

Chapter 27. Application to Politics: The Shapley-Shubik Power Index

- Problem and Definition
- Sample Calculations
- Case involving the European Parliament

Problem

- Understanding voting power is difficult
- Certainly power is not proportional to vote
- There are voting schemes where despite having a vote some players have no power
- And what about the ability to veto?
- Shapley and Shubik suggested applying the Shapley Value to the analysis of these situations.

History of Power Indices

- Von Neumann and Morgenstern used the stable set to create an early *power index*, an “economic vote-selling model, in which equilibrium prices describe the share of spoils that each player might expect to receive if he ends up on the winning side.”
- Shapley and Shubik extended the Shapley value for this purpose
- Coleman and Banzhaf later created their own measures
- Transactional models of political systems developed by Coleman and applied extensively by Stokman

(Dubey and Shapley, 1978)

Definitions

- Voting games: a coalition game where by one coalition satisfying majority rules gets one point; all others get zero points
- Weighted voting games: a voting game whereby certain players get more votes
- Proper games: games where two disjoint winning coalitions are not permitted
- Strong games: games which cannot be deadlocked

Swing Voters

- Shapley Value applied to this context means that
 - $\phi = 0$, a dummy player
 - $\phi = 1$, a dictator player
- Further, since we are studying voting games we
 - Specify a player i
 - Sum across winning coalitions S which
 - would then be losing if player i deserted
 - Divide through by all possible coalitions

Pivoting Discussion

- Today the UN Security Council contains 15 members, 5P and 10 N
 - P Permanent member of UN Security Council
 - N Non-permanent member
- Passing a resolution requires 9 votes, of which all 5 P must approve
 - (PPPPPNNN)N(NNNNNN)
 - Parentheses imply any arrangement of possible letters is allowed
 - Underline indicates that the player is the swing or pivotal vote

UNSC 2004 Proposed Reform

- Add five new permanent members to the council
- Can we design a voting procedure so that
 - The power of non-resident members is not reduced
 - There are fewer than 192 members
 - No greater consensus than two-thirds is needed
 - Permanent members retain a veto
- Experiments with Shapley-Shubik suggest no!
- A weighted voting game, or a veto-breaking procedure may be necessary

Vote Apportionment and the European Parliament

- In 2009 the European Parliament adopts a new mechanism for apportioning seats
- Is this a fair mechanism?
- How fair is this new system for the Netherlands?
- Similar discussion now occurring in U.S. House of Representatives



Picture by Alina Zienowicz Ala z (license: cc-by-sa-2.0)

