AE4536: Buckling of structures

Stability of continuous systems

Roeland De Breuker

20/09/13

Learning goal

Learning how to assess equilibrium and stability of continuous systems.

Equilibrium

First form

$$P(\underline{u}_0+\underline{u}_1) = P_0'(\underline{u}_1) + \frac{1}{2!}P_0''(\underline{u}_1,\underline{u}_1) + h.o.t.$$

$$(\underline{u}_1,\underline{u}_1) = \{u_1^2,\ u_1'^2,\ u_1 u_1'\} \qquad \underline{u}_1 = \{u_1,\ u_1'\}$$

$$P(\underline{u}_0+\delta\underline{u}) = P_0'(\delta\underline{u}) + \cancel{h.o.t.}^{\,0} = 0$$

$$\underbrace{P_0'(\delta\underline{u})}_{\delta P} = 0 \longrightarrow$$

Euler-Lagrange equation

Stability

$$P_0''(\underline{u}_1, \underline{u}_1) \quad > 0 \quad \forall \underline{u}_1 \quad \text{Stable}$$

$$= 0 \quad \exists \underline{u}_c \quad \text{critical}$$

$$\begin{cases} P_0''(\underline{u}_1, \underline{u}_1) > 0 \\ P_0''(\underline{u}_c, \underline{u}_c) = 0 \end{cases} \qquad \begin{aligned} \delta \underline{u} &\neq \alpha \cdot \underline{u}_c \\ \underline{u}_1 &= \underline{u}_c + \delta \underline{u} \end{aligned}$$

Stability

$$P_0''(\underline{u}_c+\delta\underline{u},\underline{u}_c+\delta\underline{u})>0$$

$$=\underbrace{P_0''(\underline{u}_c,\underline{u}_c)}_{=0}+2\underbrace{P_0''(\underline{u}_c,\delta\underline{u})}_{=0}+\underbrace{P_0''(\delta\underline{u},\delta\underline{u})}_{>0}>0$$

$$P_0''(\underline{u}_c,\delta\underline{u})=0$$

Trefftz criterion

$$\delta(\delta^2P)\neq\delta^3P$$

First variation of second form

$$\delta(W_0u_1^2)=2W_0u_1\delta u_1$$

Eigenvalue problem

$$\lambda_c,\underline{u}_c\rightarrow P_0''(\underline{u}_c,\delta\underline{u};\lambda_c)=0$$

↑ load parameter

Stability of critical points

$$P(\underline{u}_c; \lambda_c) = \underbrace{\frac{1}{3!} P_0'''(\underline{u}_c, \underline{u}_c, \underline{u}_c)}_{=0} + \underbrace{\frac{1}{4!} P_0''''(\underline{u}_c, \underline{u}_c, \underline{u}_c, \underline{u}_c)}_{>0}$$

Necessary

"Sufficient"

Approach

- General rule is that the potential energy functional, expanded around an equilibrium state, should be positive definite
- Expand the potential energy functional around and equilibrium position and investigate the second and consecutive forms to assess stability of the system and stability of critical points
- The first variation of the second form of the potential energy functional provides a condition to obtain critical loads and modes
- Higher order forms decide on the stability of the critical loads in the direction of the critical modes