Example: Flatwise compression of sandwich with Gr pins for core

- applied compression load
- facesheet
- pins
- undeformed
- deformed
- $F_a$
- $S$
- $M$
- $\theta$
Individual pin under flatwise load

After some algebra:

\[ M = \frac{\ell}{2} (F_a \cos \theta - S \sin \theta) \]

\[ \beta = \sqrt{\frac{P}{E_p I_p}} \]

\[ M_1 = -\frac{\ell}{2} (F_a \cos \theta - S \sin \theta) \sin \beta \ell \frac{\sin \beta \ell}{\sin \beta \ell - \beta \ell(1 + \cos \beta \ell)} \]

\[ S_1 = \beta \frac{\ell}{2} (F_a \cos \theta - S \sin \theta) \frac{1 + \cos \beta \ell}{\sin \beta \ell - \beta \ell(1 + \cos \beta \ell)} \]

\[ S_2 = \frac{2 \sin \beta \ell - \beta \ell(1 + \cos \beta \ell)}{2(\sin \beta \ell - \beta \ell(1 + \cos \beta \ell))} (-F_a \cos \theta + S \sin \theta) \]

etc…
Stiffener crippling

- Perhaps the most common stiffener failure and one of the most common failure modes in a fuselage
Fuselage failure modes by %

- **Crippling**: 25%
- **Panel buckling**: 16%
- **Bearing**: 12%
- **OHT**: 15%
- **OHC**: 24%
- **Misc.**: 8%

**Legend**:
- CAI
- TAI
- SAI
- OH
- OHT
- Bearing
- Panel buckling
- Misc.
Stiffener crippling

- in a good design, stiffeners do not fail by column buckling but by flange crippling

- if column buckling is an issue, the effective beam length is decreased and/or the bending stiffness of the stiffener is increased until flange crippling becomes the primary failure mode

**Reason**: in column buckling the entire stiffener is “gone”. In stiffener crippling, local flange failure occurs which may be confined in one flange, without immediate failure of entire stiffener, to absorb enough of a crash load to protect passengers
Stiffener crippling

- stiffener crippling occurs when one or more flanges buckle in a **local** buckling mode with wavelength unrelated to the length of the beam

**Explanatory Formula**

\[
\text{half-wave length} = f(b, t, D_{ij})
\]

- \(b\) = flange width
- \(t\) = flange thickness
- \(D_{ij}\) = bending stiffnesses of flange
Stiffener crippling

- ideally, one first obtains the individual loads on each flange
- then computes short-wavelength buckling load of the flange of interest,
- then goes through a post-buckling solution similar to the one for plate under compression
- determine load for final failure
- very cumbersome (plus modelling issues with radius regions)
Stiffener crippling

• Distinguish two cases:
  – One-Edge-Free (OEF)
  – No-Edge-Free (NEF)

If an edge is not supported or stabilized by another member of the cross-section (or via other means) it is free.
For OEF, the buckling problem has been solved before as the case of a very long plate under compression with three sides simply supported and one side free.
OEF crippling - predictions

• buckling load was found to be

\[ N_{xcrit} = \frac{12D_{66}}{b^2} \]

exact expression

\[ N_{xcrit} = \frac{4\pi^2}{b^2} \lambda^2 D_{66} + \frac{2\pi^2}{b^2} D_{12} , \lambda=5/12 \]

our approx. expression

• therefore, to maximize the crippling load of a flange of given width b, one must maximize D_{66} for the flange.

what does that imply for the flange layup? and how does this implication match our requirement for high I for the entire cross-section?
OEF Crippling: Buckling versus Final Failure; Comparisons with test results

Flange width to thickness ratio, b/t

crippling strength/ultimate compr strength

Predictions
- exact buckling
- approx. buckling

Test results from Mil-Hdbk 17-3F, vol 3, ch 5, Fig 5.7.2.h, Jun 17, 2002
OEF crippling: test vs theory

• irrespective of layup, test results follow a single curve:

\[
\frac{\sigma_{crip}}{\sigma^u_c} = \frac{2.151}{\left( \frac{b}{t} \right)^{0.717}}
\]

• predictions are a function of layup

\[
\frac{\sigma_{crip}}{\sigma^u_c} = \frac{1.63}{\left( \frac{b}{t} \right)^{0.717}}
\]

preliminary design curve (valid for at least 25%0, 25%45) b/t>2.9

• approx. predictions are close to exact theoretical predictions
OEF crippling design curve

