AE4536: Buckling of structures

Imperfect beams

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Imperfections in the governing equations

$w + \bar{w}$

$\varepsilon = u' + \frac{1}{2} w'^2$

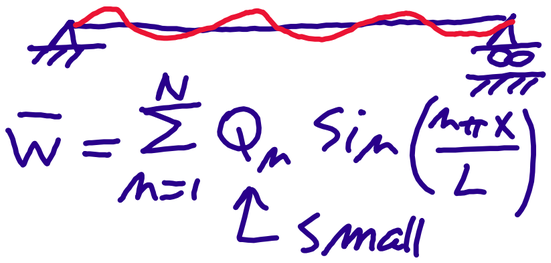
$\Rightarrow u' + \frac{1}{2}(w' + \bar{w}')^2$

$= u' + \frac{1}{2}(w'^2 + 2w'\bar{w}' + \bar{w}'^2)$ ($\bar{w}'^2 \approx 0$)

$\kappa = -w'' \Rightarrow \kappa = -w'' - \bar{w}''$ ($\bar{w}'' \approx 0$)

$EI w_0'''' + P(w_0'' + \bar{w}'') = 0$

$EI w_c'''' + P w_c'' = P \sum_{n=1}^{N} \left(\frac{n\pi}{L}\right)^2 Q_n \sin\left(\frac{n\pi x}{L}\right)$



Structural response

$$w_c'''' + k^2 w_c'' = k^2 \sum_{n=1}^{N} \left(\frac{n\pi}{L}\right)^2 \sin\left(\frac{n\pi x}{L}\right)$$

$$w_c = w_{c,h} + w_{c,p}$$

$$w_{c,p} = \sum_{n=1}^{N} C_n \sin\left(\frac{n\pi x}{L}\right)$$

$$\sum_{n=1}^{N} \left[C_n \left(\frac{n\pi}{L}\right)^4 - k^2 C_n \left(\frac{n\pi}{L}\right)^2 - k^2 Q_n \left(\frac{n\pi}{L}\right)^2 \right] \sin\left(\frac{n\pi x}{L}\right) = 0$$

$$C_n = \frac{P}{\frac{n^2\pi^2 EI}{L^2} - P} Q_n = \frac{P}{P_E - P} Q_n$$

Force-displacement diagram

$$w_{c,p} = \sum_{n=1}^{N} \frac{P}{P_E - P} Q_n \sin\left(\frac{n\pi x}{L}\right)$$