AE4536: Buckling of structures

Discrete Nonlinear Precritical Equilibrium with Snap-Through

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Problem definition

$$a_0 = \frac{L/2}{\cos\varphi}$$

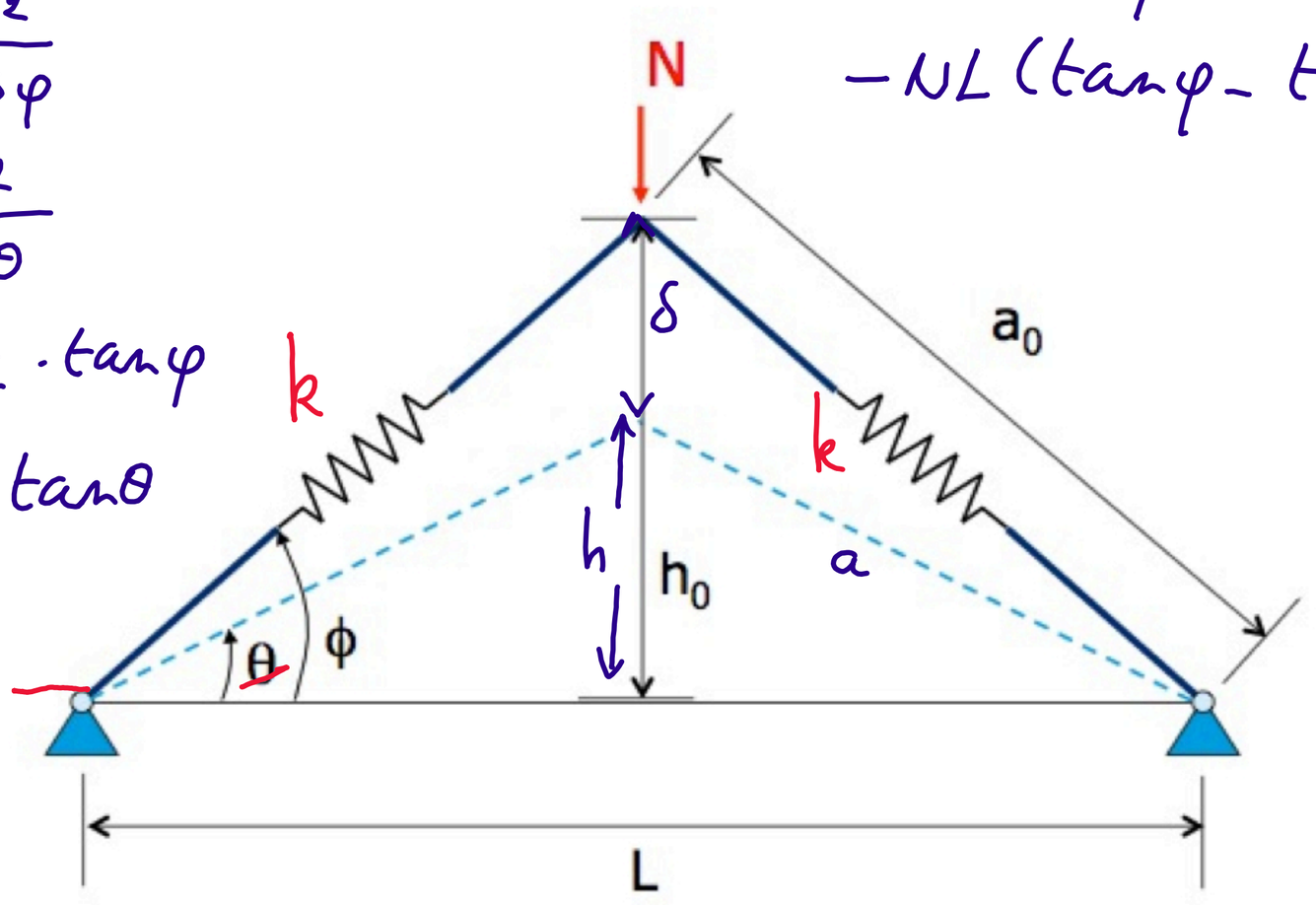
$$a = \frac{L/2}{\cos\theta}$$

$$h_0 = L/2 \cdot \tan\varphi$$

$$h = L/2 \cdot \tan\theta$$

$$P(\theta) = \frac{1}{2}k(a_0 - a)^2 \cdot 2 - N(h_0 - h)$$

$$= k\frac{L^2}{4}\left(\frac{1}{\cos\varphi} - \frac{1}{\cos\theta}\right)^2 - NL(\tan\varphi - \tan\theta)$$



