AE4536: Buckling of structures

Rayleigh quotient

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Rayleigh quotient for general structures

$$P(\underline{u}) = U(\underline{u}) - \lambda V(\underline{u}) \qquad \underline{u} = \sum_{i=1}^{\infty} a_i \varphi_i(x)$$

$$P(\underline{a}) = U(\underline{a}) - \lambda V(\underline{a})$$

$$\delta P(\underline{a}) = \frac{\partial U(\underline{a})}{\partial \underline{a}} - \lambda \frac{\partial V(\underline{a})}{\partial \underline{a}} = 0$$

$$\rightarrow \lambda_c = \frac{\partial U(\underline{a})/\partial \underline{a}}{\partial V(\underline{a})/\partial \underline{a}}$$

Rayleigh quotient for beams

$$U = \frac{1}{2}\int_0^L EI\kappa^2\,dx = \frac{1}{2}\int_0^L EI\,w''^2\,dx = \frac{1}{2}\int_0^L EI\,a_i^2\,\varphi_i''^2\,dx$$

$$V = \frac{1}{2}P\int_0^L w'^2\,dx = \frac{1}{2}P\int_0^L a_i^2\,\varphi_i'^2\,dx$$

$$\partial U/\partial a_i = a_i\int_0^L EI\,\varphi_i''^2\,dx \qquad \frac{\partial V}{\partial a_i} = a_i P\int_0^L \varphi_i'^2\,dx$$

$$P_c = EI\,\frac{\int_0^L \varphi_i''^2\,dx}{\int_0^L \varphi_i'^2\,dx}$$

Rayleigh quotient example

$$\varphi = x(L-x)$$
$$\varphi' = L-2x$$
$$\varphi'' = -2$$
$$\varphi'^2 = L^2 - 4Lx + 4x^2$$
$$\varphi''^2 = 4$$

$$\int_0^L \varphi'^2 \, dx = L^3 - 2L^3 + \frac{4}{3}L^3 = \frac{1}{3}L^3$$

$$\int_0^L \varphi''^2 \, dx = 4L$$

$$P_c = EI \frac{4L}{\frac{1}{3}L^3} = \frac{12\,EI}{L^2} \approx 1{,}2\, P_E$$

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Timoshenko quotient

$$U = \frac{1}{2}\int_0^L EI\kappa^2\,dx$$

$$= \frac{1}{2}\int_0^L \frac{M^2}{EI}\,dx$$

Only SS ↓

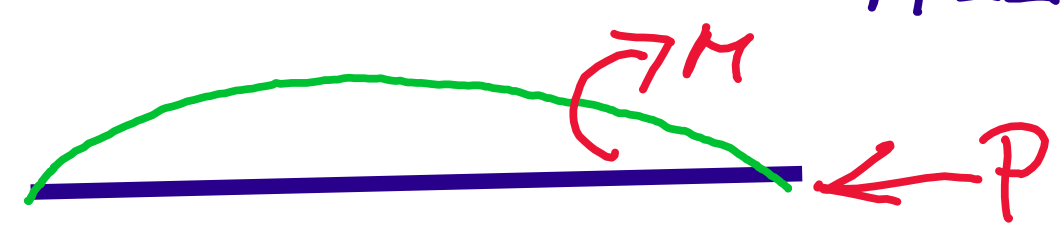
$$= \frac{1}{2}\int_0^L \frac{P^2 w^2}{EI}\,dx$$

$$P_c = EI\,\frac{\int_0^L \varphi'^2\,dx}{\int_0^L \varphi^2\,dx}$$

$$M = EI\kappa$$

$$M^2/EI = EI\kappa^2$$

$$M = -Pw$$



Timoshenko quotient example

$$\varphi = x(L-x)$$

$$\varphi^2 = x^2(L^2 - 2Lx + x^2)$$

$$= x^4 - 2Lx^3 + L^2x^2$$

$$\int_0^L \varphi^2 dx = \frac{1}{5}L^5 - \frac{1}{2}L^5 + \frac{1}{3}L^5$$

$$= \frac{1}{30}L^5$$

$$P_c = EI\frac{\frac{1}{3}L^3}{\frac{1}{30}L^5} = \frac{10EI}{L^2} \approx 1{,}02\, P_E$$

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Summary

- The Rayleigh quotient can be used to calculate the approximate buckling load of a general structure
- The Timoshenko quotient can be used to calculate the approximate buckling load of beams only
- For beams, the Timoshenko quotient is mostly more accurate than the Rayleigh quotient