

# Chapter 11.

## Nash Equilibria and Non-Cooperative Solutions

- Competition and Cooperation
- Payoff Polygons
- Pareto Optimality

# Saddle Points and Nash

- All the techniques you have learned from zero sum game theory apply to non-zero sum game theory
- Saddle points become Nash equilibrium.
- Most people call both saddle point and Nash by the name of Nash equilibrium.

# What is Cooperative Game Theory?

- Your textbook uses an older taxonomy of games involving numbers of players and conflicting objectives.
- N-Person games
- Newer ideas endorse the difference between cooperative and non-cooperative games.
- Cooperative games are a “reduced form” of problem focusing on coalitions, payoffs, agreed axioms for settlement.

# Cooperative vs Noncooperative Games

	<u>Cooperative Game Theory</u>	<u>Competitive Game Theory</u>
Concepts	Uses concepts of fairness	Uses concepts of equilibrium
Concerns	Concerns outcomes	Concerns process
Commitment	Binding commitments possible based on fairness or suasion	Binding commitments possible based upon the design of the game
Conflict	May or may not involve conflict	May or may not involve conflict
Formation	Strategically Ill-formed. What game should we be playing?	Strategically well-formed. How do we play this game to advantage?
Mediation	Mediate through play	Mediate by changing the game

# Examples of Conflict and Cooperation

- *A cooperative game without conflict.* Members of a workforce choose which of equally arduous tasks to undertake to best coordinate with each other.
- *A cooperative game with conflict.* Bargaining over price between a monopolist and a monopsonist.
- *A noncooperative game with conflict.* The prisoner's dilemma.
- *A noncooperative game without conflict.* Two companies set a product standard without communication.

Rasmussen (2007)

# Steps in Constructing a Pay-Off Polygon

- Step 1. Plot all pay-offs in a 2D space of row and column.
- Step 2. Fill in the polygon.
  - This involves imagining all possible mixed strategies and filling in the polygon, a convex hull.
  - Some strategies will be inside the hull
- Step 3. Circle the equilibrium outcome or outcomes
- Step 4. Identify Pareto optimal strategies.

# Pareto Optimality

- Pareto described optimal solutions as outcomes which make all players better off, without making any player worse off.
- This allows a definition of optimal which is consistent with our capabilities of measuring utility.
- It can be difficult sometimes to imagine a series of unilateral changes which make everyone better off.
- The chief difficulty is that you must have a reference solution in mind before you can evaluate Pareto optimality.

# Nash and Pareto Optimality

- Nash equilibria are not necessarily Pareto optimal
- A very few game theorists do not like Nash equilibria because it recommends an inadequate solution
- Others argue that with non-zero sum games we must make the best we can of an unfair situation



# Identifying Pareto Optimal Strategies

- Advance along the polygon until you can do no better for Colin.
- Note this strategy.
- Advance along the polygon until you can do no better for Rose.
- Note this strategy.
- Connect the two best strategies with a dotted line.
- There may be intermediate strategies. These strategies will generally be north-easterly.

# Multiple Equilibria

- Some game theorists dislike Nash equilibria because there are multiple equilibria in games
- Multiple equilibria become a threat because these analysts are seeking unique recommendations
- Others argue that there are multiple equilibria in our problem settings, so why should there not be multiple equilibria in our games?

# Evaluating Nash Equilibria

- Broadly and widely accepted as the leading solution concept
- Dilemmas are to be blamed, not an inadequate solution concept
- Many have extended the game concept, and extensions of Nash are thereby naturally suggested
  - Trembling hand
  - Two-step deviations

# Nash and Rationality

- Game theorists are still a little dissatisfied with the computability and rationality of Nash equilibrium
- It seems futile to attempt to study irrationality
- Cognitive psychology is a useful and complementary endeavor; useful “hacks” for acting rational without effort
- The chief difficulty stems from the combinatorics of adding more players and strategies to games.
- Perhaps the game representation or conceptualization needs to be modified, not Nash?