Chapter 19. An Introduction to N-Person Games

- Representing Three Person Games
- Prudential Strategies and Security Levels
- Questions and Assumptions of N-Person Games
- Coalitions and Characteristic Functions

Questions of Cooperative Game Theory

- Which coalition or coalitions should form?
- How should a coalition which forms divide its winnings among its members?

- Von Neumann and Morgenstern argued that the grand coalition (coalition of all coalitions) should always form, and therefore focused on dividing the winnings
- We can reasonably question their assumptions for policy analysis

Normal Form for Three Players

Third player Larry, two separate games based on Larry's choice

Larry A

Rose \ Colin	A	B
A	(1, 1, −2)	(−4, 3, 1)
B	(2, −4, 2)	(−5, −5, 10)

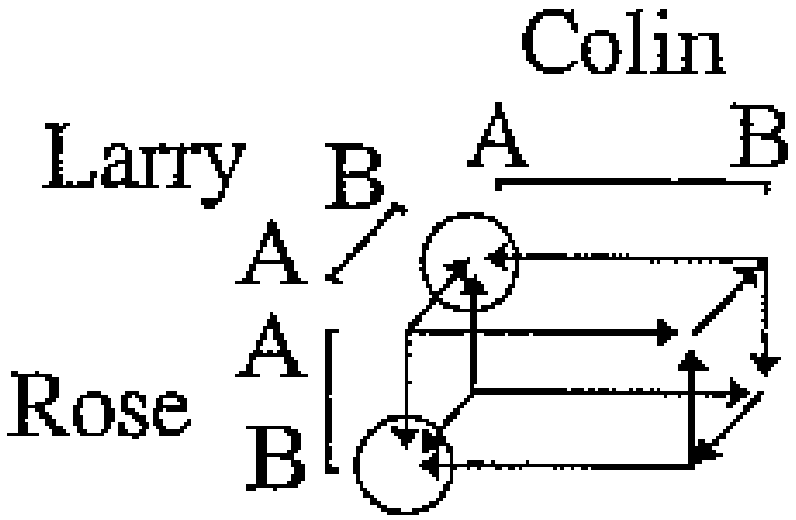
Larry B

Rose \ Colin	A′	B
A	(3, −2, −1)	(−6, −6, 12)
B	(2, 2, −4)	(−2, 3, −1)

Payoffs (Rose, Colin, Larry)

Tables from Game Theory and Strategy (Straffin 1993) p.127

Movement Diagram in Three Dimensions

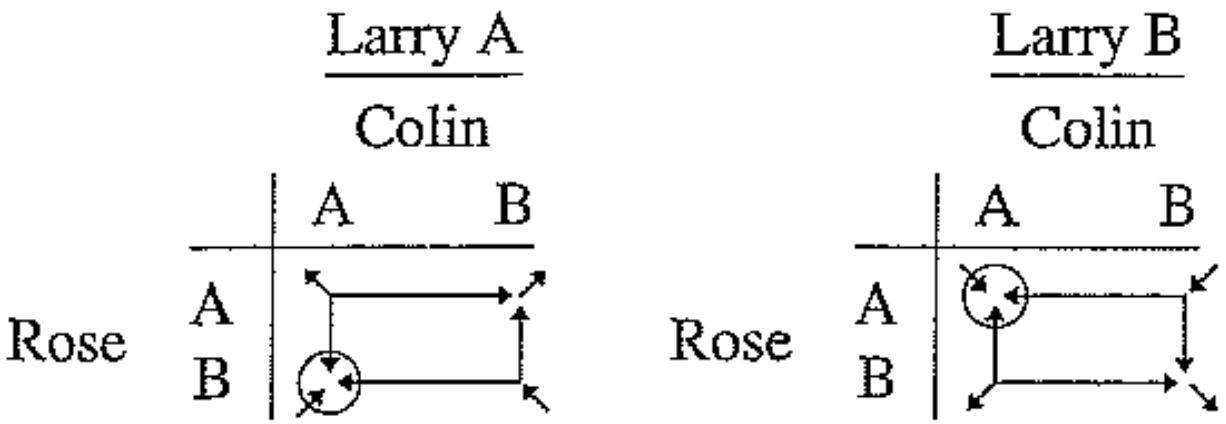


Easy to visualize, but hard to draw well. So we do it in parts.

Diagram from Game Theory and Strategy (Straffin 1993) p.127

Movement Diagram in Easy Parts

The in-flows and out-flows represent Larry's choices.



Payoffs (Rose, Colin, Larry)

Diagram from Game Theory and Strategy (Straffin 1993) p.127

Prudential Strategies

- First discussed in Chapter 11
- Involves cautious play
- Extension to N-Player
  - Assume the worst: Other players have ganged up on you to secure themselves the best possible outcomes
  - You counter by a mixed strategy which secures you at least a minimum payment
  - This is known as your security level

Counter-Prudential Play

- Suppose Rose were known to play using prudential strategies
- How would Colin and Larry respond?
- This is the counter-prudential strategy.
- Prudential play is not the best response to counter-prudential play
- In general prudential and counter-prudential strategies are out of equilibrium

Assumptions of N-Person Games

- Also known as *coalition games* or more generally *cooperative game theory*
- Players can communicate and form coalitions with other players
- They can't do this unless explicitly stated in non-cooperative games!
- The value of the game changes according to coalition structures
- Players can make sidepayments to other players – we assume utility is transferrable

Sidepayments

- A significant assumption used in forthcoming chapters
- Recall how we said that interpersonal comparisons of utility were, in general, not possible?
- Perhaps in some games there are transferrable units of utility such as currency
- Indeed some public administrations and political scientists suggest that decision-makers swap issues and legislative solutions creating a sort of currency
- Aumann (1967) offers a general theory of N-person games without sidepayments

Characteristic Function

- The characteristic function enumerates possible coalitions and their value
- For each coalition you can calculate the marginal value created or destroyed when forming

$$v(\phi) = 0$$

$$v(\mathrm{R}) = -4.4 \qquad v(\mathrm{C}) = -4 \qquad v(\mathrm{L}) = -1.43$$

$$v(\mathrm{CL}) = 4.4 \qquad v(\mathrm{RL}) = 4 \qquad v(\mathrm{RC}) = 1.43$$

$$v(\mathrm{RCL}) = 0$$

- What happens if players refuse to play the game?
- ϕ Is the null set. By convention we set the value of the null set to zero.

A Reduced Form?

- You can envisage a process of group formation leading to coalition values, but you need not
- Is the characteristic function a reduced form of the non-cooperative game?
- Yes. The non-cooperative game provides a useful justification for the values listed in the function.
- No. Shared values in groups are in themselves a fundamental construct of interest.