Equilibrium

$$P(\theta_0 + \underbrace{\delta\theta}_{\varepsilon\theta_1}) = \underbrace{\left.\frac{dP}{d\theta}\right|_{\theta_0}}_{=0} \delta\theta + \frac{1}{2!}\left.\frac{d^2P}{d\theta^2}\right|_{\theta_0}^{0} \delta\theta^2 + h.o.t.^{0} = 0 = \delta P$$

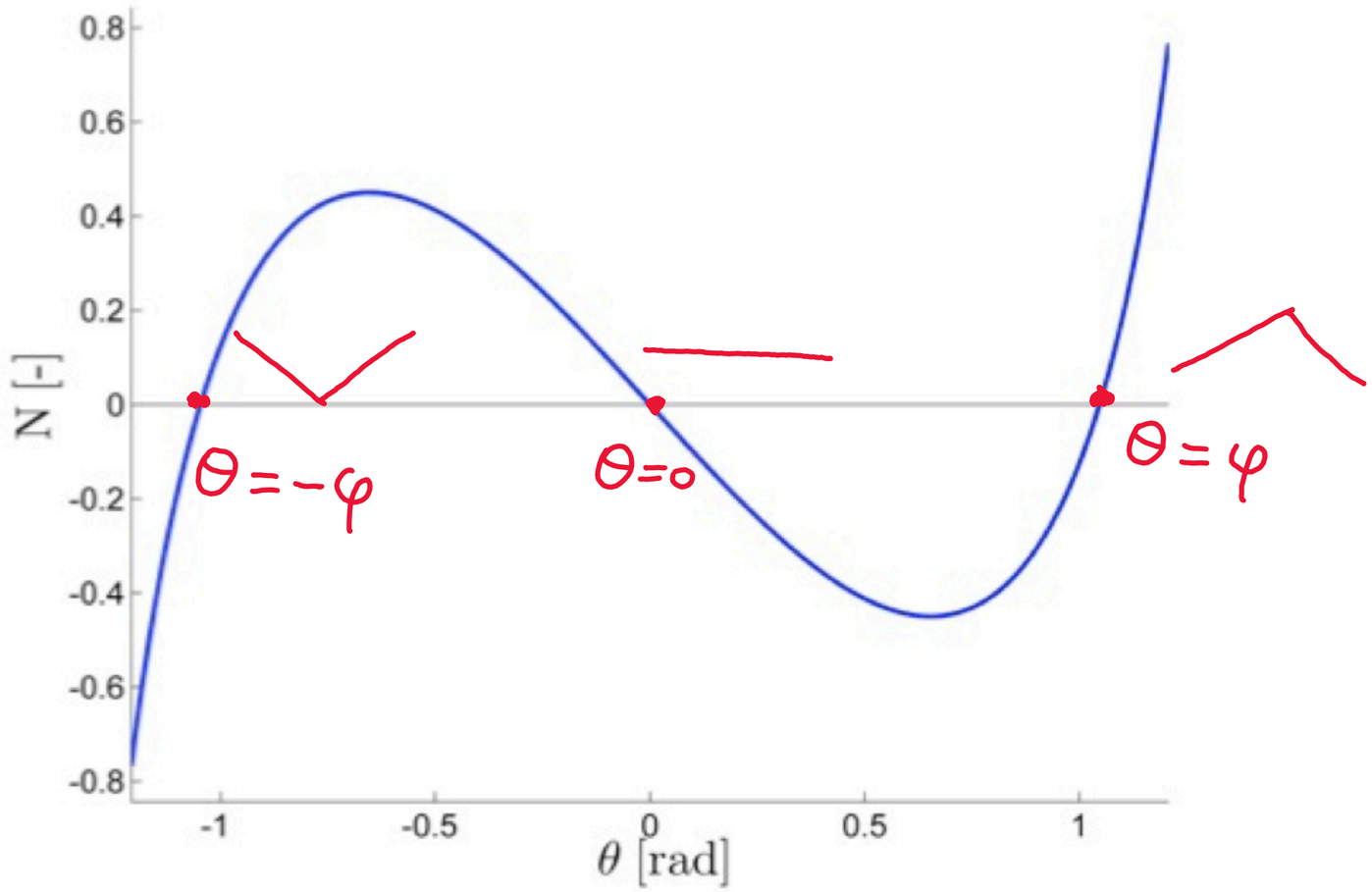
$$\left.\frac{dP}{d\theta}\right|_{\theta_0} = \frac{kL^2}{4} \cdot 2\left(\frac{1}{\cos\varphi} - \frac{1}{\cos\theta_0}\right)\left(-\frac{0 - -\sin\theta_0}{\cos^2\theta_0}\right)$$

