AE4536: Buckling of structures

Discrete Symmetrical Stable Postcritical Behaviour

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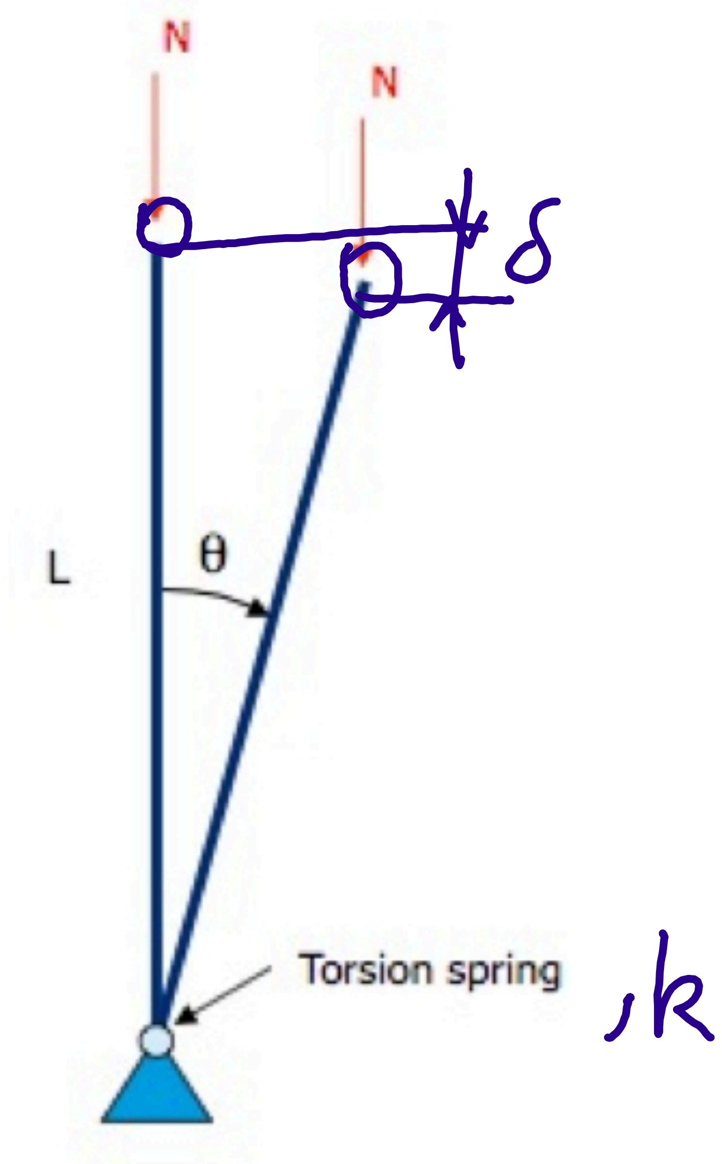
Problem definition

$$\delta = L - L\cos\theta$$
$$= L(1-\cos\theta)$$



$$P = U - V$$
$$= \frac{1}{2}k\theta^2 - N\delta$$
$$= \frac{1}{2}k\theta^2 - NL(1-\cos\theta)$$

Equilibrium

$$P(\theta) = \frac{1}{2}k\theta^2 - NL(1-\cos\theta)$$

$$P(\underbrace{\theta_0 + \delta\theta}_{\varepsilon\theta_1}) = \underbrace{\left.\frac{dP}{d\theta}\right|_{\theta_0}}_{=0} \delta\theta + \cancel{\frac{1}{2!}\frac{d^2P}{d\theta^2}}^{\,0}\delta\theta^2 + \cancel{h.o.t.}^{\,0} = 0$$

