AE4536: Buckling of structures

Beam with flexible boundaries

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Governing flexible BC equation

$w' = -\frac{M_1}{k_{\theta_1}}$

$w' = \frac{M_2}{k_{\theta_2}}$

$w = 0$

$w = 0$

$w'''' + k^2 w'' = 0$

$w = A \sin kx + B \cos kx + Cx + D$

Flexible BC equation solution

① $w(0)=0 \qquad B+D=0$

② $w(L)=0 \qquad A\sin kL + B\cos kL + CL + D = 0$

③ $w'(0) = -\dfrac{M_1}{k_{\theta_1}} \qquad Ak + C = -\dfrac{M_1}{k_{\theta_1}}$

④ $w'(L) = \dfrac{M_2}{k_{\theta_2}} \qquad Ak\cos kL - Bk\sin kL + C = \dfrac{M_2}{k_{\theta_2}}$

$M = -EIw''$

$M_1 = EIBk^2$

$M_2 = EIk^2(A\sin kL + B\cos kL)$

$w'' = -Ak^2\sin kx - Bk^2\cos kx$

$w''(0) = -Bk^2$

$w''(L) = -Ak^2\sin kL - Bk^2\cos kL$

Flexible BC equation solution

① $B + D = 0$

② $A\sin kL + B\cos kL + CL + D = 0$

③ $Ak + C = \frac{-EIk^2}{k_{\theta_1}} B$

④ $Ak\cos kL - Bk\sin kL + C = \frac{EIk^2}{k_{\theta_2}}(A\sin kL + B\cos kL)$

$\phi = kL;\ \lambda_1 = \frac{EI}{k_{\theta_1} L};\ \lambda_2 = \frac{EI}{k_{\theta_2} L}$

③ ; ④ $\cancel{k}L$

Flexible BC equation solution

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 & 1 \\ \sin\phi & \cos\phi & L & 1 \\ \phi & \phi^2\lambda_1 & L & 0 \\ \phi\cos\phi-\phi^2\lambda_2\sin\phi & -\phi\sin\phi-\phi^2\lambda_2\cos\phi & L & 0 \end{bmatrix}}_{\det = 0}\begin{Bmatrix} A \\ B \\ C \\ D \end{Bmatrix} = \mathbf{0}$$

$$\left(1-\lambda_1-\lambda_2-\lambda_1\lambda_2\phi^2\right)\phi\sin\phi+\left(2+\lambda_1\phi^2+\lambda_2\phi^2\right)\cos\phi-2=0$$

Special cases

$\underline{\lambda = \lambda_1 = \lambda_2}$ $\quad \underbrace{\left(\tan\left(\frac{\phi}{2}\right) + \lambda\phi\right)}_{=0} \underbrace{\left[\left(2+\lambda\phi^2\right)\tan\left(\frac{\phi}{2}\right) - \phi\right]}_{=0} = 0$

$$\phi = kL \,, \quad k^2 = \frac{P_{cr}}{EI} \rightarrow \frac{\phi^2}{L^2} EI = P_{cr}$$

$$\tan\left(\frac{\phi}{2}\right) + \lambda\phi = 0 \rightarrow \tan\left(\frac{\phi}{2}\right) = -2\lambda\left(\frac{\phi}{2}\right)$$

Special cases

SS $\quad k_{\theta_1} = k_{\theta_2} = 0 \rightarrow \lambda = \infty$

$\tan\left(\frac{\phi}{2}\right) = \infty \rightarrow \frac{\phi}{2} = \frac{\pi}{2} \rightarrow P_{cr} = \frac{\pi^2 EI}{L^2}$

CC $\quad k_{\theta_1} = k_{\theta_2} = \infty \rightarrow \lambda = 0$

$\tan\left(\frac{\phi}{2}\right) = 0 \rightarrow \frac{\phi}{2} = \pi \rightarrow P_{cr} = 4\frac{\pi^2 EI}{L^2}$

Special cases

CS $\quad k_{\theta_1} = \infty \quad \rightarrow \lambda_1 = 0$

$\quad k_{\theta_2} = 0 \quad \rightarrow \lambda_2 - \infty$

$$(1-\lambda_2)\phi \sin\phi + (2+\lambda_2\phi^2)\cos\phi - 2 = 0 \quad \Big| \; \lambda_1 = 0$$

$$\left(\frac{1}{\lambda_2} - 1\right)\phi \sin\phi + \left(\frac{2}{\lambda_2} + \phi^2\right)\cos\phi - \frac{2}{\lambda_2} = 0 \quad \Big| \; 1/\lambda_2$$

$$-\phi\sin\phi + \phi^2\cos\phi = 0 \rightarrow \phi = 0 \vee \tan\phi = \phi$$

$$P_{cr} = 2.04\, P_E$$

Summary

- An equation was derived for a beam under axial load with flexible boundary conditions
- The equation can be used to calculate the critical load of the beam
- When the spring stiffnesses are known, the equation can be solved for Φ
- The solution for Φ can be used to calculate kL, and furthermore P_{cr}