

Chapter 16.

The Nash Arbitration Scheme and Cooperative Solutions

- Criteria for a Cooperative Solution
- Fallback Options
- Procedures for Maximizing Joint Utility

The Problem

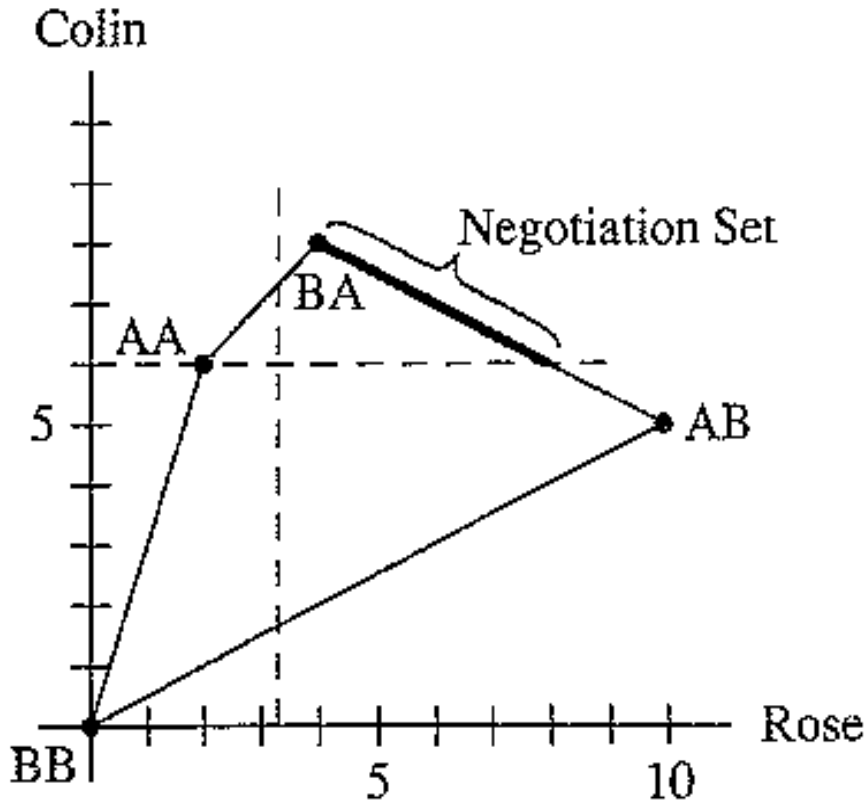


Diagram from Game Theory and Strategy (Straffin 1993) p.103

- Players in a game achieve outcomes which may be far from Pareto optimal
- Suppose the players were to communicate with one another, and further were to seek outside arbitration
- Can we as policy analysts recommend a solution which is fair to all and which restores optimal outcomes?
- Negotiation set: Pareto optimal, securing both players at least a minimum value for agreeing to arbitration

Axioms and the Axiomatic Method

- John Nash proposed that a solution to this problem of arbitration should satisfy certain requirements or axioms
 - **Axiom 1.** Rationality
 - **Axiom 2.** Linear Invariance
 - **Axiom 3.** Symmetry
 - **Axiom 4.** Independence of Irrelevant Alternatives
- Nash in 1956 showed that his solution is the only one which is possible and satisfies the stated requirements

Independence of Irrelevant Alternatives

- Fair outcomes remain fair even as we close in on a solution
- So for instance,
 - Suppose Rose and Colin are agreeing on how to share rooms in a rented house
 - They agree that it is fair that the kitchen becomes a shared space
 - Then in arbitration Rose is assigned the large bedroom
 - Under these circumstances, Colin cannot go back and later say it is unfair to have shared the kitchen

Nash Arbitration

- Also known as Nash bargaining
- Suppose that there is a status quo point (x_0, y_0)
- Nash proves that the only arbitration which satisfies his definitions of fairness uses the following procedure

$\max(U)$ where $U = (x-x_0)(y-y_0)$

Setting the Status Quo

- How to set it?
- **Nash** argues that it should be based upon threats from the associated non-cooperative games
- **Shapley** argues that it should be based on the mixed strategy equilibrium of the associated non-cooperative game
- **Muthoo** argues that it should be based on the fallback options of the players – the best alternative to negotiated agreement (BATNA)

Real World Example of Escalation Prior to Bargaining



- November 2009 North Korean military vessel ventures into South Korean waters and is then fired upon
- Periodic deadly conflicts 1999, 2002
- Why the conflict?
- Said to be part of a cat-and-mouse game of international negotiation as North Korea seeks concessions in direct and six-party talks

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7

Image adapted from picture by Mass Communication Specialist 1st Class Darryl Wood, released in public domain (usgov).