Absolute and Relative Numbers of Votes in the Parliament

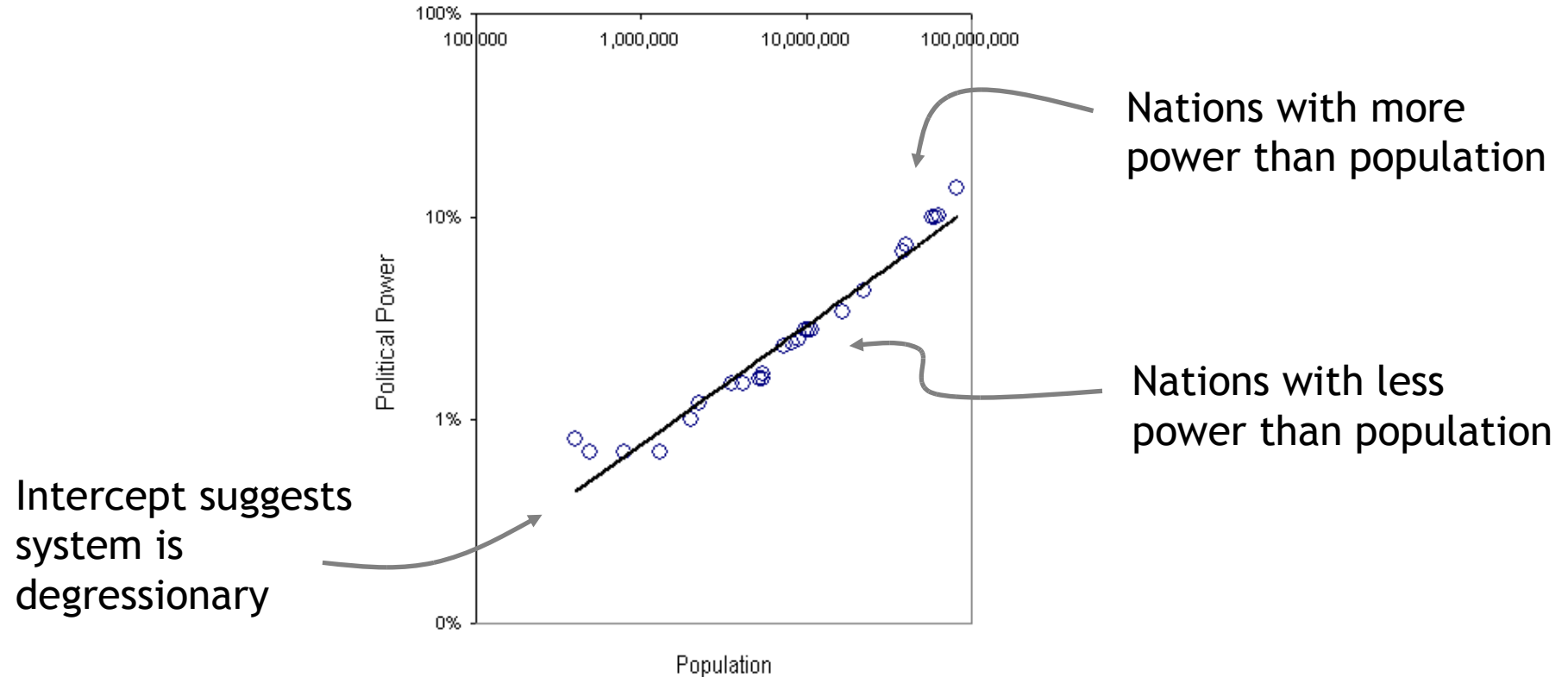
Nation	Absolute Number of Votes	Relative Number of Votes
Austria	19	2.5%
Belgium	22	2.9%
Bulgaria	18	2.4%
Cyprus	6	0.8%
Czech Republic	22	2.9%
Denmark	13	1.7%
Estonia	6	0.8%
Finland	13	1.7%
France	74	9.9%
Germany	96	12.8%
Greece	22	2.9%
Hungary	22	2.9%
Ireland	12	1.6%

Italy	73	9.7%
Latvia	9	1.2%
Lithuania	12	1.6%
Luxembourg	6	0.8%
Malta	6	0.8%
Netherlands	26	3.5%
Poland	51	6.8%
Portugal	22	2.9%
Romania	33	4.4%
Slovakia	13	1.7%
Slovenia	8	1.1%
Spain	54	7.2%
Sweden	20	2.7%
United Kingdom	73	9.7%
Total	751	100%

Shapley Value as a Measure of Political Influence

Run 1		Run 2		Run 3	
<u>Nation</u>	<u>Cumulative Vote</u>	<u>Nation</u>	<u>Cumulative Vote</u>	<u>Nation</u>	<u>Cumulative Vote</u>
France	74	Greece	22	Slovakia	13
Romania	107	Portugal	44	Ireland	25
Bulgaria	125	Luxembourg	50	Luxembourg	31
Ireland	137	Germany	146	Germany	127
Portugal	159	Romania	179	Latvia	136
Germany	255	Poland	230	Czech Republic	158
Hungary	277	Hungary	252	Belgium	180
Lithuania	289	Ireland	264	Lithuania	192
Cyprus	295	Belgium	286	United Kingdom	265
Greece	317	Slovenia	294	Italy	338
Poland	368	Netherlands	320	Netherlands	364
Belgium	390	Malta	326	Finland	377
		Spain	380		

Member States: Power and Population



Conclusions About Power and Representation

- Since the system is degressionary, small states are disproportionately represented (Malta, Luxembourg, Cyprus)
- However large states are also disproportionately powerful (Spain, France, United Kingdom, Italy, Germany). Poland also over-represented.
- The Netherlands (like other medium-sized nations) is being under represented. 30 votes instead of 26 needed for a proportional, degressionary system.