Chapter 23.
Imputations, Domination, and Stable Sets

- Imputation
- Domination
- Stable Sets

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Characteristic Function

- The characteristic function enumerates possible coalitions
- For each coalition, it lists the value created or destroyed

$$\nu(\phi) = 0$$

$$\nu(R) = -4.4 \qquad \nu(C) = -4 \qquad \nu(L) = -1.43$$

$$\nu(CL) = 4.4 \qquad \nu(RL) = 4 \qquad \nu(RC) = 1.43$$

$$\nu(RCL) = 0$$

- What happens of the players refuse to play the game?
- ϕ is the null set. By convention the value of the null set is zero.

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Viable Alternative Structure for Representation

$$\nu(\phi) = 0$$

$$\nu(R|C,L) = \nu(C|R,L) = \nu(R|C,L) = -1$$

$$\nu(R|C \cup L) = \nu(C|R \cup L) = \nu(R|C \cup L) = 2$$

$$\nu(C \cup L|R) = \nu(R \cup L|C) = \nu(C \cup L|R) = 0$$

$$\nu(R,C,L) = 3$$

- Characteristic function with partitions
- Prisoner’s dilemma
- Payoff according to the coalition structure of the other players

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Which Representation is More True?

Normal or Extensive Form

- Shows the process by which outcomes are reached
- May be needed to fully understand the valuation of the game
- Payoffs may be dependent on those not in your coalition
- Endorsed by Straffin

Characteristic Function

- Based on payoffs, which are more fundamental than strategies
- The game may not yet be clear to all players
- Choice of representation may depend upon questions
- Characteristic functions may be useful for encouraging cooperation

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What is an Imputation?

- An imputation is a proposed division of resources
- An n-tuple of numbers (for n players) describing how much is awarded to the nth player
- Based on a solution concept and also a characteristic function of a game
- The solution concept we are discussing in this chapter is the von Neumann and Morgenstern *stable set*
- Also known as the *solution*

What is Domination?

In analytical form: An imputation $x = \{x_1, ..., x_n\}$ dominates an imputation $y = \{y_1, ..., y_n\}$ if there is some coalition S such that:

i) $x_i > y_i$ for all i in S, and

ii) $\sum_{i \in S} x_i \leqslant v(S)$

- **In words:** One imputation dominates the other if all entries are greater, and if the imputation delivers less than or equal to the value of the coalition according to the specified game

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Necessary Components of Domination

- requires two sets, one dominated and one dominating
- is dependent on a specific characteristic function
- requires we specify a coalition for whom the imputation dominates

Strange Properties of Domination

- Given two imputations, its possible that neither dominates the other
- Cycles of domination are possible. For individual decision-making we wouldn’t like this, but of course domination refers to group decision-making.
- It's possible that every imputation is dominated by another.

Requirements for a Solution to Cooperative Game

- Von Neumann-Morgenstern stable set or solution
- Multiple alternative arrangements should be possible
- Logically distinct ways of solving the problem should be permitted
- All solutions entertained should be equally good
- Any solution entertained should be better than those rejected

Issues with the Stable Set

- Every imputation possible may be in one of the available stable sets;
- This makes it hard to prove that the stable set has predictive validity
- There may be no stable set possible
- It may be impossible to calculate the stable set
- Non-zero sum games result in some clearly better solutions; these should be studied

Stable Set

- **In analytical form:** A *stable set* (sometimes called a *von Neumann-Morgenstern solution*) for a game G is a set I of imputations such that:

  i) I is internally stable: no imputation in I is dominated by any other imputation in I, and

  ii) I is *externally stable*: every imputation not in I is dominated by some imputation in I

- **In words:** To be considered in the stable set, we shouldn't be able to either add or remove elements from the set because they are dominated

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