Chapter 10.
Games Against Nature

- Decision Making Under Risk
- Criteria for Decision Making Under Uncertainty
- Axioms for Selection of Criteria

Decision-Making Under Risk
Decision-Making Under Uncertainty

- There is an important distinction between decision-making under risk and uncertainty
- The distinction dates at least to the 1950s from early systems analysts
- Risk means we can attach quantitative measures of likelihood to possible outcomes.
- The principal of expected utility applies
- Uncertainty means we cannot necessarily determine the likelihood of possible outcomes.

Laplace's Criteria

- The principal of insufficient reason
- If we are under complete uncertainty, then all outcomes are equally likely
- In practice, sum across the states of nature and choose the alternative providing maximum payoff
- Useful for those who can enumerate all possibilities and are unlikely to be surprised by new outcomes

Wald's Criteria

- Wald was a pessimist
- Choose the strategy which minimizes the worst outcome
- In practice you should calculate the minimum entry in each row, and choose the row with the largest minimum
- Useful criteria for those in charge of health, security or safety
- Wald's perspective also relates to a game-playing strategy

Hurwicz Criteria

- If Wald has the pessimists strategy, then what is the optimists strategy?
- For each row find the maximum possible outcome
- Across rows select the best of the best strategies
- This strategy would be good for those exposed to large upsides, such as those working with intellectual property or social networks
- Hurwicz suggests mixing the optimist and pessimists (Wald's) strategies with a coefficient of optimism (α)

Example Regret Calculations

		Nature			
		A	B	C	D
You	A	2	2	0	1
	B	1	1	1	1
	C	0	4	0	0
	D	1	3	0	0
	max	2	4	1	1

Game Against Nature

subtract from the best possible payoffs for each column the actual payoffs for each strategy

		Nature			
		A	B	C	D
You	A	0	2	1	0
	B	1	3	0	0
	C	2	0	1	1
	D	1	1	1	1

Game Transformed to Regret Matrix

Tables from Game Theory and Strategy (Straffin 1993) p.56, 58

Savage's Criteria

- Make decisions you will later not regret
- Choose the row with the minimum possible regret
- Savage's criteria is good for politically elected decision-makers who may be asked to justify their decisions after the fact
- Used a lot by RAND policy analysts in the form of exploratory models

Axiomatic Method

- How do we select from our available criteria?
- Might we even invent new criteria?
- The axiomatic method sets logical requirements for possible criteria
- The axiomatic method is used to defend utility, and we will see it again later in the course

Axioms All Criteria Obey

- **Axiom 1 (Symmetry)** – It should not matter in which order we present the rows or columns
- **Axiom 2 (Strong Domination)** – If every entry in row X is larger than the corresponding entry in row Y, a method should not recommend strategy Y.
- **Axiom 3 (Linearity)** – The recommended strategy should not change if all entries are multiplied by a positive constant or a constant is added to all entries.
- **Axiom 4 (Column Duplication) -** The recommended strategy should not change if we add to the matrix a new column which is a duplicate of a column already in the matrix.

Axiom 5 (Bonus Invariance)

- The recommended strategy should not change if a constant is added to every entry in a column
- This criteria says that if Nature should give you a bonus, or exact a penalty, it should not change your choice
- In other words, you should maximize your own choices regardless of what nature does
- Both the Wald and Hurwicz criteria violate this axiom

Axiom 6: Row Adjunction

- Suppose a method chooses a row X as the best strategy to follow in a game against nature, and then a new row Z is added to the matrix.
- The method can choose Z, but otherwise cannot choose any other strategy than X.
- The idea here is that if X is really the best strategy available, new strategies should not have affected your decision.
- Savage's criteria violates this method

Conclusions

- All criteria for decision-making have their failings.
- For our domain, it seems to me that Axiom 6 is particularly important, Axiom 5 less so, and Axiom 4 not at all.
- Therefore I favor Laplace's criteria. Hurwicz's criteria is also good since it encompasses both optimistic and pessimistic strategies.