

Chapter 2.

Matrix Games

- What is Dominance and Higher Order Dominance?
- What are Saddle Points?
- What is the Value of the Game?
- How do you Solve Using the Minimax Criteria?

Dominance

- The principal of dominance is easy:
- If a strategy is never in your best interest (or that of the decision-maker) then it simply should never be played
- Eliminate these strategies from the game; they are never part of an optimum play
- Both row and column strategies may be dominated
- It is common knowledge for both players that these strategies are dominated

Higher-Order Dominance

- Sometimes when eliminating dominated strategies, it becomes apparent that there are more strategies which are dominated
- Higher-order dominance means that we can continue eliminating dominated strategies as long as it is possible
- This may dramatically simplify the game!
- This ability to eliminate stems from *common knowledge*

Strategy Profiles

- We create strategy profiles by taking a given strategy, and examining the profile of possible outcomes given the range of strategic choices open to the other player
- One of the strategy profiles of Rose is shown below (strategy profile A):

		Colin	
		A	B
Rose	A	(3, -3)	(8, -8)
	B	(5, -5)	(5, -5)
	C	(3, -3)	(-8, 8)

Criteria for Dominance

- To demonstrate dominance all we must do is demonstrate that there is at least one strategy profile which is uniformly better than another
- We say “Strategy A dominates strategy B” or “Strategy B is dominated by strategy A”
- Even if the dominant strategy profile disappears we were correct to eliminate the dominated strategy
- Don’t worry about the order of elimination; ultimately it doesn’t matter

Weak Dominance and Strong Dominance

- Weakly dominated strategies are strategies with payoffs which are **less than or equal** to other strategies across the entirety of the strategy profile
- Strongly dominated strategies are strategies with payoffs which are **less than** strategies across the entirety of the strategy profile
- If you eliminate weakly dominated strategies, you may eliminate or throw out some potential equilibrium points

Recommended Play of the Game

- Game theorists recommend solutions which are at equilibrium.
- Equilibrium plays are the best response to the other player.
- Another way of saying this is that players are never made better off by unilateral defection.
- These solutions guarantee a certain minimum payment to the game (the value)
- If you play away from the equilibrium, you will not play as effectively as you could

Finding the Recommended Solution

- The recommended solution is known as the **saddle point**
- We found saddle points in the previous chapter using movement diagrams
- Saddle points are defined as an entry in a game (with payoffs to the row player) as those entries which are
 - a) Less than or equal to any entry in its row, AND
 - b) Greater than or equal to any entry in its column

Value of the Game

- For any game there is a value v
- Rational play of the game can guarantee at least this value (and perhaps even more if the other player is limitedly rational)
- If there are multiple saddle points, they all provide the same value of the game

Solving Games Using the Minimax Criteria

- First find the maximum entry in all columns
- Then find the minimum entry in all rows
- Then find the minimum of the maximum entries in the column
- Then find the maximum of the minimum rows
- This is the minimax value
- The maximin and minimax values are identical when there are saddle points
- The intersections of maximin and minimax values are saddle points

No Equilibrium / Multiple Equilibrium

- **Not all games have saddle points!**
- In these situations solution concepts we discuss will not find an equilibrium point
- Other concepts are needed, and can guarantee the value of the game
- **Some games have multiple equilibria!**
- The solution concepts we define will find multiple equilibria
- All equilibria provide the same value (or they would not be equilibria)

Example Solution Using Minimax Criteria

		Colin				Row minimum	Max of the Column Min
		A	B	C	D		
Rose	A	-8	-7	0	-3	-8	0
	B	-10	-3	-6	4	-10	
	C	-2	-3	-8	-1	-8	
	D	3	0	7	0	0	
Column max		3	0	7	4		
Min of the Column Max		0					

An arrow points from the value 0 in the 'D' column to the 'Max of the Column Min' value 0. Another arrow points from the value 0 in the 'D' column to the 'Min of the Column Max' value 0.