AE4536: Buckling of structures

Stability of frames

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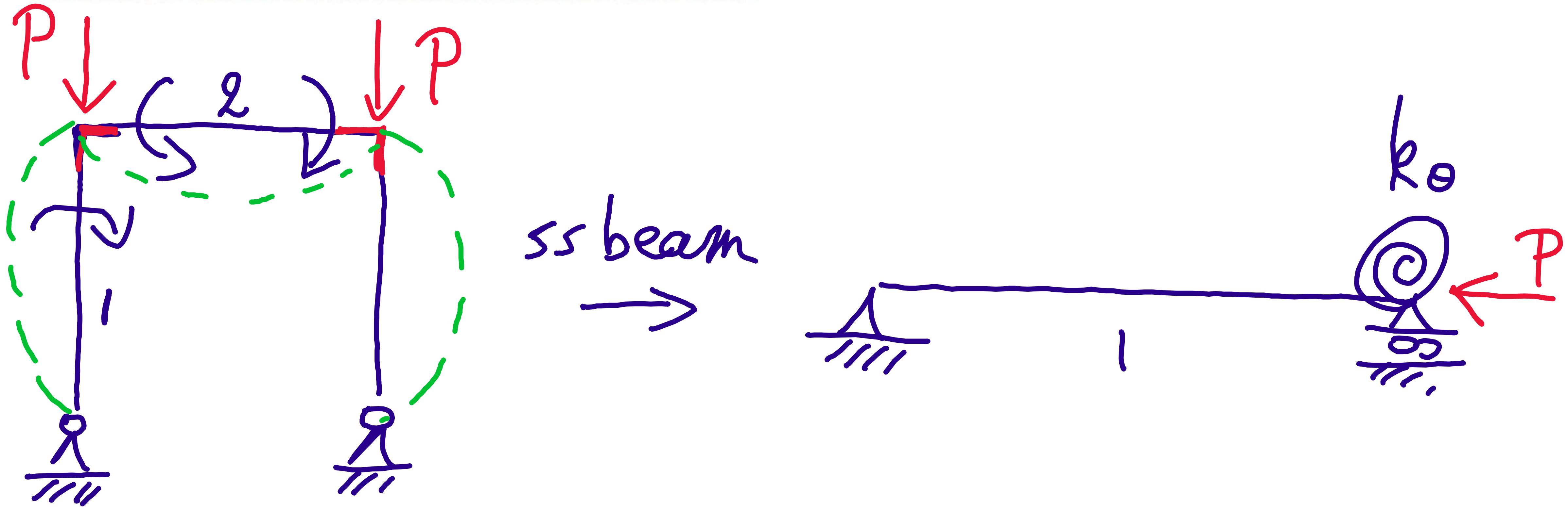
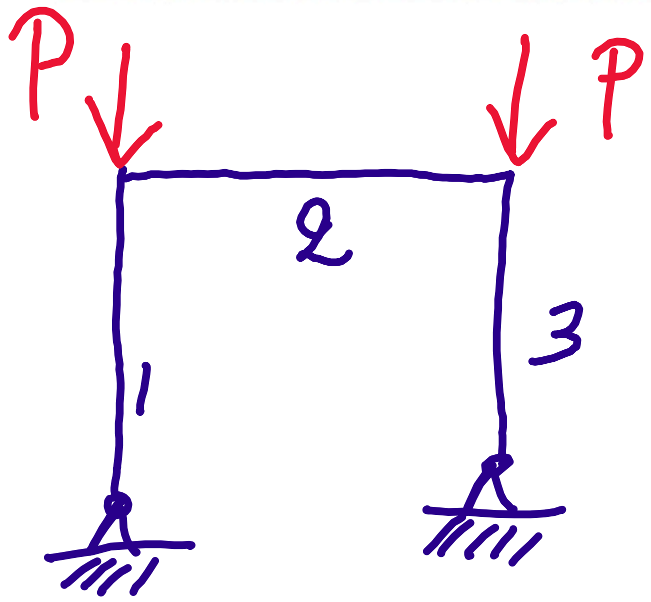
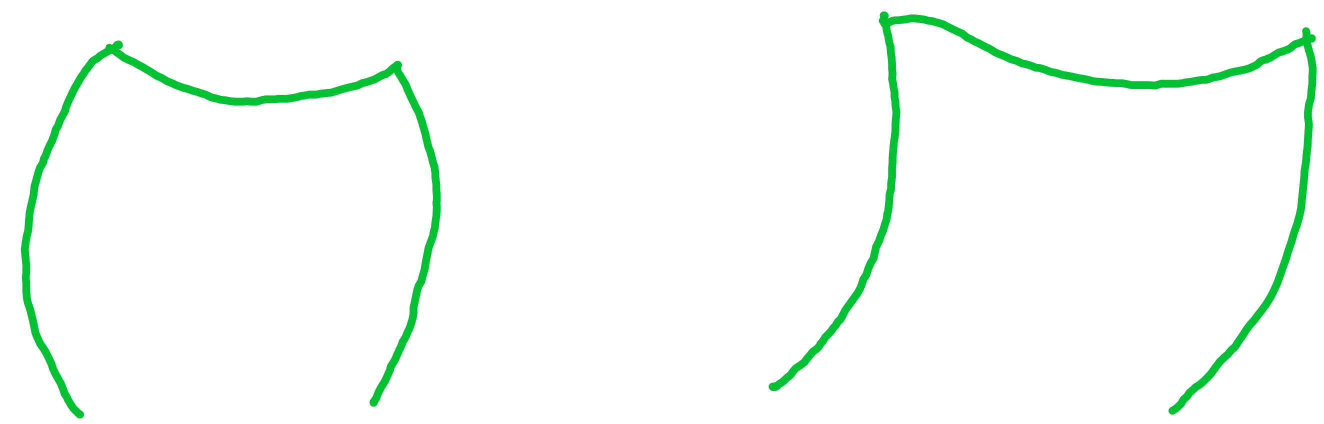
Challenge the future

