_0^{(k)} = \left\| \mathbf{x}^{(k)} - \mathbf{x}_0 \right\| + b_0$$

$$\frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{v}} = \begin{bmatrix} \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial x} \\ \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial y} \\ \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial z} \\ \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial b} \end{bmatrix}^T = \begin{bmatrix} -\frac{x^{(k)} - x_0}{\left\| \mathbf{x}^{(k)} - \mathbf{x}_0 \right\|} \\ -\frac{y^{(k)} - y_0}{\left\| \mathbf{x}^{(k)} - \mathbf{x}_0 \right\|} \\ -\frac{z^{(k)} - z_0}{\left\| \mathbf{x}^{(k)} - \mathbf{x}_0 \right\|} \\ 1 \end{bmatrix}^T = \begin{bmatrix} (-\mathbf{1}^{(k)})^T & 1 \end{bmatrix}$$

Linearized code observation equations

$$\boldsymbol{\delta\rho} = \begin{bmatrix} \delta\rho^{(1)} \\ \delta\rho^{(2)} \\ \vdots \\ \delta\rho^{(K)} \end{bmatrix} = \underbrace{\begin{bmatrix} (-\mathbf{1}^{(1)})^T & 1 \\ (-\mathbf{1}^{(2)})^T & 1 \\ \vdots & \vdots \\ (-\mathbf{1}^{(K)})^T & 1 \end{bmatrix}}_{\boldsymbol{G}} \begin{bmatrix} \delta\mathbf{x} \\ \delta b \end{bmatrix} + \tilde{\boldsymbol{\varepsilon}}_\rho$$

$$\delta\mathbf{x} = \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix}$$

$$\delta\mathbf{b} = b - b_0$$

Least-squares estimation - BLUE

linear model

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \varepsilon; \qquad \mathbf{Q}_{\mathbf{yy}}$$

variance matrix

$$\hat{\mathbf{x}} = \underbrace{\left(\mathbf{A}^T\mathbf{Q}_{\mathbf{yy}}^{-1}\mathbf{A}\right)^{-1}}_{\mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}}}\mathbf{A}^T\mathbf{Q}_{\mathbf{yy}}^{-1}\mathbf{y}$$

$$\mathbf{W} = \mathbf{Q}_{\mathbf{yy}}^{-1}$$

See appendix 6.A

Least-squares estimation

linearized model $\boldsymbol{\delta\rho} = \mathbf{G}\delta\mathbf{v} + \tilde{\boldsymbol{\varepsilon}}; \quad \mathbf{Q}_{\rho\rho}$

iteration required, Gauss-Newton method:

$\mathbf{v}_0 = \ldots$

while $\left\|\delta\hat{\mathbf{v}}\right\|^2_{Q_{\hat{v}\hat{v}}} \geq \eta$

$$\delta\boldsymbol{\rho} = \boldsymbol{\rho} - \mathbf{H}(\mathbf{v}_0)$$

$$\mathbf{G} = \frac{\partial \mathbf{H}(\mathbf{v}_0)}{\partial \mathbf{v}}$$

$$Q_{\hat{v}\hat{v}} = \left(\mathbf{G}^T \mathbf{Q}_{\rho\rho}^{-1} \mathbf{G}\right)^{-1}$$

$$\delta\hat{\mathbf{v}} = Q_{\hat{v}\hat{v}} \cdot \mathbf{G}^T \mathbf{Q}_{\rho\rho}^{-1} \delta\boldsymbol{\rho}$$

$$\hat{\mathbf{v}} = \mathbf{v}_0 + \delta\hat{\mathbf{v}}$$

$$\mathbf{v}_0 = \hat{\mathbf{v}}$$

end

Velocity estimation

- use position estimates from subsequent epochs (*next lectures*)
- use Doppler shift observations → pseudorange rate

$$\dot{\rho}^{(k)} = \dot{r}^{(k)} + (\dot{b} - \dot{b}^{(k)}) + \dot{I}^{(k)} + \dot{T}^{(k)}$$
$$= (\mathbf{v}^{(k)} - \mathbf{v}) \cdot \mathbf{1}^{(k)} + \dot{b} + \varepsilon_{\dot{\varphi}}^{(k)}$$