$$+ \frac{NL}{2}\frac{1}{\cos^2\theta_0} = 0$$

$$N = kL\sin\theta_0\left(\frac{1}{\cos\varphi} - \frac{1}{\cos\theta_0}\right)$$

Equilibrium path

$\varphi = 1$



Stability of equilibrium

$$P(\theta_0+\theta_1) = \cancel{\left.\frac{dP}{d\theta}\right|_{\theta_0}}^{0}\theta_1 + \frac{1}{2!}\left.\frac{d^2P}{d\theta^2}\right|_{\theta_0}\theta_1^2 + \frac{1}{3!}\left.\frac{d^3P}{d\theta^3}\right|_{\theta_0}\theta_1^3 + \text{h.o.t.}$$

$$\left.\frac{d^2P}{d\theta^2}\right|_{\theta_0} = -\frac{kL^2}{2}\frac{1}{\cos\varphi}\cdot\frac{\cos^3\theta_0 + 2\cos\theta_0\sin^2\theta_0}{\cos^4\theta_0}$$

$$+\frac{kL^2}{2}\cdot\frac{\cos^4\theta_0 + 3\cos^2\theta_0\sin^2\theta_0}{\cos^6\theta_0}$$

$$+\frac{NL}{2}\frac{2\cos\theta_0\sin\theta_0}{\cos^4\theta_0}$$

Stability of equilibrium

$$= kL^2 \left[\frac{1}{\cos^2\theta_0} (1 + 3\tan^2\theta_0) - \frac{1}{\cos\theta_0 \cos\varphi} (1 + 2\tan^2\theta_0) \right]$$