What is a frame?

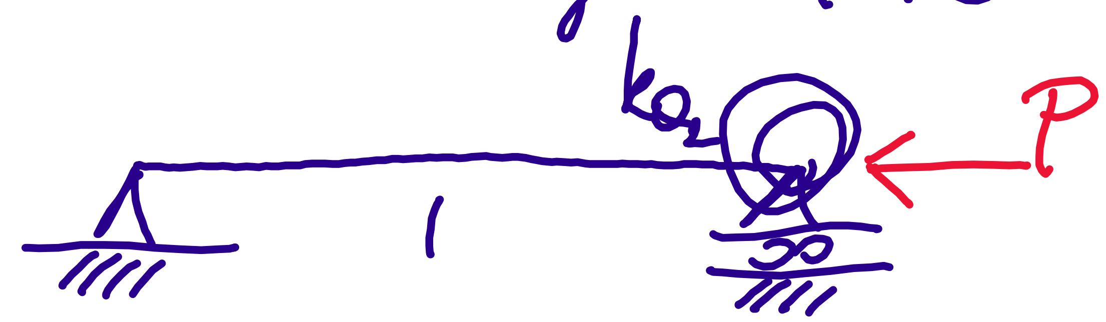


Frame buckling example





1st symmetric 1st asymmetric



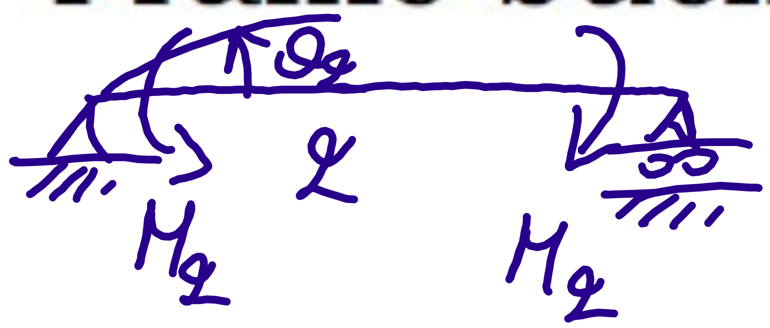
<u>Mode</u>

$$k_{\theta_1}=0 \rightarrow \lambda_1=\infty$$

$$k_{\theta_2}=? \rightarrow \lambda_2=?$$

$$(1+\lambda_2\phi^2)\sin\phi=\phi\cos\phi \rightarrow \tan\phi=\frac{\phi}{1+\lambda_2\phi^2}$$

