AE4536: Buckling of structures

Discrete Symmetrical Unstable Postcritical Behaviour

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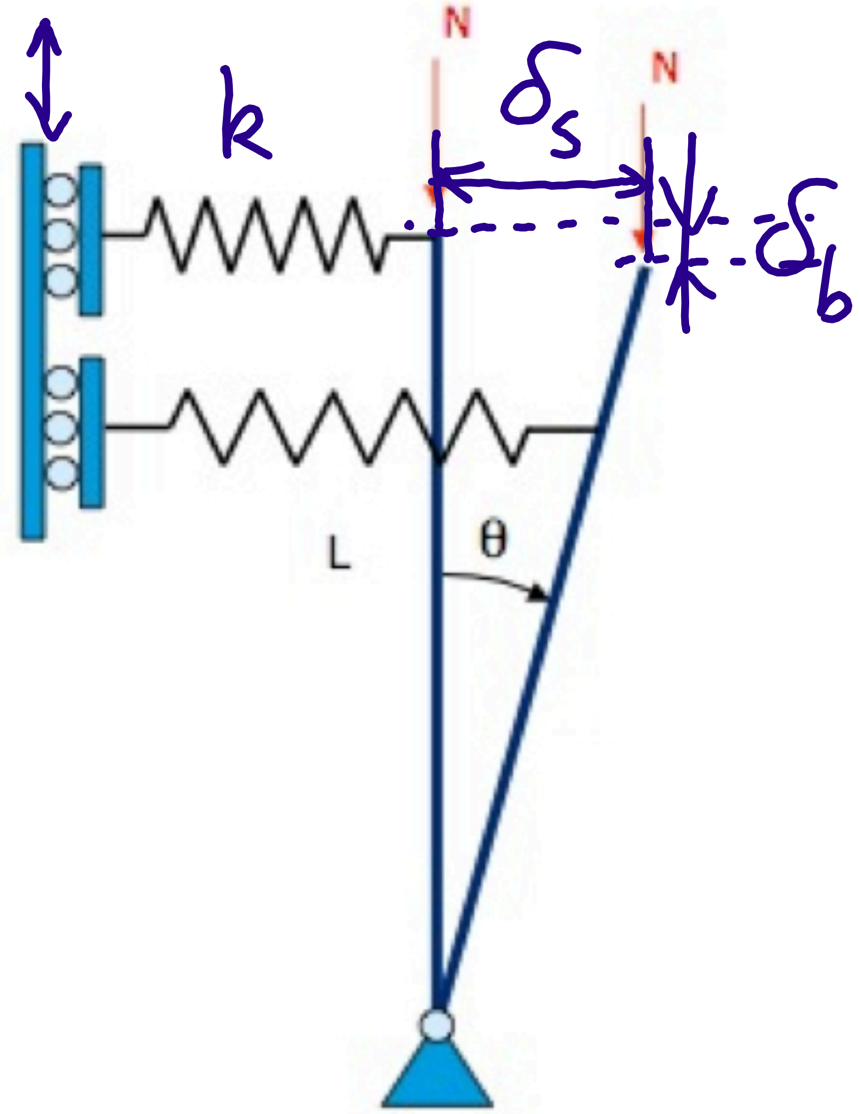
Problem definition

$$P(\theta) = \frac{1}{2} k \delta_s^2 - N \delta_b$$

$$= \frac{1}{2} k L^2 \sin^2\theta - NL(1-\cos\theta)$$

$\delta_s = L \sin\theta$

$\delta_b = L(1-\cos\theta)$



Equilibrium

$$P(\underbrace{\theta_0+\delta\theta}_{\varepsilon\theta_1}) = \underbrace{\left.\frac{dP}{d\theta}\right|_{\theta_0}}_{=0}\delta\theta + \frac{1}{2!}\cancel{\left.\frac{d^2P}{d\theta^2}\right|_{\theta_0}}^{0}\delta\theta^2 + \cancel{h.o.t.}^{0} = 0 = \delta P$$