$\mathbf{v}^{(k)}$	satellite velocity vector (ECEF), known from navigation message
$\mathbf{v}$	user velocity vector (ECEF)

$$\dot{\rho}^{(k)} - \mathbf{v}^{(k)} \cdot \mathbf{1}^{(k)} = -\mathbf{1}^{(k)} \cdot \mathbf{v} + \dot{b} + \varepsilon_{\dot{\varphi}}^{(k)}$$

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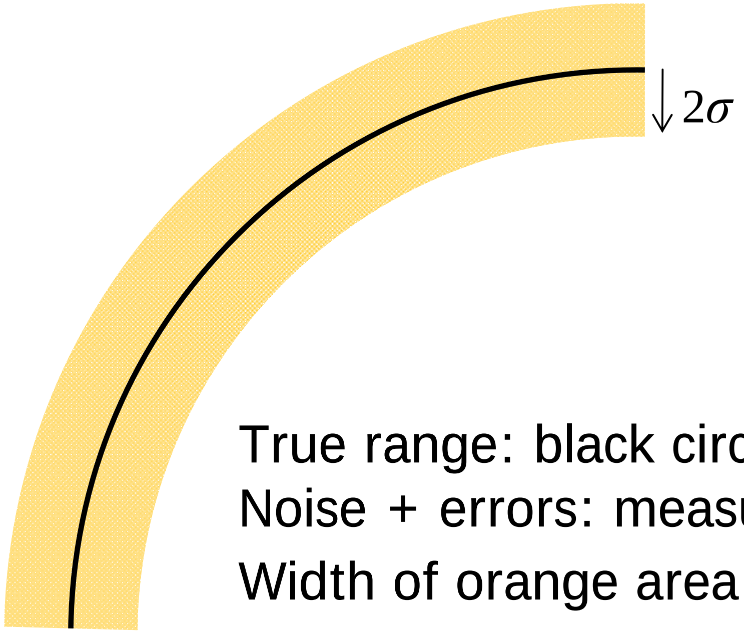
Satellite geometry (2D)



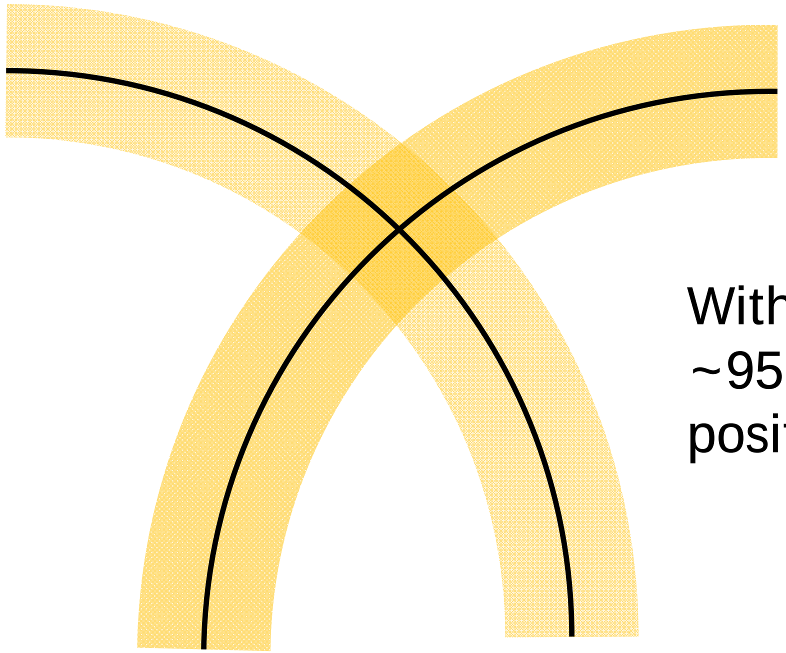
True range: black circle

Noise + errors: measured range is in orange area

Width of orange area depends on σ^2 (URE)