$$+ NL \frac{\tan\theta_0}{\cos^2\theta_0}$$

$> 0 \quad < 0$

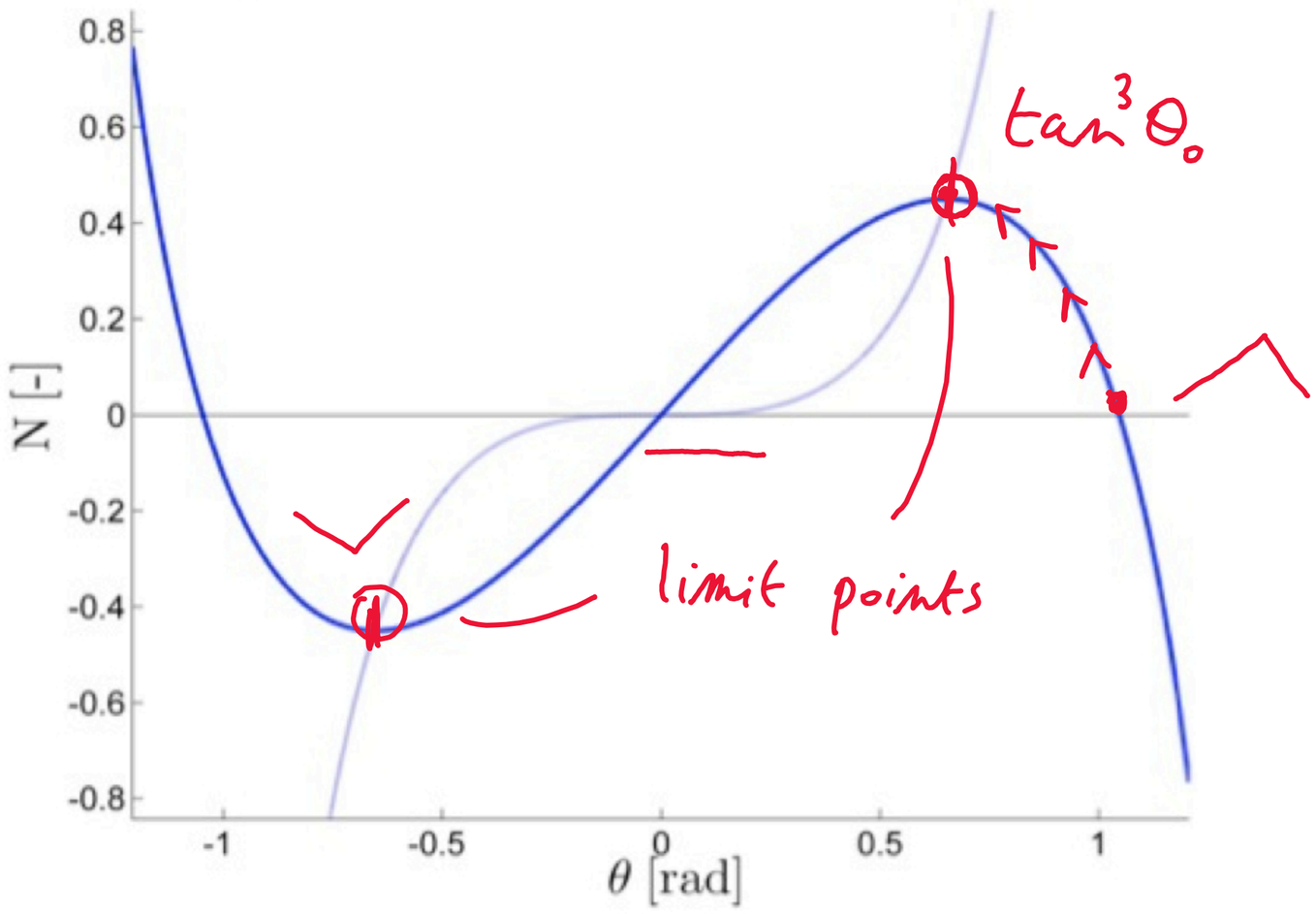
eliminate φ $\longrightarrow$ $\left.\frac{d^2 p}{d\theta^2}\right|_{\theta_0} = kL^2 \frac{1}{\sin 2\theta_0} \left(\tan^3\theta_0 - \frac{N}{kL} \right)$