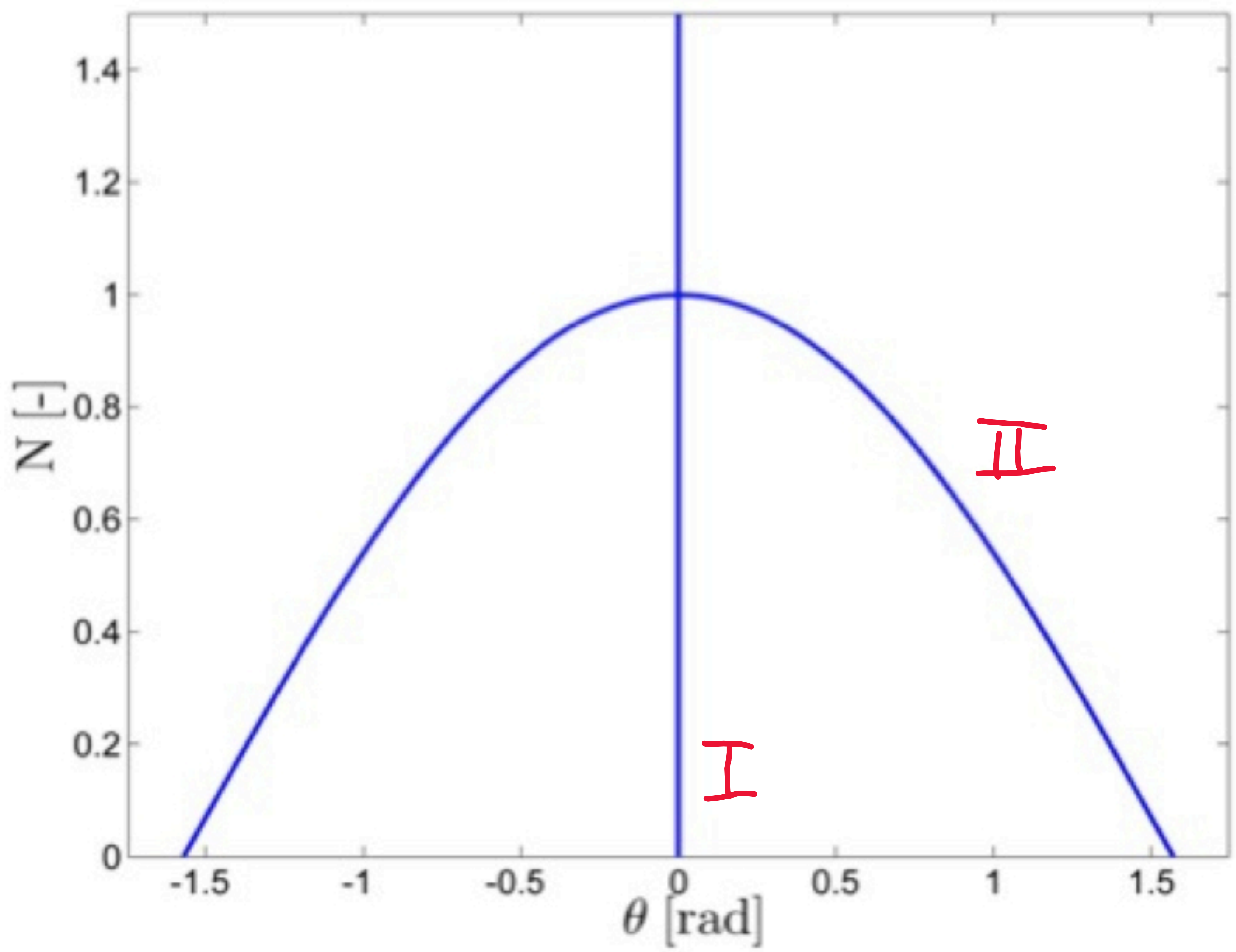
$$\left.\frac{dP}{d\theta}\right|_{\theta_0} = kL^2\sin\theta_0\cos\theta_0 - NL\sin\theta_0 = 0$$

$$= L(kL\cos\theta_0 - N)\sin\theta_0 = 0$$

(1) $\theta_0 = 0 \quad \forall N$

(2) $N = kL\cos\theta_0$

Equilibrium path



Stability of equilibrium

$$P(\theta_0+\theta_1) = \cancel{\left.\frac{dP}{d\theta}\right|_{\theta_0}}^{\,0}\theta_1 + \frac{1}{2!}\frac{d^2P}{d\theta^2}\theta_1^2 + \frac{1}{3!}\frac{d^3P}{d\theta^3}\theta_1^3 + h.o.t.$$