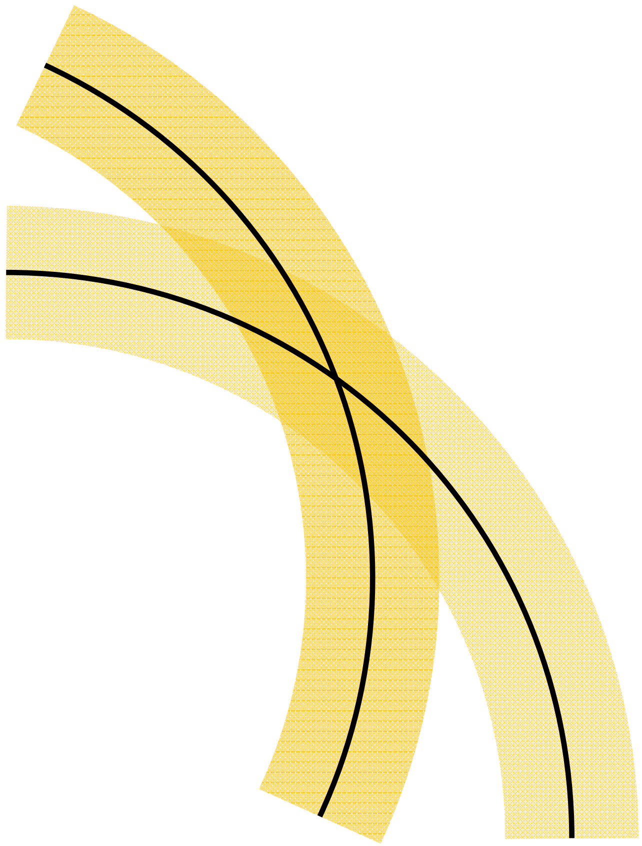
$$P(|\rho - \rho_{true}| \leq 2\sigma) = 95.4\%$$

Satellite geometry (2D)



With 2 observations:
~95% probability that measured position will be in dark orange area

Satellite geometry



With 2 observations:
~95% probability that measured position will be in dark orange area

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Satellite geometry

With 2 observations:
~95% probability that measured position will be in dark orange area

Satellite geometry

With 3 observations:
~95% probability that measured position will be in dark orange area

Satellite geometry

$$\begin{pmatrix} \delta\hat{\mathbf{x}} \\ \delta\hat{b} \end{pmatrix} = \underbrace{\left(G^{\mathrm{T}}\mathbf{Q}_{\rho\rho}^{-1}G\right)^{-1}}_{\mathbf{Q}_{\hat{\mathbf{v}}\hat{\mathbf{v}}}} G^{\mathrm{T}}\mathbf{Q}_{\rho\rho}^{-1}\delta\mathbf{p}$$

If: $\mathbf{W}^{-1} = \mathbf{Q}_{\rho\rho} = \sigma^2 I_K$ ⇒

$$\mathbf{Q}_{\hat{\mathbf{v}}\hat{\mathbf{v}}} = \sigma^2 (G^T G)^{-1} = \sigma^2 H = \begin{pmatrix} \sigma_x^2 & & \mathrm{cov} & \\ & \sigma_y^2 & & \\ & & \sigma_z^2 & \\ & \mathrm{cov} & & \sigma_b^2 \end{pmatrix}$$

RMS position error = $\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} = \sigma \underbrace{\sqrt{H_{11} + H_{22} + H_{33}}}$

influence of satellite geometry on precision
= Position Dilution of Precision (PDOP)

Satellite geometry

$$\mathbf{Q}_{\hat{\mathbf{v}}\hat{\mathbf{v}}} = \sigma^2 (G^T G)^{-1} = \sigma^2 H = \begin{pmatrix} \sigma_x^2 & & \text{cov} & \\ & \sigma_y^2 & & \\ & & \sigma_z^2 & \\ & \text{cov} & & \sigma_b^2 \end{pmatrix}$$

RMS position error = $\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} = \sigma \sqrt{H_{11} + H_{22} + H_{33}} = \sigma \cdot \text{PDOP}$

RMS clock bias error = $\sqrt{\sigma_b^2} = \sigma \sqrt{H_{44}} = \sigma \cdot \text{TDOP}$

