Chapter 29. Bargaining Sets

- Bargaining Sets in Short
- Example Game
- General Procedures

Bargaining Set in Short

- When who you know becomes a bargaining chip to secure values in multi-actor setting
- The logic of mutual exclusion
- Every *objection* is met with a *counter-objection*
- When no counter-objection can be launched then the solution must be stable
- Recall that an imputation is any division of the pay-offs that is individual and group rational

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Consider the Following Game

$v\{A\} = v\{B\} = v\{C\} = 0$

$v\{AB\} = 60 \quad v\{AC\} = 80 \quad v\{BC\} = 100$

$v\{ABC\} = 105$

Formulas from Game Theory and Strategy (Straffin 1993) p.190

Coalition Structures

- A coalition structure is a *partition* of the grand coalition into *disjoint* coalitions
- Partition: A separation or division
- Disjoint: Non-overlapping
- The possible coalition structures for the game are
  - One player {A}{B}{C}
  - Two players {AB}{C}, {AC}{B}, {BC}{A}
  - Three players {ABC}

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Coalition {A}{B}{C}

- This coalition is uninteresting!
- Nothing to share, no-one to bargain with
- The answer is trivial
- X_A=20 X_B=40 X_C=60

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Coalition Structure {AB}{C}

- Players A and B bargain with each other
  - based on their outside options with player C
  - They have $\nu\{AB\}$ to share and thus 60 units
- Player A's outside options with C
  - Player A and C together can earn $\nu\{AC\}$ or 80
  - Player A could claim up to $\nu\{AC\} - X_A$
- Player B's outside options with C
  - Player B and C together can earn $\nu\{BC\}$ or 100
  - Player B could claim up to $\nu\{BC\} - X_B$
- The stable solution sets these claims equal:
  - $\nu\{AC\} - X_A = \nu\{BC\} - X_B$
  - $X_A + 20 = X_B$
  - There are 60 units to share, thus $X_A = 20$, $X_b = 40$

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Reflection

- The two player coalition structures all award the same amount
- $X_A=20 \quad X_B=40 \quad X_C=60$
- This is not surprising since
  - There were six equations with three unknowns
  - All players face the same characteristic function
- Straffin calls this the *balancing property*

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Coalition Structure {ABC}

- Players A and B bargain with each other based on their outside options with player C
- Players A and C bargain with each other based on their outside options with player B
- Players B and C bargain with each other based on their outside options with player A
- These are the same bargaining problems as the two-player coalitions; the answer remains the same
- They have $\nu\{ABC\}=105$ to divide
  - $X_A+40 = X_B+20 = X_C$
  - $X_A=15$ $X_B=35$ $X_C=55$

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Allocating Cost in the Grand Coalition

- Note that there are plenty of ways to object to the division $X_A=15$ $X_B=35$ $X_C=55$!
- This is a disbursement of cost
- Since no coalition likes it, it is stable!
- Proofs demonstrate there is always atleast one imputation in the bargaining set
- This is intuitive: we can always divide a tax n-ways

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Allocating Benefit in the Grand Coalition

- On the other hand we can freely allocate benefit and find no meaningful opposition, other than
  - by propositions to reallocate
  - or feasible allocations with more benefits
- Thus, there may be multiple bargaining sets
- Straffin demonstrates this using the core

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General Algebraic Procedure for Solving Three-Player Bargaining Sets

1. Enumerate all coalition structures
2. Discard the one-player structures if pay-offs are zero, or reformulate to the strategic equivalence
3. Establish equalities for two-player structures and for each player in each structure
4. When the grand coalition is subadditive, tax each player strategically
5. If the grand coalition is superadditive then there are multiple possible stable solutions; the core

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