$$\left.\frac{d^2P}{d\theta^2}\right|_{\theta_0} = L\cos\theta_0\,(kL\cos\theta_0 - N) + L\sin\theta_0\,(-kL\sin\theta_0)$$

$$= kL^2(\cos^2\theta_0 - \sin^2\theta_0) - NL\cos\theta_0$$

$$(1)\quad \left.\frac{d^2P}{d\theta^2}\right|_{\theta_0} = kL^2 - NL$$

$N < kL$ stable

$N > kL$ unstable

$N = kL$ critical

$$(2)\quad \left.\frac{d^2P}{d\theta^2}\right|_{\theta_0} = kL^2(\cos^2\theta_0 - \sin^2\theta_0) - kL^2\cos^2\theta_0 < 0 \quad \text{unstable}$$

Stability of critical points

$$P(\theta_0+\theta_1) = \frac{1}{3!}\frac{d^3P}{d\theta^3}\theta_1^3 + \frac{1}{4!}\frac{d^4P}{d\theta^4}\theta_1^4 + h.o.t.$$

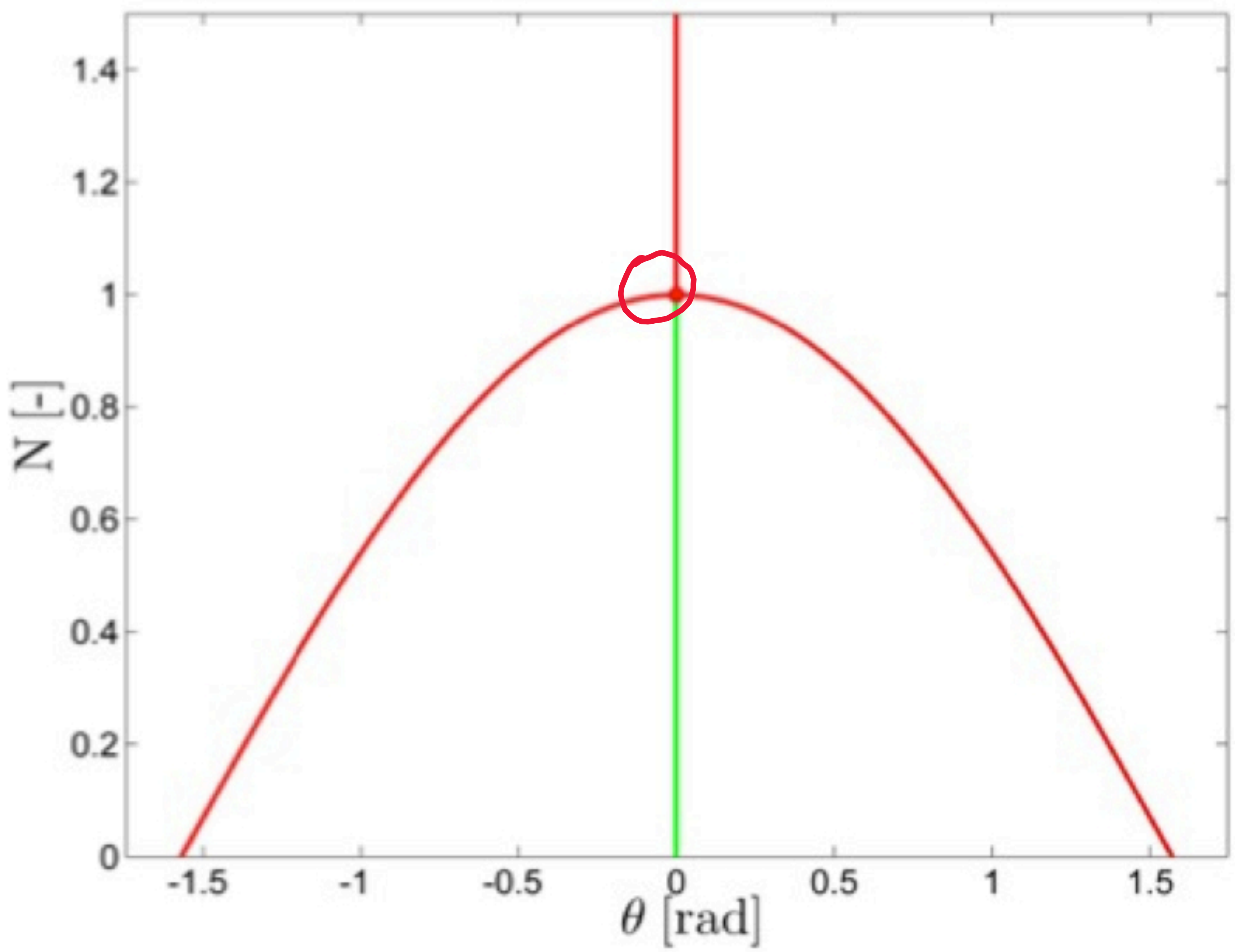
$$\left.\frac{d^3P}{d\theta^3}\right|_{\theta_0} = kL^2(-2\cos\theta_0\sin\theta_0 - 2\sin\theta_0\cos\theta_0) + NL\sin\theta_0$$