RMS overall error =

$$\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 + \sigma_b^2} = \sigma \sqrt{H_{11} + H_{22} + H_{33} + H_{44}} = \sigma \cdot \text{GDOP}$$

Performance measures

Accuracy:

error in meters + associated statistical value

E.g.: difference true horizontal position and the radius of a circle which encloses 95% of the position estimates indicated by the GNSS receiver if measurement would be repeated large number of times, i.e. there is a 95% probability that the measured position will be inside the circle centered at the true position

Performance measures

Accuracy:

- DOP : Dilution of Precision ≠ precision
  geometrical strength (depends on satellite geometry)
  the lower, the better
- URE : User Range Error
  depends on satellite ephemeris and clock errors, ionosphere and troposphere, multipath, noise, and other error sources
- HPL/VPL : Horizontal/Vertical Protection Level
  horizontal/vertical position is assured to be within region defined by HPL/VPL
- RMS error : *see previous slide*
  = formal precision

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Performance measures

Integrity:

ability of a system to provide timely warnings to users when the system should not be used

- alarm limit: position error that should result in an alarm being raised
- Time To Alarm (TTA): time between occurrence of integrity event (position error too large) and alarm being raised

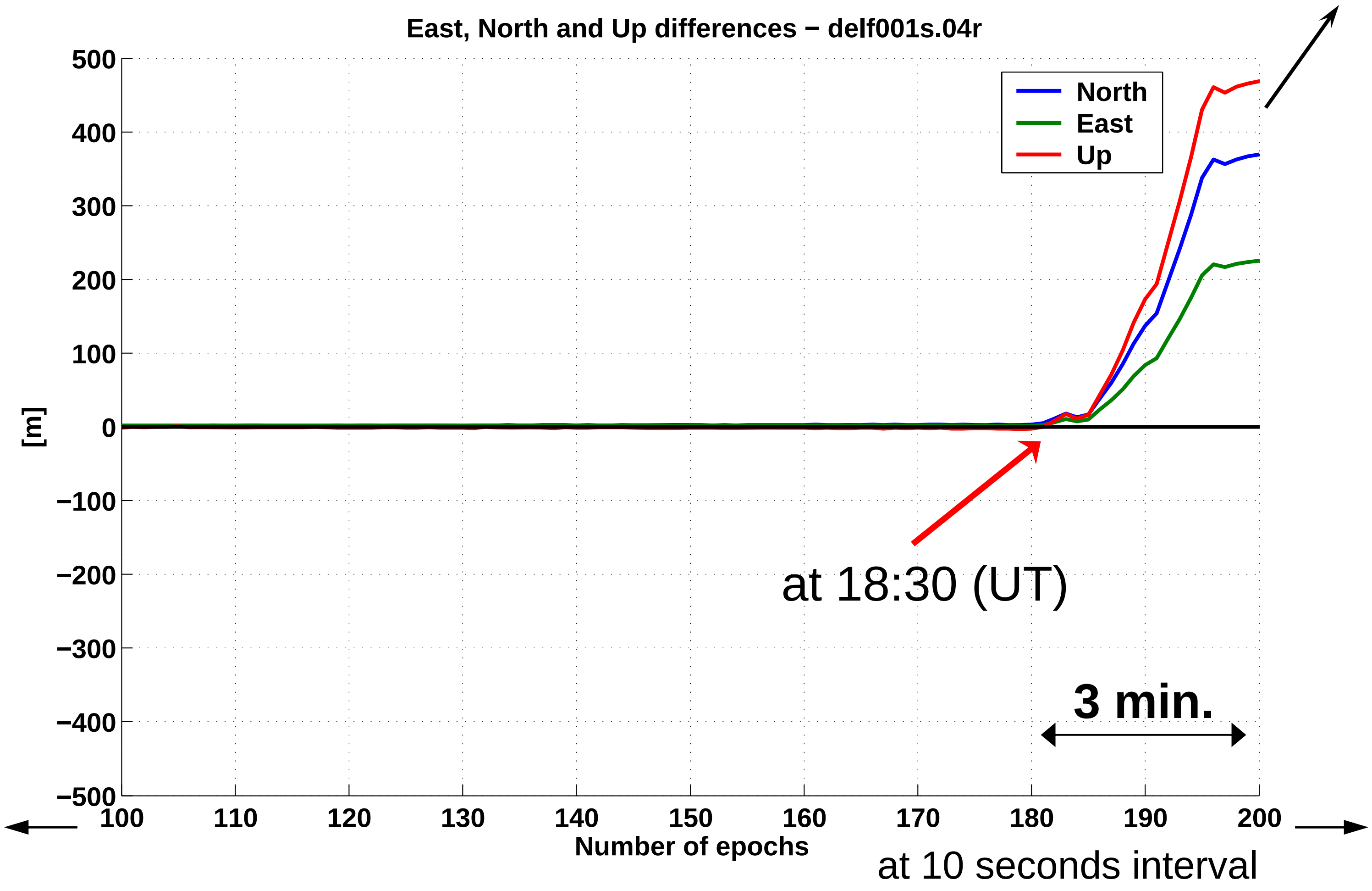
Continuity:

