

AE4536: Buckling of structures

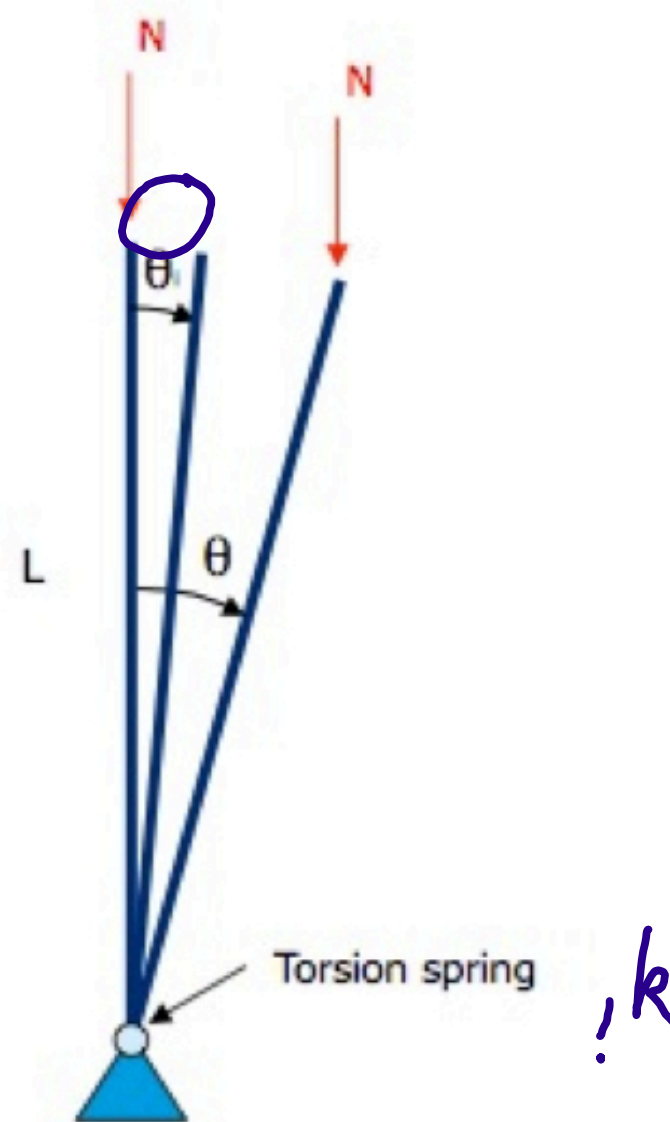
Discrete Symmetrical Stable Postcritical Behaviour of
an Imperfect Structure

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Imperfect column

$$\begin{aligned}
 P(\theta, \theta_i) &= \frac{1}{2} k (\theta - \theta_i)^2 \\
 &\quad - NL(1 - \cos \theta) \\
 &\quad + NL(1 - \cos \theta_i) \\
 &= \frac{1}{2} k (\theta^2 - 2\theta\theta_i + \theta_i^2) \\
 &\quad - NL(1 - \cos \theta) \\
 &\quad + NL(1 - (1 - \frac{1}{2}\theta_i^2 + \dots)) \\
 &= \frac{1}{2} k (\theta^2 - 2\theta\theta_i) \\
 &\quad - NL(1 - \cos \theta)
 \end{aligned}$$