$$= 0$$

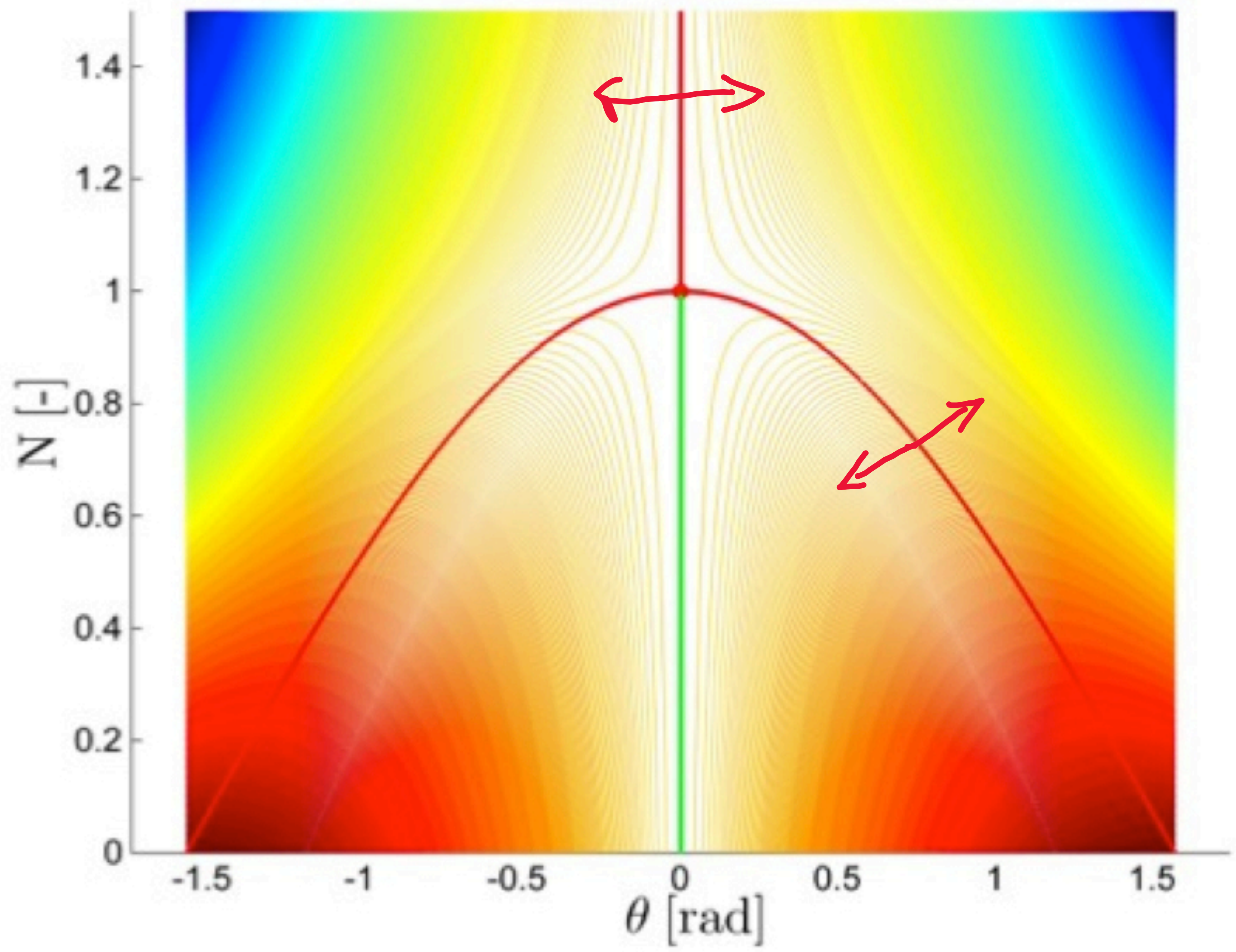
$$\left.\frac{d^4P}{d\theta^4}\right|_{\theta_0} = kL^2(-4(-\sin^2\theta_0 + \cos^2\theta_0)) + NL\cos\theta_0$$

