AE4536: Buckling of structures

Stability of discrete systems

Roeland De Breuker

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TU Delft Delft University of Technology
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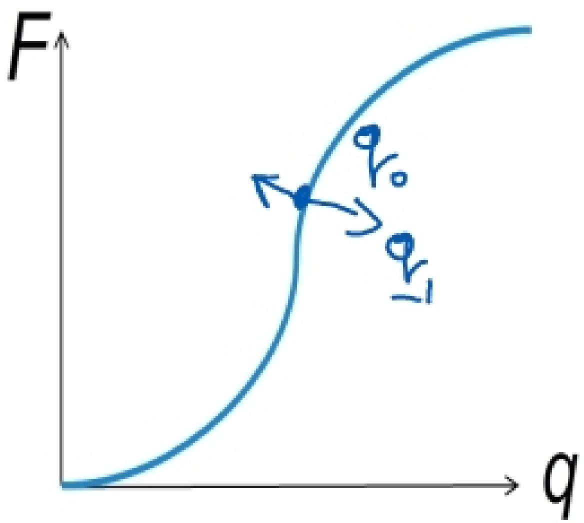
Learning goal

Learning how to assess equilibrium, stability, and post-critical behaviour of discrete systems.

$$P(q_i+\delta q_i) = \underbrace{\frac{\partial P(q_i)}{\partial q_i}}_{\sum F=0}\delta q_i + \frac{1}{2!}\frac{\partial^2 P(q_i)}{\partial q_i \partial q_j}\cancelto{0}{\delta q_i \delta q_j} + \cancelto{0}{h.o.t.}$$

Solve for $\underline{q}_0 \rightarrow$ equilibrium DOFs



$$\frac{\partial P(q_{0,i})}{\partial q_i} = P_{,i}$$

$$P(q_{0,i} + q_{1,i}) = \underbrace{P_{,i}\, q_{1,i}}_{=0} + \frac{1}{2!} \overbrace{P_{,ij}}^{K_{ij}} q_{1,i}\, q_{1,j} + \frac{1}{3!} P_{,ijk}\, q_{1,i}\, q_{1,j}\, q_{1,k} + h.o.t.$$

When we found equilibrium, assess the next leading terms

@ equilibrium : K_{ij} = positive definite stable
K_{ij} = negative definite unstable
K_{ij} = semi positive definite critical

$$\frac{1}{2!} P_{,ij}\, q_{1,i}\, q_{1,j} \xrightarrow{\text{Matrix format}} \frac{1}{2!}\, \underline{q}_1^t\, \underline{\underline{K}}\, \underline{q}_1$$

Let's simplify

$$\underline{q}_1 = \underline{\underline{a}}\, \underline{\varphi}_1$$

$$\rightarrow \frac{1}{2!}\, \underline{\varphi}_1^t\, \underbrace{\underline{\underline{a}}^t\, \underline{\underline{K}}\, \underline{\underline{a}}}_{\underline{\underline{\bar{K}}}}\, \underline{\varphi}_1$$

diagonal

$$P(\varphi_{0,i} + \varphi_{1,i}) = \frac{1}{2!} \bar{K}_{ii}\, \varphi_{1,i}^2 + \frac{1}{3!} \bar{K}_{ijk}\, \varphi_{1,i}\, \varphi_{1,j}\, \varphi_{1,k}$$

$$+ \text{h.o.t.}$$

$\bar{K}_{11} = 0$ then $\underline{\tilde{\varphi}}_1 = \{\hat{\varphi}_1, 0, 0, \ldots, 0\}$

critical

buckling mode

$$P(\varphi_{0,i} + \tilde{\varphi}_{1,i}) = \frac{1}{3!}\bar{K}_{111}\hat{\varphi}_1^3 + \frac{1}{4!}\bar{K}_{1111}\hat{\varphi}_1^4$$

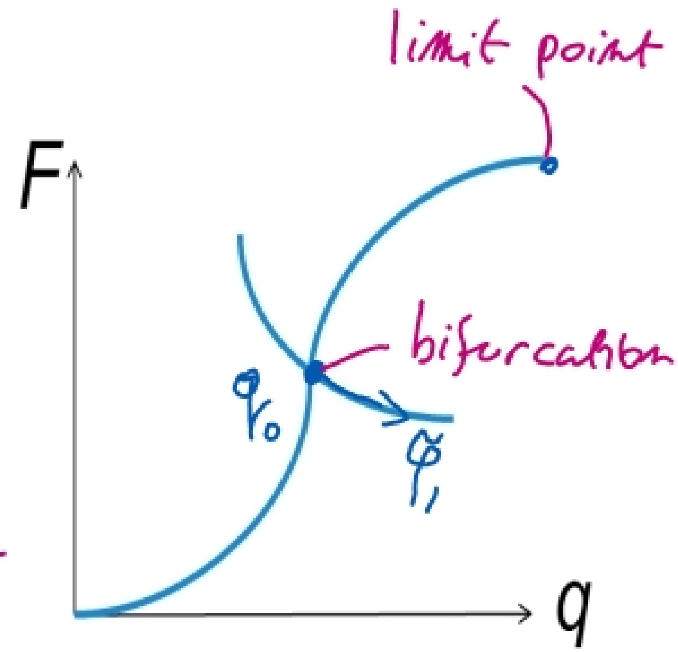
Stability

$\bar{K}_{111} = 0$

$\bar{K}_{1111} > 0$ necessary

$$> 3\sum_{i=2}^{n} \frac{\bar{K}_{i11}^2}{\bar{K}_{ii}}$$

sufficient



Approach

- General rule is that the potential energy function, expanded around an equilibrium point, should be positive definite

- To get the equilibrium position, perturb the discrete displacement field with an infinitesimally small variation and solve the first derivative of the potential energy function with respect to the degrees-of-freedom

- To assess stability of the system, investigate the higher order terms:
  - Second derivative negative means instability, positive means stability, equal to zero indicates a critical point
  - Third derivative should be zero for stability
  - Fourth derivative should be larger or equal than zero for stability