Frame buckling example



$$\theta_2 = \frac{M_2 L}{2EI} \qquad \text{integrate } EIw'''' = 0$$

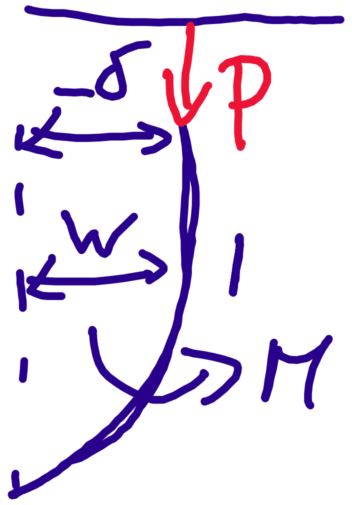
$$\rightarrow k_{\theta_2} = \frac{2EI}{L} \rightarrow \lambda_2 = \frac{1}{2}$$

$$\rightarrow \tan\phi = \frac{\phi}{1+\frac{1}{2}\phi^2} \rightarrow \phi = 3.59$$

$$P_{cr} = 1.302\, P_E$$

Frame buckling example

Mode 2



$$M = Pw = -EIw''$$

$$w'' + k^2 w = \frac{M}{EI}$$

$$w(0) = 0$$

$$w(L) = -\delta$$

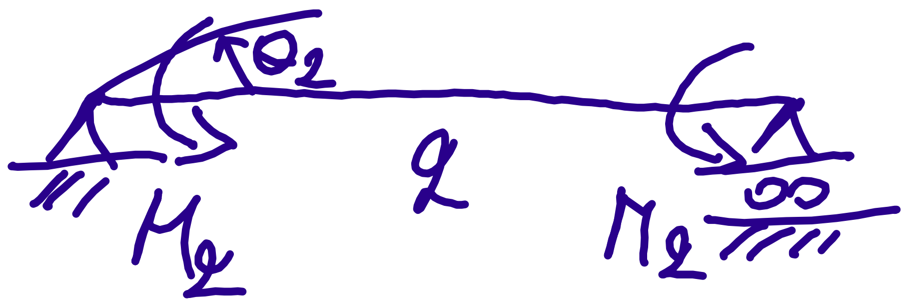
$$w = -\frac{\delta}{\sin kL}\sin kx \qquad w' = -\frac{\delta k}{\sin kL}\cos kx$$

$$w'(L) = -\frac{\delta k}{\tan kL}$$

$$w'(L) = \frac{k}{\tan kL}\,\frac{M(L)}{EIk^2}$$

$$M(L) = -EIw''(L) = -EI\delta k^2 \rightarrow k_\theta = EIk\tan kL$$

Frame buckling example



$$\theta_2 = \frac{M_2 L}{6EI} \rightarrow k_\theta = \frac{6EI}{L}$$

$$\rightarrow \frac{6EI}{L} = EI\,k \tan kL \rightarrow \phi = 1.35$$

$$P_{cr} = 0.184\, P_E$$

Sidesway

Summary

- The buckling load of a frame is dependent on the buckling mode. First determine the buckling modes of interest and back-calculate the corresponding buckling loads
- Buckling of frames can be reduced to the buckling of individual members with flexible boundary conditions
- In order to determine the effect of a member on another member, an equivalent spring stiffness needs to be calculated
- The equivalent spring stiffness can be calculated by relating the rotation of the intersection point between the two beams of interest to the applied moment