$$= -4kL^2 + kL^2 = -3kL^2 < 0$$ unstable

Stability of equilibrium



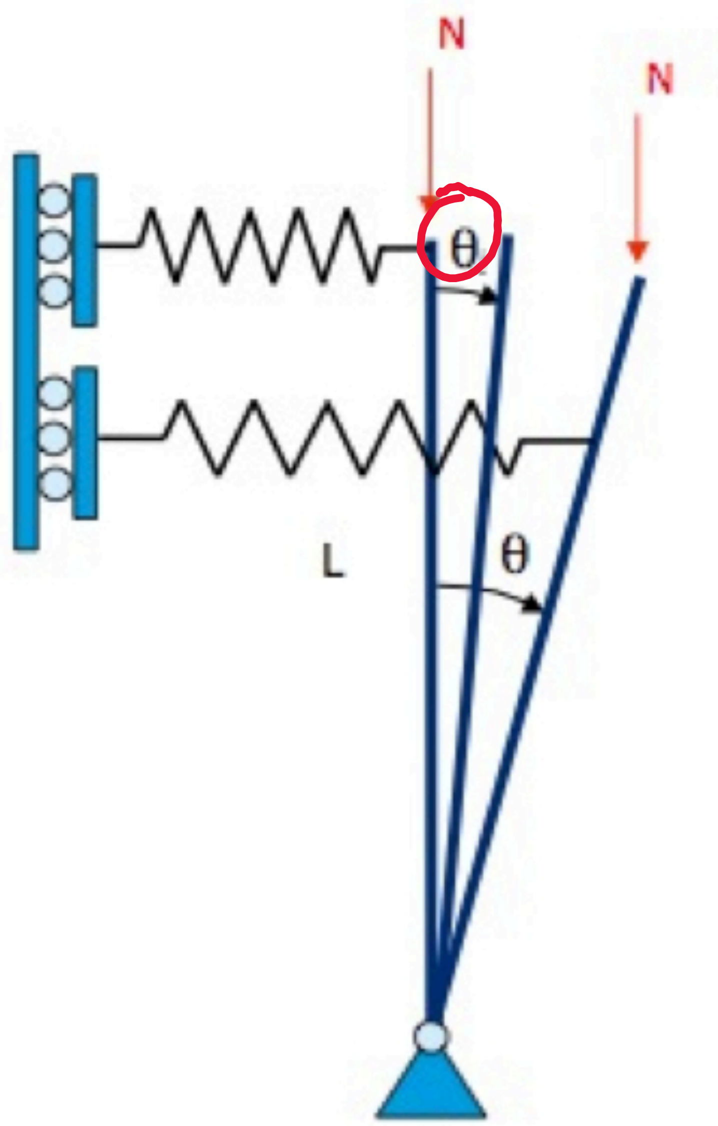
Potential energy plot



Imperfect column

$$P(\theta,\theta_i) = \frac{1}{2}kL^2(\sin\theta - \sin\theta_i)^2$$
$$- NL(1-\cos\theta)$$
$$+ NL(1-\cos\theta_i)$$

$$= \frac{1}{2}kL^2(\sin^2\theta - 2\sin\theta.\theta_i)$$
$$- NL(1-\cos\theta)$$