probability that a system will provide the specified level of accuracy and integrity throughout an operation of specified period (assuming accuracy and integrity requirements are met at the start of the operation)

Availability:

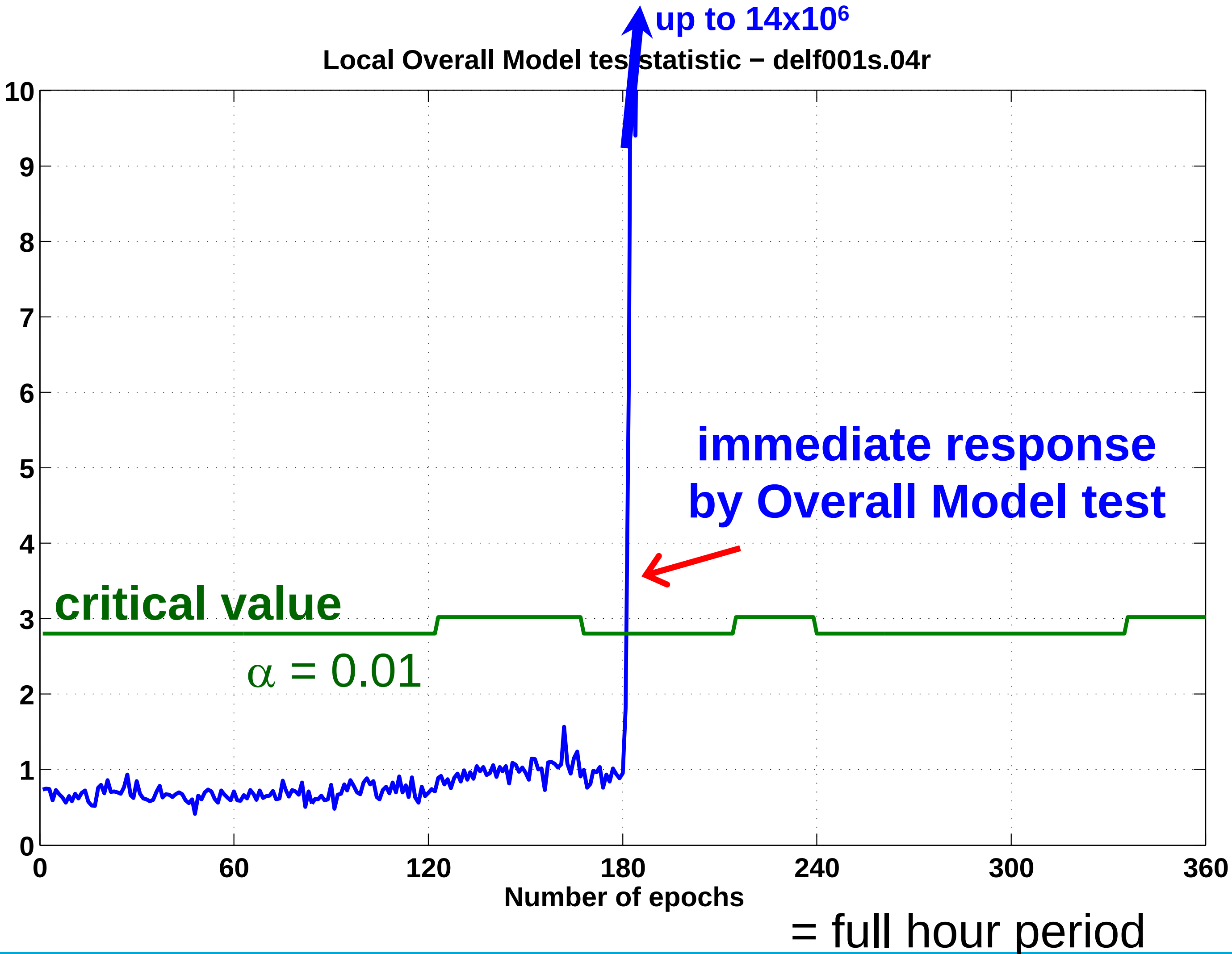
percentage of time that system is operating satisfactory, i.e. required accuracy, integrity and continuity can be provided