![Graph showing OEF crippling design curve with flange width to thickness ratio on the x-axis and crippling strength/compression ultimate strength on the y-axis. The best fit design curve is indicated.]
• predictions are very conservative especially for high b/t values (recall plates have high post-buckling ability)

• for low b/t values, effect of radii and transverse shear effects (not included in the predictions) may be important
This is the case of a long plate under compression, simply-supported all around which was solved before as part of the biaxially loaded plate.
NEF crippling - predictions

• buckling load was found to be

\[ N_o = \frac{\pi^2}{2} \left( D_{11}m^4 + 2(D_{12} + 2D_{66})m^2n^2(AR)^2 + D_{22}n^4(AR)^4 \right) \]

\[ \frac{a^2(m^2 + kn^2(AR)^2)}{a^2(m^2 + kn^2(AR)^2)} \]

• which for k=0 and n=1 can be rearranged to:

\[ N_{xcrit} = \frac{\pi^2}{b^2} \sqrt{D_{11}D_{22}} \left[ \frac{m^2b^2}{a^2} \left( \frac{D_{22}}{D_{11}} \right) + \frac{2(D_{12} + 2D_{66})}{\sqrt{D_{11}D_{22}}} + \frac{\sqrt{D_{22}}}{\sqrt{D_{11}}} \frac{a^2}{b^2m^2} \right] \]

• and for a very long plate,

\[ N_{xcrit} = \frac{2\pi^2}{b^2} \left[ \sqrt{D_{11}D_{22}} + (D_{12} + 2D_{66}) \right] \]

≈1 for long plate (hint: find m that minimizes Nxcrit)

it still “pays” to have ±45 plies away from the mid-plane as in the OEF condition
NEF Crippling: Buckling versus Final Failure; Comparisons with test results

Test results from Mil-Hdbk 17-3F, vol 3, ch 5, Fig 5.7.2.f, Jun 17, 2002
NEF crippling: test vs theory

• irrespective of layup, test results follow a single curve:

\[
\frac{\sigma_{\text{crip}}}{\sigma_u} = \frac{14.92}{\left(\frac{b}{t}\right)^{1.124}}
\]

• predictions are a function of layup

• predictions are very conservative especially for high b/t values (recall plates have high post-buckling ability)

• for low b/t values, effect of radii and transv shear effects (not included in the predictions) may be important

\[
\frac{\sigma_{\text{crip}}}{\sigma_u} = \frac{11.0}{\left(\frac{b}{t}\right)^{1.124}}
\]

preliminary design curve

b/t\geq8.5
NEF crippling design curve

Ratio of Flange Width to thickness (b/t)
Stiffener crippling – Other considerations

- closed section stiffeners:
  - hollow: analysis as before
  - filled (with foam for example): see section on sandwich
Stiffener crippling – Other considerations

- stiffeners under bending moment loads => flanges in bending
Crippling of flanges in bending

- Determine the normal stress distribution on the flange of interest
Crippling of flanges in bending

- determine the portion that is under compression and find the extreme stresses $\sigma_{cmax}$ $\sigma_{cmin}$
Crippling of flanges in bending

• determine average compr. stress and analyze as a
flange in compression

\[ \sigma_{\text{cmin}} \]

\[ \sigma_{\text{cmax}} \]

\[ \frac{\sigma_{\text{min}} + \sigma_{\text{max}}}{2} \]

\[ \frac{\sigma_{\text{min}} + \sigma_{\text{max}}}{2} \]
Crippling: importance of radius regions

without special provisions, this region fills with resin
=> weaker
Crippling: importance of radius regions

- significant improvement is obtained by filling the radius region with stiff material (uni-directional tape for example)

\[
A_f = 2 \left[ R_i + \frac{t}{2} \right]^2 \left( 1 - \frac{\pi}{4} \right)
\]

\[
F_f = \frac{E_f A_f}{\sum E_j A_j} F_{TOT}
\]

Filler of area \( A_f \) and stiffness \( E_f \)
Crippling: Effect of filler material in radius regions

- $E_f$ ranges from 3GPa (pure resin) to 138 GPA (0 degree tape)
- $R_i$ ranges from 2.5mm-6.35mm

$E_1 = 89.6$GPa
$E_2 = 31.0$GPa
$E_3 = 48.3$GPa
Crippling: Effect of filler stiffness and radius

- Filler material is all resin
- Filler material is UD tape

\[ Ri = \text{filler stiffness (Pa)} \]

\[ \text{Filler force/Total force} \]

- \( Ri = 6.35 \text{ mm} \)
- \( 4.8 \text{ mm} \)
- \( 3.2 \text{ mm} \)
- \( 2.5 \text{ mm} \)