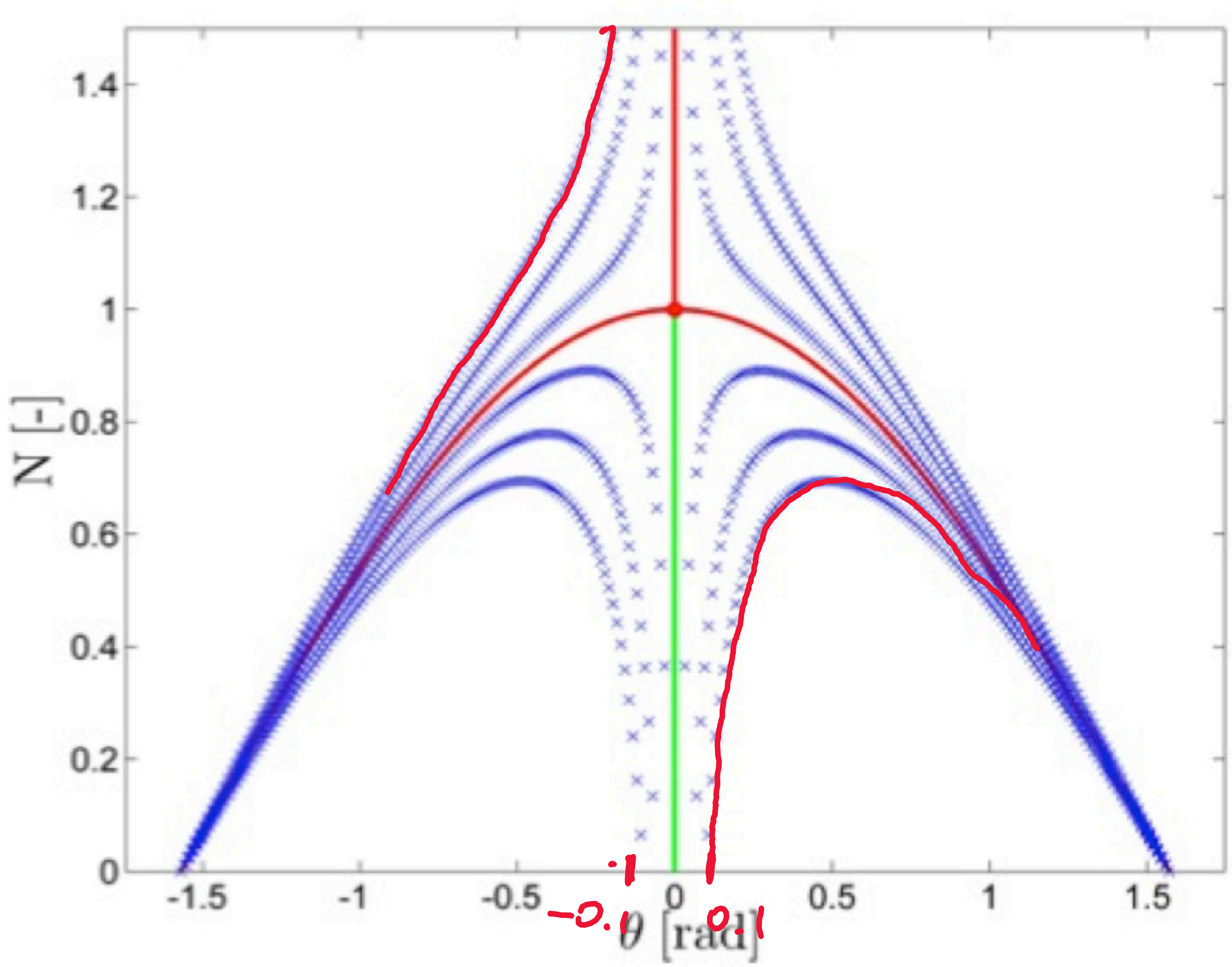


Equilibrium

$$\left.\frac{dP}{d\theta}\right|_{\theta_0} = \frac{1}{2}kL^2\left(2\sin\theta_0\cos\theta_0 - 2\,\theta_i\cos\theta_0\right) - NL\sin\theta_0 = 0$$

$$N = \frac{kL^{\cancel{2}}\left(\sin\theta_0\cos\theta_0 - \theta_i\cos\theta_0\right)}{\cancel{L}\sin\theta_0} = \frac{kL\cos\theta_0\left(\sin\theta_0 - \theta_i\right)}{\sin\theta_0}$$

Equilibrium path



Stability of equilibrium

$$\left.\frac{d^2P}{d\theta^2}\right|_{\theta_0} = kL^2\left(\cos^2\theta_0 - \sin^2\theta_0 + \theta_i \sin\theta_0\right) - NL\cos\theta_0$$

