Chapter 2. Matrix Games

- What is Dominance and Higher Order Dominance?
- What are Saddle Points?
- What is the Value of the Game?
- How do you Solve Using the Minimax Criteria?



Dominance

- The principal of dominance is easy:
- If a strategy is never in your best interest (or that of the decision-maker) then it simply should never be played
- Eliminate these strategies from the game; they are never part of an optimum play
- Both row and column strategies may be dominated
- It is common knowledge for both players that these strategies are dominated

Higher-Order Dominance

- Sometimes when eliminating dominated strategies, it becomes apparent that there are more strategies which are dominated
- Higher-order dominance means that we can continue eliminating dominated strategies as long as it is possible
- This may dramatically simplify the game!
- This ability to eliminate stems from *common knowledge*



Strategy Profiles

- We create strategy profiles by taking a given strategy, and examining the profile of possible outcomes given the range of strategic choices open to the other player
- One of the strategy profiles of Rose is shown below (strategy profile A):

| | | Colin | |
|------|---|--------|--------|
| | | А | В |
| Rose | А | (3,-3) | (8,-8) |
| | В | (5,-5) | (5,-5) |
| | С | (3,-3) | (-8,8) |



Criteria for Dominance

- To demonstrate dominance all we must do is demonstrate that there is at least one strategy profile which is uniformly better than an other
- We say "Strategy A dominates strategy B" or "Strategy B is dominated by strategy A"
- Even if the dominant strategy profile disappears we were correct to eliminate the dominated strategy
- Don't worry about the order of elimination; ultimately it doesn't matter

Weak Dominance and Strong Dominance

- Weakly dominated strategies are strategies with payoffs which are **less than or equal** to other strategies across the entirety of the strategy profile
- Strongly dominated strategies are strategies with payoffs which are **less than** strategies across the entirety of the strategy profile
- If you eliminate weakly dominated strategies, you may eliminate or throw out some potential equilibrium points

Recommended Play of the Game

- Game theorists recommend solutions which are at equilibrium.
- Equilibrium plays are the best response to the other player.
- Another way of saying this is that players are never made better off by unilateral defection.
- These solutions guarantee a certain minimum payment to the game (the value)
- If you play away from the equilibrium, you will not play as effectively as you could



Finding the Recommended Solution

- The recommended solution is known as the saddle point
- We found saddle points in the previous chapter using movement diagrams
- Saddle points are defined as an entry in a game (with payoffs to the row player) as those entries which are
 a) Less than or equal to any entry in its row, AND
 - b) Greater than or equal to any entry in its column

Value of the Game

- For any game there is a value *v*
- Rational play of the game can guarantee at least this value (and perhaps even more if the other player is limitedly rational)
- If there are multiple saddle points, they all provide the same value of the game



Solving Games Using the Minimax Criteria

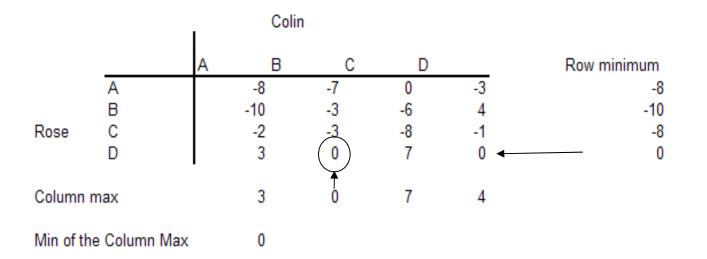
- First find the maximum entry in all columns
- Then find the minimum entry in all rows
- Then find the minimum of the maximum entries in the column
- Then find the maximum of the minimum rows
- This is the minimax value
- The maximin and minimax values are identical when there are saddle points
- The intersections of maximin and minimax values are saddle points

No Equilibrium / Multiple Equilibrium

- Not all games have saddle points!
- In these situations solution concepts we discuss will not find an equilibrium point
- Other concepts are needed, and can guarantee the value of the game
- Some games have multiple equilibria!
- The solution concepts we define will find multiple equilibria
- All equilibria provide the same value (or they would not be equilibria)



Example Solution Using Minimax Criteria



Max of the Column Min 0

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