

Chapter 28. Application to Politics: The Banzhaf Index and the Canadian Constitution

- Banzhaf Index
- Canadian Constitution

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- b. 1940
- Earned degrees in electrical engineering (MIT) and patent law (Columbia University)
- Full Professor, George Washington University 1971
- Sued school boards for fair representation
- Sued tobacco companies
- Sued McDonalds and Coca-Cola in an effort to prevent obesity



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Understanding the Swing Voter

- Swing voter: A voter who can change the outcome of an election by voting differently
- The swing total for a fixed number of players can vary widely according to the voting rules being used

The Power of the Swing Voter

- Represents the “volatility” or “degree of suspense” in the decision rule
- An indication of the likelihood of a close decision – one in which a single voter could tip the scales
- A kind of democratic participation index, measuring the decision rule’s sensitivities to the desires of the “average voter” or the “public will”

Axioms: Shapley-Shubik vs Banzhaf

Shapley-Shubik and Banzhaf share the following axioms:

- **Dummy Player:** A player which can never help a coalition win should be accorded no power
- **Permutation:** The measure should not be sensitive to the ordering of the players
- **Additivity:** The power of a player is directly proportional to the marginal value they bring to any coalition

The Differing Axiom

Shapley-Shubik and Banzhaf differ on a fourth axiom:

- Power is proportional to the probability of a coalition having a swing voter (Shapley-Shubik)
- Power is proportional to the expected number of swing voters in the coalition (Banzhaf)

Dubey and Shapley (1979)

Other Ways of Understanding

Shapley-Shubik

- Uses permutation calculations
- Coalition arrangement matters
- Coalitions have aligned perspectives

Banzhaf

- Uses combinatoric calculations
- Arrangements do not matter
- Coalitions have varied perspectives

Canadian Provinces, Territories and Coalitions

The Constitutional Act encourages the formation of coalitions to increase collective power.



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Canadian Provinces: Facts and Figures

	Receipts (millions)	Payments (million)	Population (thousands)
Newfoundland and Labrador	5956	7951	509
Prince Edward Island	554	1512	141
Nova Scotia	8046	9797	938
New Brunswick	2355	8254	750
Quebec	13924	87994	7829
Ontario	16565	109761	13069
Manitoba	3936	12778	1222
Saskatchewan	1709	14836	1030
Alberta	4185	43253	3688
British Columbia	5989	41777	4455

The Problem

- Canada is a vast, diverse and multi-lingual country
- How should Canada apportion its political power, and maintain and update its Constitution?
- Note that until 1982, when Canada was patriated, the Parliament of the United Kingdom was ultimately responsible for Canada

Admendment Formula

- the **7/50 formula**.
- Constitution Act **38(1)(b)**
- Amendments can only be passed by the Canadian House of Commons, the Senate, and a two-thirds majority of the provincial legislatures representing at least 50% of the national population

Recent Changes in the Banzhaf Index

	Victoria (1971)	Constitution Act (1982)	Constitution Act (2009)
	Banzhaf Index β	Banzhaf Index β	Banzhaf Index β
Newfoundland and Labrador	5.94	9.29	9.01
Prince Edward Island	5.94	9.29	9.01
Nova Scotia	5.94	9.54	9.01
New Brunswick	5.94	9.29	9.02
Quebec	21.78	11.32	10.52
Ontario	21.78	12.34	14.35
Manitoba	5.45	9.54	9.02
Saskatchewan	5.45	9.54	9.02
Alberta	5.45	9.54	10.52
British Columbia	16.37	10.31	10.52

Quebec has lost power, confirming earlier concerns (Kilgour and Levesque, 1984)

Ontario and Alberta have been net beneficiaries.