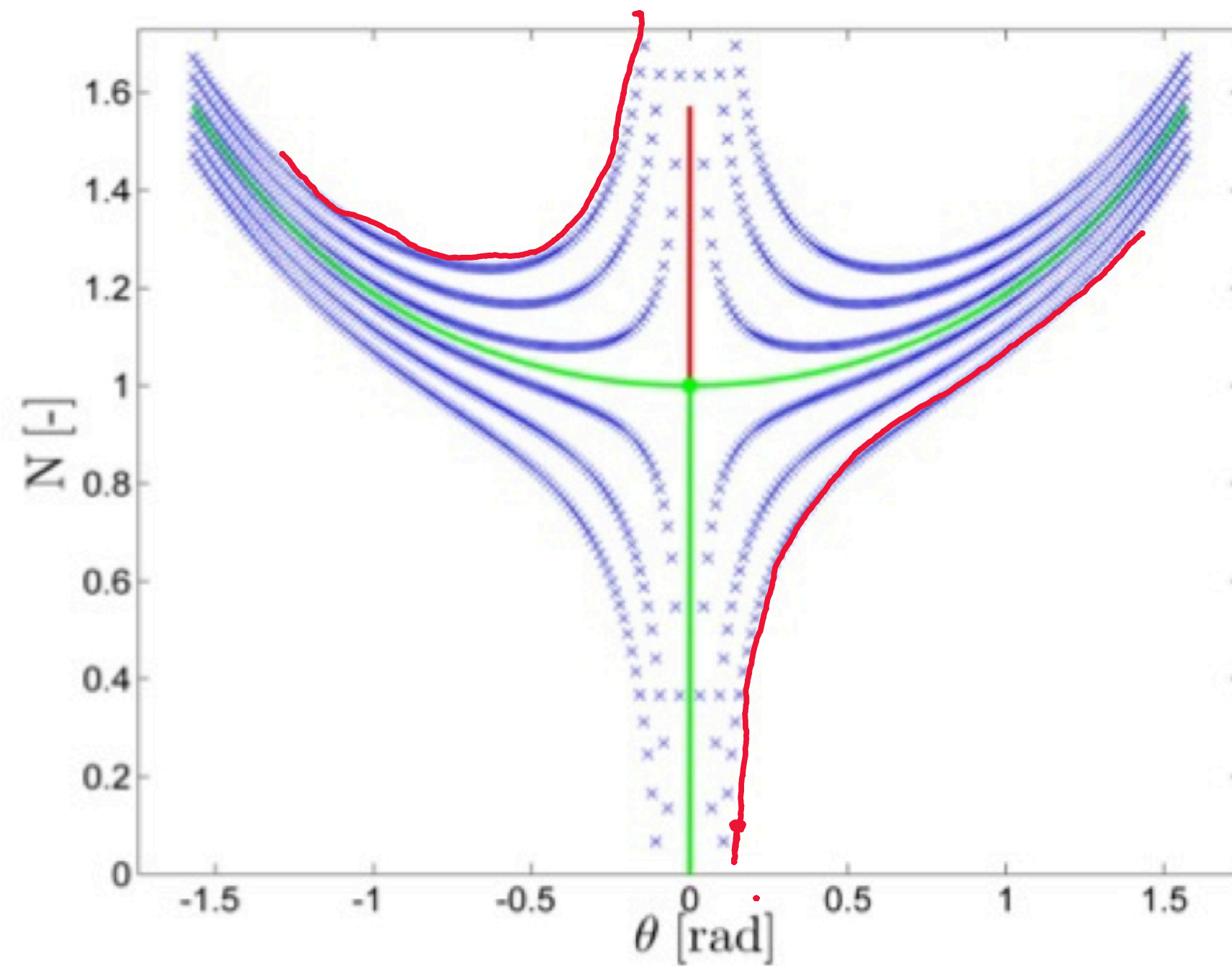
Equilibrium

$$\left. \frac{dP}{d\theta} \right|_{\theta_0} = k\theta_0 - k\theta_i - NL \sin\theta_0 = k(\theta_0 - \theta_i) - NL \sin\theta_0 = 0$$