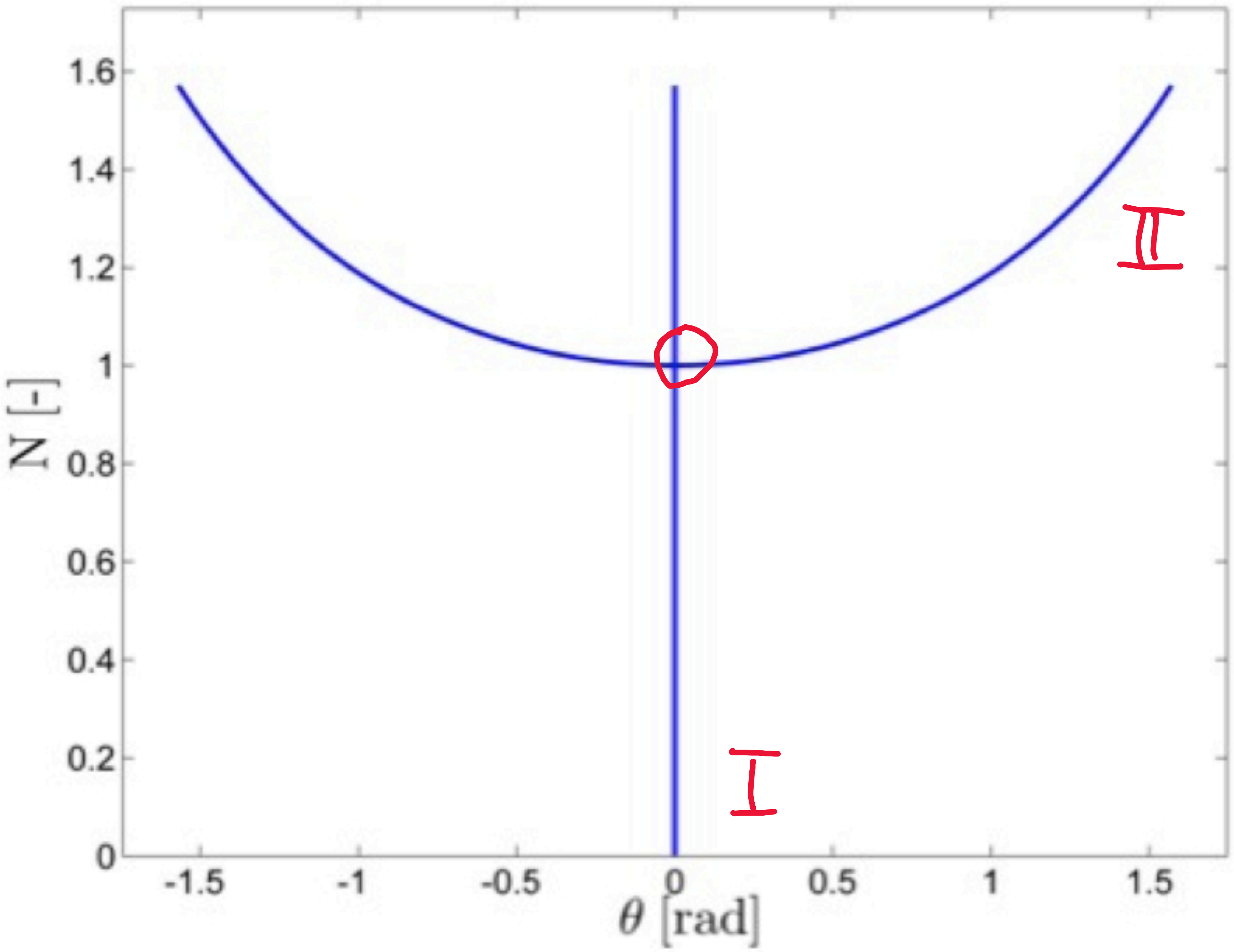
$$\left.\frac{dP}{d\theta}\right|_{\theta_0} = k\theta_0 - NL\sin\theta_0 = 0$$

(1) $\theta_0 = 0 \quad \forall N$

(2) $N = \frac{k}{L}\frac{\theta_0}{\sin\theta_0}$

$$\lim_{\theta_0\to 0}\frac{k}{L}\frac{\theta_0}{\sin\theta_0} = \lim_{\theta_0\to 0}\frac{k}{L}\frac{1}{\cos\theta_0} = \frac{k}{L}$$

