State space models

AE4-E08 Satellite Navigation

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State vector description

Dynamic systems

first order vector differential equation (state equation)

- general form (possibly non-linear, and dependent on time t)

$$\dot{x}(t) = f(x(t), z(t), t)$$

state-vector input

- linear time-varying

$$\dot{x}(t) = F(t)\,x(t) + G(t)z(t)$$

- linear time-invariant

$$\dot{x}(t) = Fx(t) + Gz(t)$$

Linear time-invariant state equation

$$\dot{x}(t) = Fx(t) + Gz(t)$$

with $\dot{\Phi}(t,t_0) = F\Phi(t,t_0)$

$$\Phi(t,t_0) = e^{F(t-t_0)} \qquad \text{where} \qquad e^{Ft} = \sum_{i=0}^{\infty} \frac{F^i t^i}{i!}$$ (matrix exponential)

full solution

$$x(t) = \underbrace{e^{F(t-t_0)}}_{\Phi(t,t_0)} x(t_0) + \int_{t_0}^{t} \underbrace{e^{F(t-\tau)}}_{\Phi(t,\tau)} Gz(\tau)\,d\tau$$