GPS PRN 23 Anomaly, 1 Jan, 2004



Not noticed by US for 3 hours
Picked up by EGNOS
Alternative: check @receiver

Receiver Autonomous Integrity monitoring (RAIM)



RAIM - Overall model test

RAIM: detect and correct for errors in GPS data @receiver

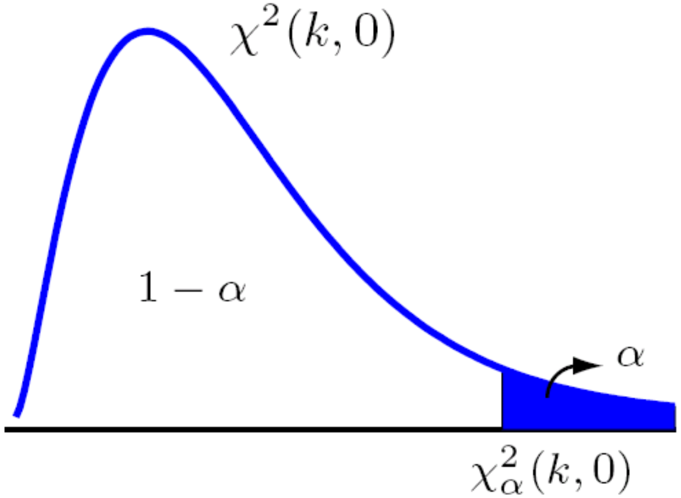
Overall model test: does H_o provide good model?

$$\hat{e}_o = y - A\hat{x}_o$$

$$\underline{T}_{q=m-n} = \underline{\hat{e}}_o^T Q_{yy}^{-1} \underline{\hat{e}}_o$$

$$H_o : \ \underline{T}_q \sim \chi^2(m-n, 0)$$

$$H_a : \ \underline{y} \in \mathbb{R}^m$$



Design computations

internal reliability

size of model error that can just be detected by using the appropriate test statistics:

Minimal **D**etectable **B**ias (**MDB**)

$$H_0 : E\{y\} = Ax; \qquad D\{y\} = Q_{yy}$$

$$H_a : E\{y\} = Ax + c\nabla; \qquad D\{y\} = Q_{yy}$$

$$MDB = |\nabla| = \sqrt{\frac{\lambda_0}{c^T Q_y^{-1} P_A^{\perp} c}}$$

c : specifies type of model error: e.g. cycle slip or code outlier

Design computations

external reliability

impact of undetected bias, $c\nabla$, on the unknown parameters:

Minimal **D**etectable **E**ffect (**MDE**)

$$\nabla\hat{x} = (A^T Q_{yy}^{-1} A)^{-1} A^T Q_{yy}^{-1} c\nabla$$

if baseline unknowns parameterized as $b = (N \quad E \quad U)^T$

it is also possible to use **M**inimal **D**etectable **P**osition **E**ffect (**MDPE**):

$$MDPE = \sqrt{\nabla\hat{N}^2 + \nabla\hat{E}^2 + \nabla\hat{U}^2}$$

Summary and outlook

- GPS system overview, signals, receivers
- GPS measurements and error sources
- PVT estimation

Next:

Positioning in dynamic environments (*Christian Tiberius*)

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