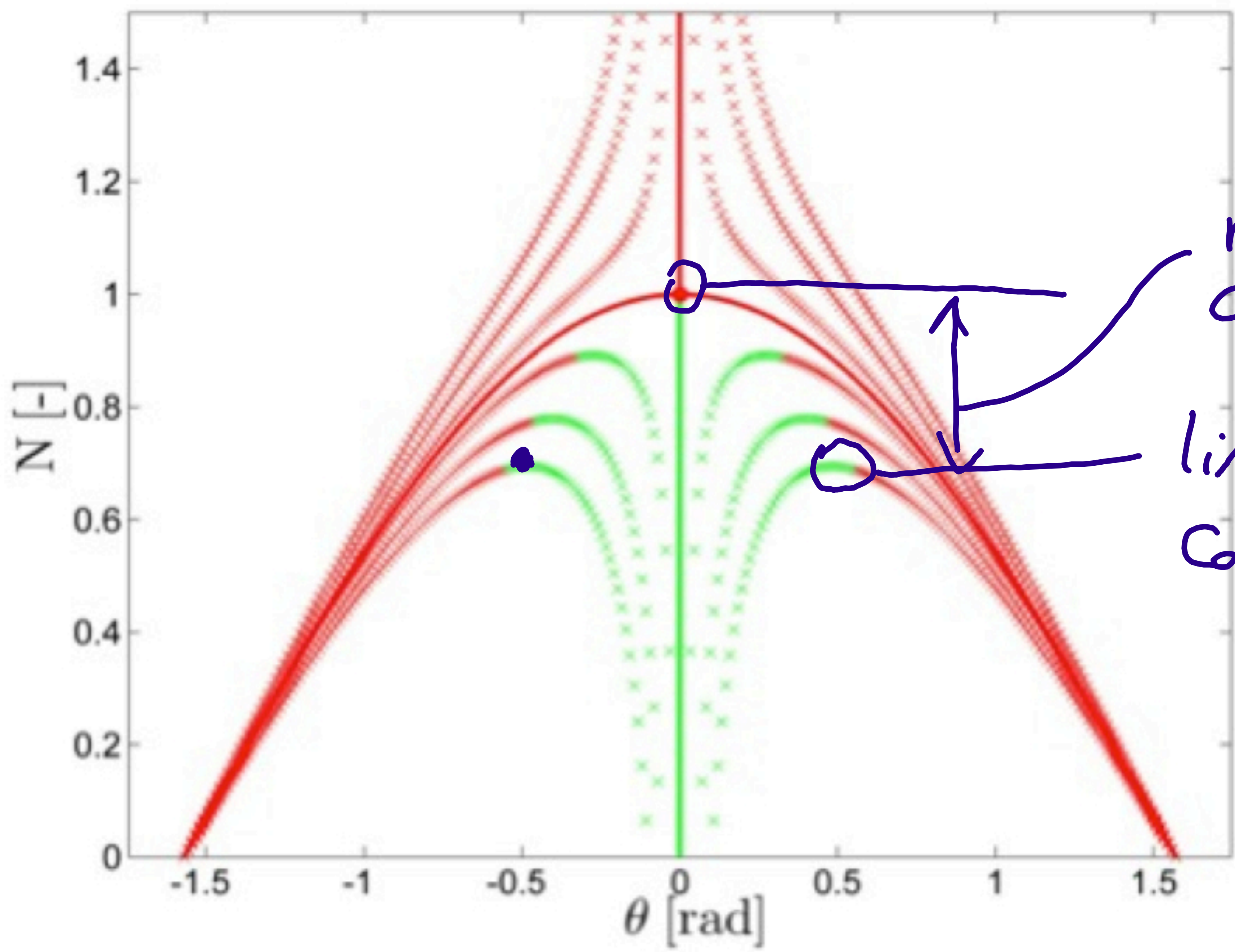
Substitute equilibrium

Stability of equilibrium

$\theta_i = -0.1$

$\theta_o = -0.561$

$N = 0.687$



reduction in critical load

limit point

Collapse load

Stability of critical points

$$\left.\frac{d^3P}{d\theta^3}\right|_{\theta_0} = kL^2\left(-2\cos\theta_0\sin\theta_0 - 2\sin\theta_0\cos\theta_0 + \theta_i\cos\theta_0\right)$$

$$+NL\sin\theta_0$$

$$= 1.35 \neq 0$$ unstable

Summary

- Equilibrium, stability and stability of critical points of a single degree-of-freedom structure with unstable symmetrical post-critical equilibrium path was analysed
- The effect of imperfections of an unstable post-critical behaviour was shown
- It was shown that a load can exist for which not stable equilibrium point exists. This load is called the collapse load