Maximization Procedure

1. **Parameterize** the negotiation set in terms of y , using one or more lines bounded by the domain
2. **Subtract** the status quo for y from the equation
3. **Multiply** by the quantity x minus the status quo x
4. **Differentiate** this quadratic with respect to x , set to zero, and solve for x
5. **Solve** for the optimum using x and the boundaries
6. **Verify** that the negotiated solution lies on the negotiation set
7. **Calculate** the highest possible value while remaining within the negotiation set if the solution is outside
8. **Repeat** for all piece-wise components of the negotiation set, and seek the compromise point

Example Maximization

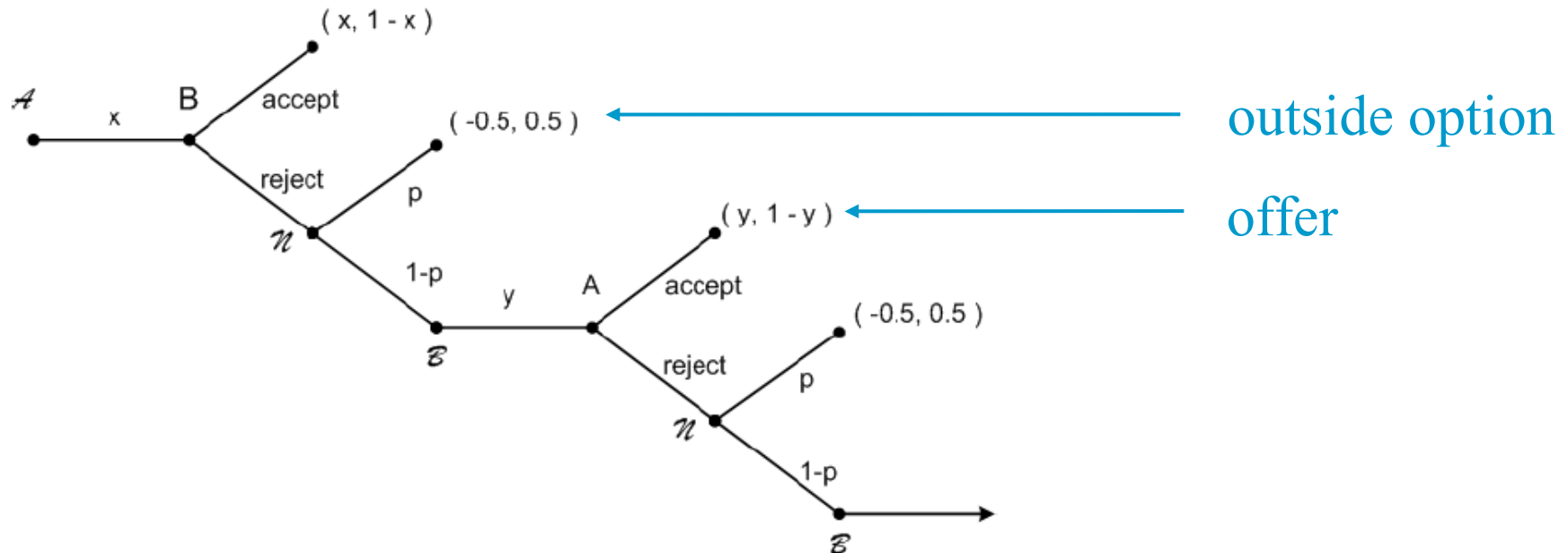
- Suppose the negotiation set lies along
 $y=10 - x$, such that $2 \leq x \leq 7$
- Suppose that the status quo is set to $(0,0)$
- Nash's quantity is
 $U=(10-x-0)(x-0) = 10x-x^2$
- Maximizing U and solving
- $dU/dx = 10-2x = 0$
 $x = 5, y=5$
- The negotiation solution does lie within the negotiation set

More Complex Example

- Suppose the negotiation set lies along
 $y_1 = 10 - 2x$, such that $5 \leq x \leq 9$
 $y_2 = 10 - 0.5x$, such that $2 \leq x \leq 5$
- Suppose that the status quo is $(0,0)$
 $U_1 = 10x - 2x^2$, $DU_1/dx = 10 - 4x$, $(2.5, 5)$, out of bounds
 $U_2 = 10x - 0.5x^2$, $DU_2/dx = 10 - x$, $(10, 1)$, out of bounds
- However we know that $(5, 5)$, which is in both sets, is an available solution which maximizes Nash's product.

Rubinstein Bargaining

- A further reason why many people find Nash arbitration compelling is that we know how it should be implemented



Things We've Learned About Bargaining Processes

- Patient bargainers do better
- Risk propense bargainers do better
- First movers do better if it's a one-shot game
- Last movers do better if it's an n-shot game
- Bargainers with inside options do better
- Bargainers with outside options do better
- Information symmetry makes things better for all
- The process affects the outcome

Adapted from Gintis (2000)