

Chapter 18. Applications to Economics: The Duopoly Problem

- Structure of the Problem
- Game Setting
- Discussion

Statement of the Problem

- How should two firms coordinate to set prices and production?
- What are the consequences of such behavior for consumer welfare and for the firms themselves?

Review of Economic Terms

- **Monopoly:** A single firm free to set prices in the market to maximize profit
- **Duopoly:** Two firms, whose choices of pricing and product affect each others' profit. Oligopoly, a small number of firms.
- **Perfect Competition:** Firms which price at marginal cost, strategically induced by a multitude of competitors. This is known in your text as the efficient point, because it maximizes social welfare.

Quantities of Interest

- **q**: number of items (in thousands) to be produced
 - **P**: price at which the product is to be sold
 - **AC**: average production cost
 - **TC**: total cost, or quantity times the average $q \cdot AC$
 - **MC**: marginal cost, cost for slightly increasing product, or $d(TC)/dq$
 - **R**: revenue, $p \cdot q$
 - **P**: profit, or revenues minus total cost
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- These quantities may vary by company, and are therefore indexed by company i

Demand Functions

- Function relating price to quantity produced
- Linear model
- Inverse demand, relates quantity produced to appropriate price-point

$$p=160-8(q_1+q_2)$$

- Linear models are not very realistic but are useful for modeling purposes
- For instance, they can result in negative prices
- More elaborate functions entail elasticities – double the price, halve the demand

Cost Functions

- Describe the costs of producing goods
- Twice differentiable
 - First differential, increasing marginal costs
 - Second differential, optimum scale
- Parabolic curve useful for qualitative models capturing these essential features of economic operation

Game Situation

- We are in a game situation. This is because
- Total demand is a joint function of production
 $p=160-8(q_1+q_2)$
- Price enters profit calculations
Profit = [quantity][price]-[quantity][average cost]
 $P=q_1(160-8q_1-8q_2)-q_1(64-4q_1+4q_1^2)$
- Profit-maximizing firm encounters an interaction term
Interaction: $-8q_1q_2$
- This is a non-zero sum game situation

Nash Arbitration in Price Setting

- The book examines what would happen if firms cooperated in price setting
- Nash arbitration is the chosen settlement procedure
- Nash threat points are used as the status quo
- Optimum selection of threats itself a strategic game
- Book gives threat point, arbitrated settlement point, and slope of the negotiation set
- In exercise use the arbitrated settlement point and the slope to parameterize the negotiation set

Thoughts on the Homework Exercises

- Excel very helpful here in confirming answers
- Some basic differential calculus needed here
- Note that these are full differentials here since companies are making quantity decisions independent of the price decisions of the other firm
- You may need the quadratic equation to solve for the roots of a parabolic equation
- You can also use Solver for instance to minimize average costs subject to quantity produced

Order of Play

- **Bertrand:** Firms simultaneously and independently choose the price
- **Cournot:** Firms simultaneously and independently choose the quantity
- **Stackleberg:** Leader firm chooses quantity. Follower firm chooses quantity, knowing the choices of the leader

Bertrand's Paradox

- Coordinating on price results in firms which attempt to undercut each other, even if there are few competing firms
- Even small reductions in price result in complete capture of the marketplace
- Ultimately firms go down to their marginal costs – these are the same outcomes as a full and competitive market
- Perhaps Bertrand's model is unrealistic since there are non-price related aspects of demand, such as differentiation by quality or location

Policy Conclusions

- Consumer surplus stems from purchasing goods at a price lower than they would willingly pay
- Limited numbers of sellers can collude on price
- Thus, setting price high harms consumer welfare
- Cooperation or arbitration in a duopoly setting is called “collusion” and often prohibited by law