$N < kL \tan^3\theta_0$ stable

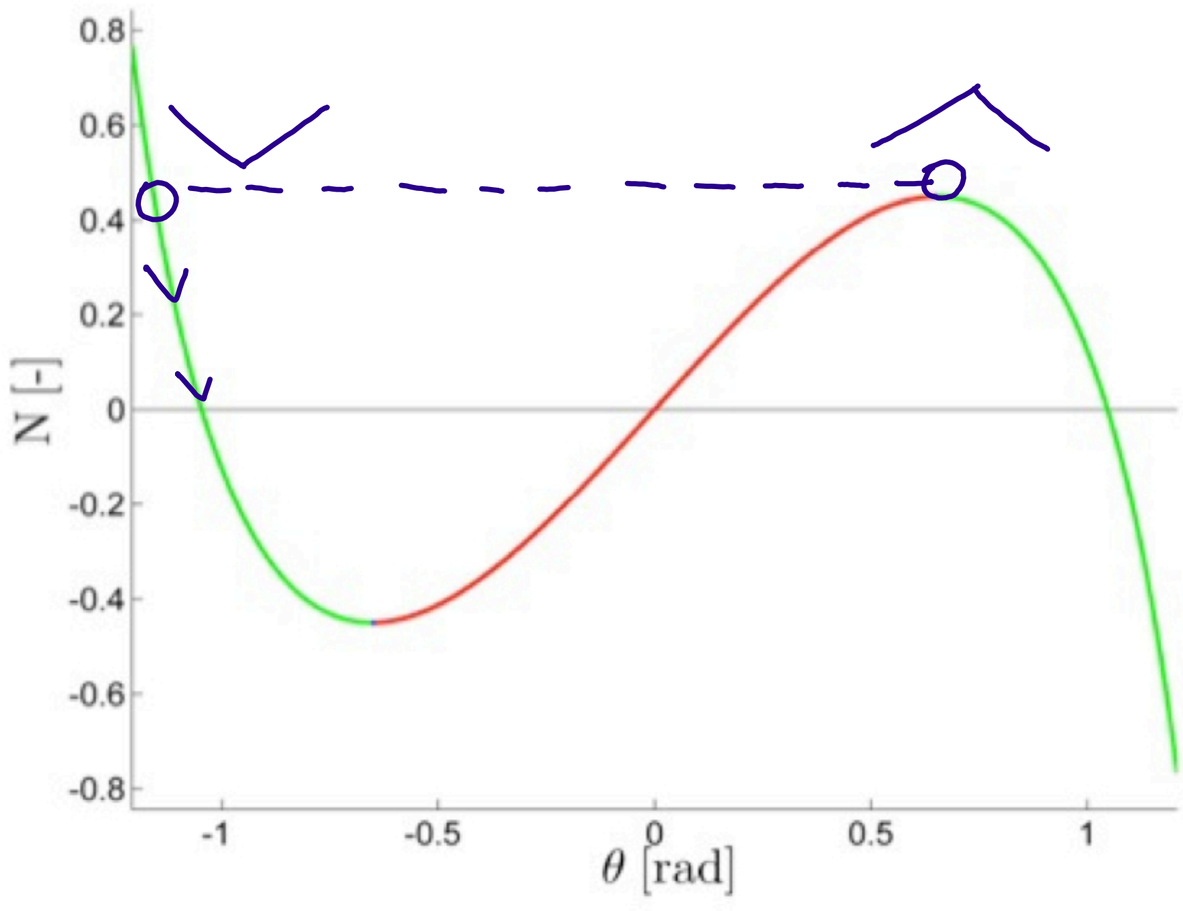
$N > kL \tan^3\theta_0$ unstable

$N = kL \tan^3\theta_0$ critical

Stability of equilibrium



Stability of equilibrium



Stability of critical points

$$\left.\frac{d^3P}{d\theta^3}\right|_{\theta_0} = 3\,\frac{\tan\theta_0}{\cos^4\theta_0} \neq 0$$

unstable

Summary

- Equilibrium and stability analysis of a single degree-of-freedom structure with nonlinear precritical equilibrium path
- The snap-through phenomenon was introduced
- It was shown that in case of snap-through, the limit point is unstable