Equilibrium path



Stability of equilibrium

$$P(\theta_0+\theta_1) = \cancel{\left.\frac{dP}{d\theta}\right|_{\theta_0}}\theta_1 + \frac{1}{2!}\left.\frac{d^2P}{d\theta^2}\right|_{\theta_0}\theta_1^2 + \frac{1}{3!}\frac{d^3P}{d\theta^3}\theta_1^3 + h.o.t.$$

$$\left.\frac{d^2P}{d\theta^2}\right|_{\theta_0} = k - NL\cos\theta_0$$

① $\frac{d^2P}{d\theta^2} = k - NL$

>0	$N<\frac{k}{L}$	stable
<0	$N>\frac{k}{L}$	unstable
$=0$	$N=\frac{k}{L}$	critical

② $\frac{d^2P}{d\theta^2} = k - \frac{k}{L}\frac{\theta_0}{\sin\theta_0}L\cos\theta_0 = k\left(1-\frac{\theta_0}{\tan\theta_0}\right) \geq 0$ stable / critical

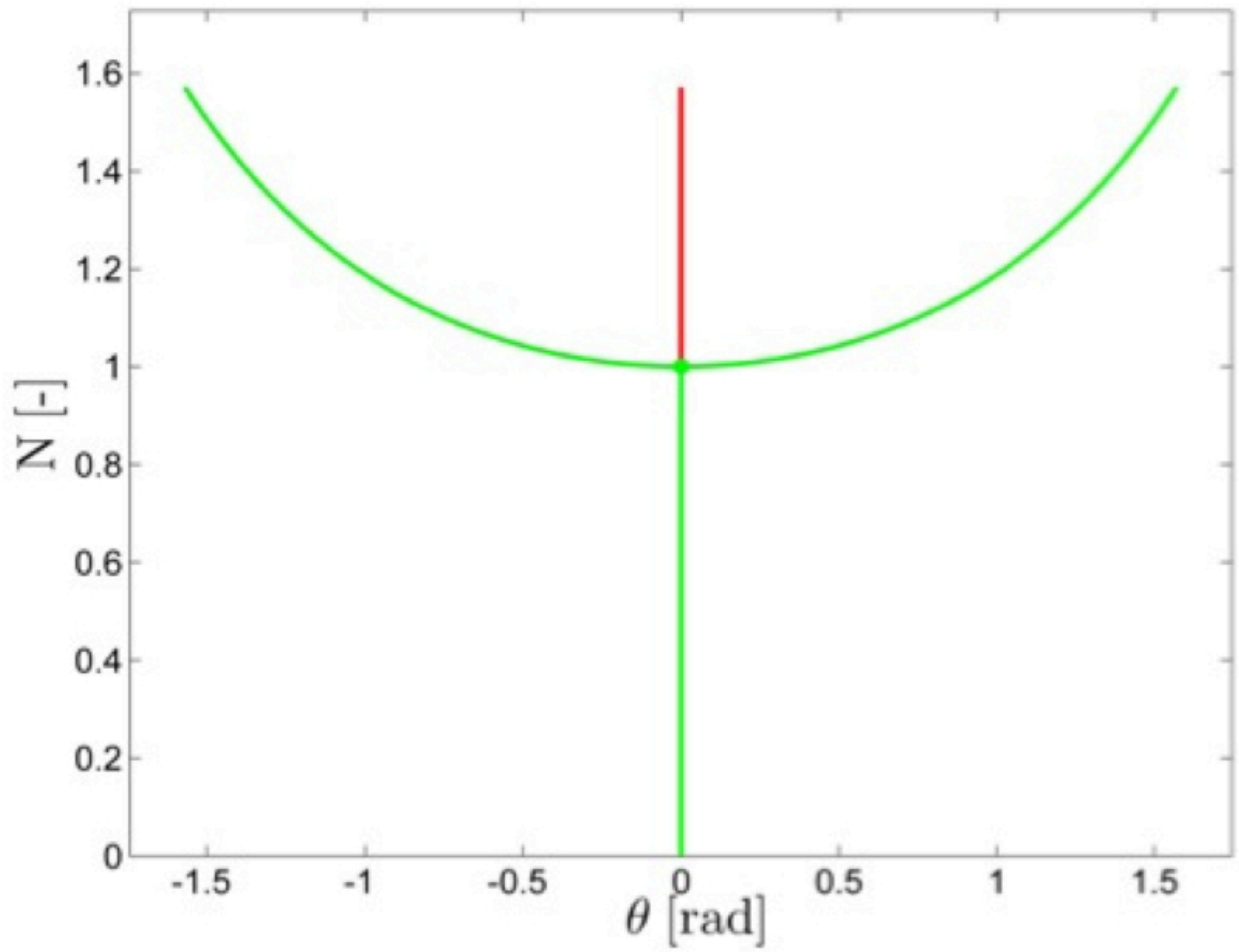
Stability of critical points

$$P(\theta_0+\theta_1) = \frac{1}{3!}\left.\frac{d^3P}{d\theta^3}\right|_{\theta_0}\theta_1^3 + \frac{1}{4!}\left.\frac{d^4P}{d\theta^4}\right|_{\theta_0}\theta_1^4 + h.o.t.$$

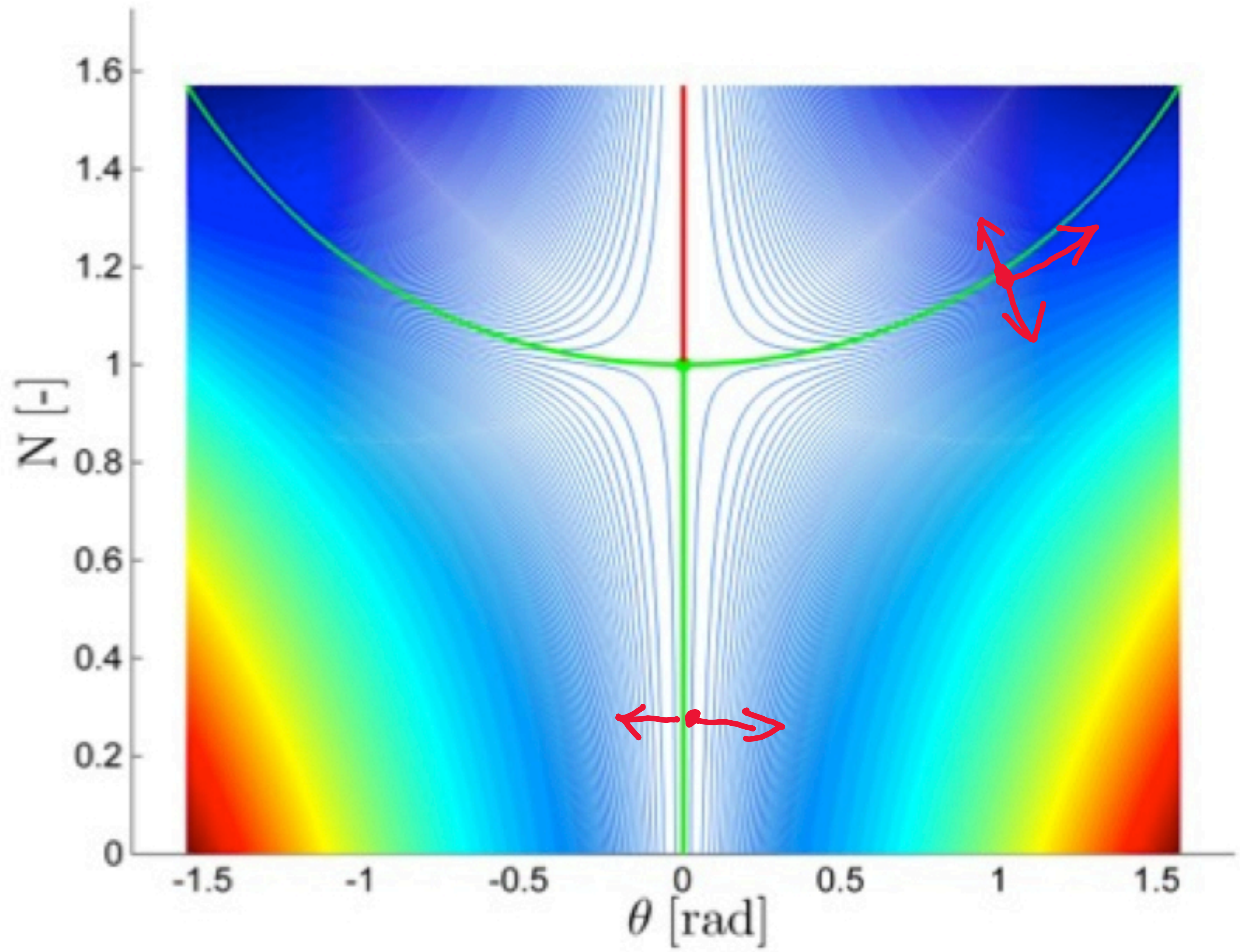
$$\frac{d^3P}{d\theta^3} = NL\sin\theta_0 = 0 \qquad \checkmark$$