$$k(\theta_0 - \theta_i) - NL \sin\theta_0 = 0$$

$$N = \frac{k}{L} \frac{\theta_0 - \theta_i}{\sin\theta_0}$$

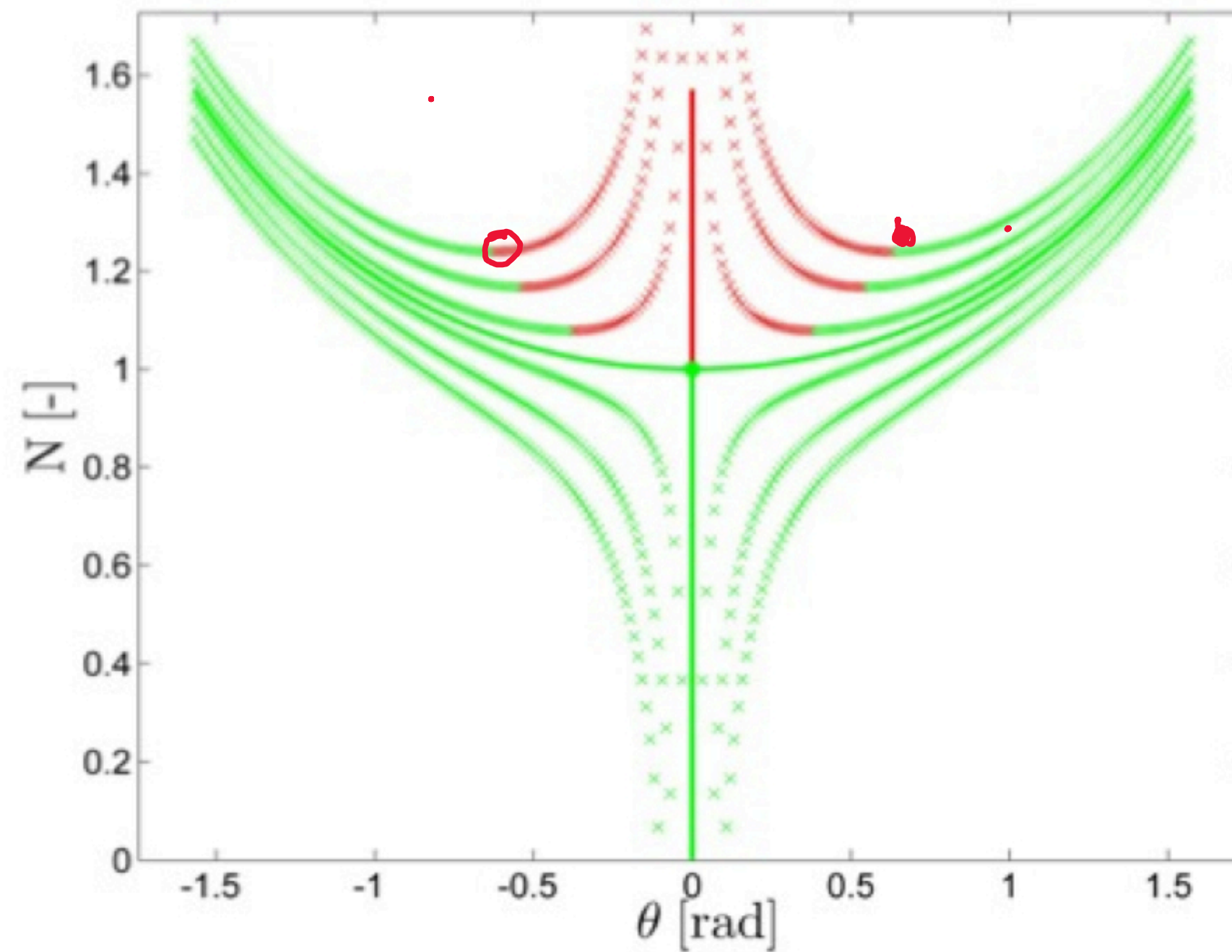
Equilibrium



Stability of equilibrium

$$\begin{aligned}\frac{d^2P}{d\theta^2}\bigg|_{\theta_0} &= k - NL \cos\theta_0 \\ &= k - \frac{k}{L} \frac{\theta_0 - \theta_i}{\sin\theta_0} L \cos\theta_0 = k \left(1 - \frac{\theta_0 - \theta_i}{\tan\theta_0} \right)\end{aligned}$$


Stability of equilibrium



$$\theta_i = -0.1$$
$$\theta_o = 0.631$$
$$N = 1.239$$

Stability of critical points

$$\frac{d^3 \mathcal{P}}{d\theta^3} = NL \sin \theta_0 = 0.73 \neq 0 \quad \text{unstable}$$



Summary

- The effect of imperfections on the equilibrium and stability of a discrete structure was shown
- The difference between a bifurcation and a limit point was shown by investigating the third derivative of the potential energy function