$$\frac{d^4P}{d\theta^4} = NL\cos\theta_0 = k > 0 \qquad \text{stable}$$

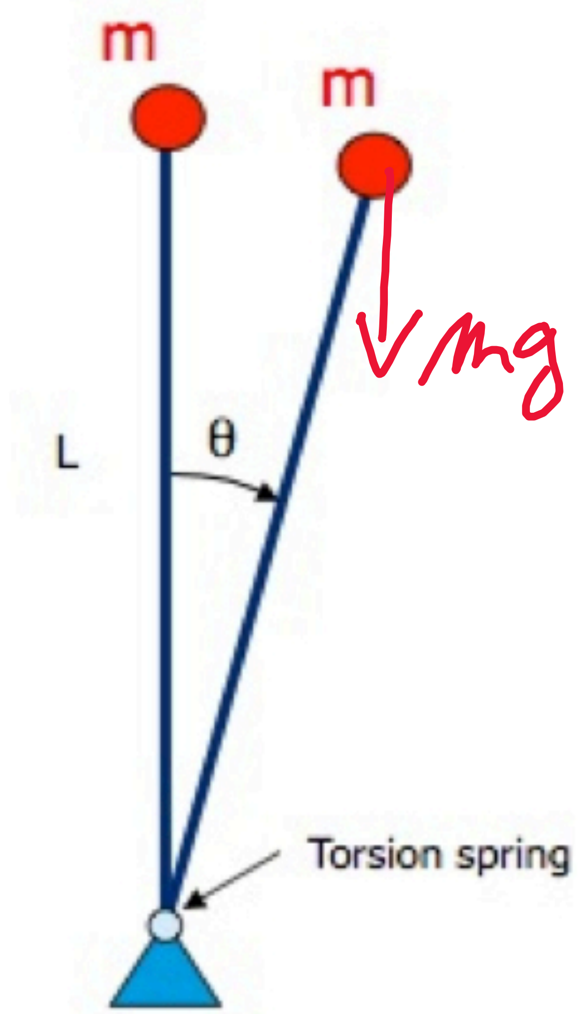
Stability of equilibrium



Potential energy plot



Dynamic analysis



$$P = \frac{1}{2}k\theta^2 - mgL(1-\cos\theta)$$

$$T = \frac{1}{2}m(L\dot{\theta})^2$$

$$\frac{d}{dt}\frac{dT}{d\dot{\theta}} + \frac{dP}{d\theta} = 0$$

$$\frac{d}{dt}\frac{dT}{d\dot{\theta}} = mL^2\ddot{\theta}$$

$$\frac{dP}{d\theta} = k\theta - mgL\sin\theta$$

$$= (k - mgL)\theta$$

$$mL^2\ddot{\theta} + \theta(k - mgL) = 0$$

$$\theta = \hat{\theta}\, e^{\lambda t}$$

$$\underbrace{\left(\lambda^2 m L^2 + (k - mgL)\right)}_{=0} \hat{\theta}\, e^{\lambda t} = 0$$

$$\lambda = \pm\sqrt{\frac{mgL - k}{mL^2}}$$

λ = real	$mgL > k$	unstable
λ = imag	$mgL < k$	stable
$\lambda = 0$	$mgL = k$	undecided

Summary

- Equilibrium, stability and analysis of critical points of a single degree-of-freedom structure with stable symmetrical post-critical equilibrium path
- Analogy with a dynamic analysis and linearised stability analysis was demonstrated