Chapter 3.
Matrix Games: Mixed Strategies

- What is the Minimax Theorem?
- What are Mixed Strategies?
- How do you Solve for Mixed Strategies?
- Solving Arbitrary N by N Game Matrices

Pure Strategies and Mixed Strategies

- **Pure strategies** involve choosing one and only one of the potential strategies available to the player
- **Mixed strategies** involve choosing a mixture of the possible strategies available to the player, such that the mixing proportions sum to 100%

Recommending Mixed Strategies in Play

- Optimum play in some games entails mixed strategies
- Players choose among their strategies in the recommended fixed proportion of the mixed strategy
- Some game theorists are troubled by the concept of mixed strategies since they believe it lacks descriptive validity
- Others suggest that there are other criteria for decision-making being used, not modeled in play
- Further, and as we will see, mixed strategies are dependent on utility measurements

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Active Strategies

- Mixed strategies need not mix in all the strategies which are available to the player
- Those strategies which are recommended to be used are **active strategies**
- When all strategies are active, the resultant mixed strategy is **totally mixed**

Expected Value of a Strategy

- Mixing proportions are effectively probabilities
- We calculate the expected value of a mixed strategy by multiplying probabilities from the mixed strategies by pay-off, and summing

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Example Calculation of Expected Value

		Colin	
		A	B
	A	2	-3
Rose	B	0	2
	C	-5	10

Suppose Colin adopts a mixed strategy of 71.4% A / 28.6% B

- Expected Value of Rose Strategy A:

71.4% (2) + 28.6% (-3) = 0.571

- Expected Value of Rose Strategy B:

71.4% (0) + 28.6% (2) = 0.571

- Expected Value of Rose Strategy C

71.4% (-5) + 28.6% (10) = -0.710

Example taken from Game Theory and Strategy (Straffin 1993) p.16

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Three Methods for Solving Mixed Strategy Solutions

- **Method of Equalizing Expectation:** A mixed strategy for the column player must result in equivalent payoff in active row strategies. Results in solving simultaneous linear equations.
- **William's Oddments:** A shorthand algebraic calculation for finding mixed strategies
- **Graphical Method:** Plotting pay-offs which result from a mix of opponent strategies. Good for Nx2 or 2xN game matrices.

Equal Values of Expected Strategies

- Note how Rose's two active strategies offer the same expected value – 0.571
- This is no coincidence
- The expected values of all active strategies must be equivalent, or they would not be part of the equilibrium
- By definition, an equilibrium cannot be bettered; if one of the active strategies offered a better expected pay-off it should be chosen exclusively

Example of Equalizing Expectations

		Colin	
		A	B
	A	1	3
Rose	B	2	-6

- Is there a mixed strategy for Colin (x, 1-x)?
- Rose strategy A results in expected value 1(x)+3(1-x) or 3-2x
- Rose strategy B results in expected value 2(x)-6(1-x) or 8x-6
- Can we find x such that 3-2x = 8x-6? Yes.

  10x = 9

  x = 0.90

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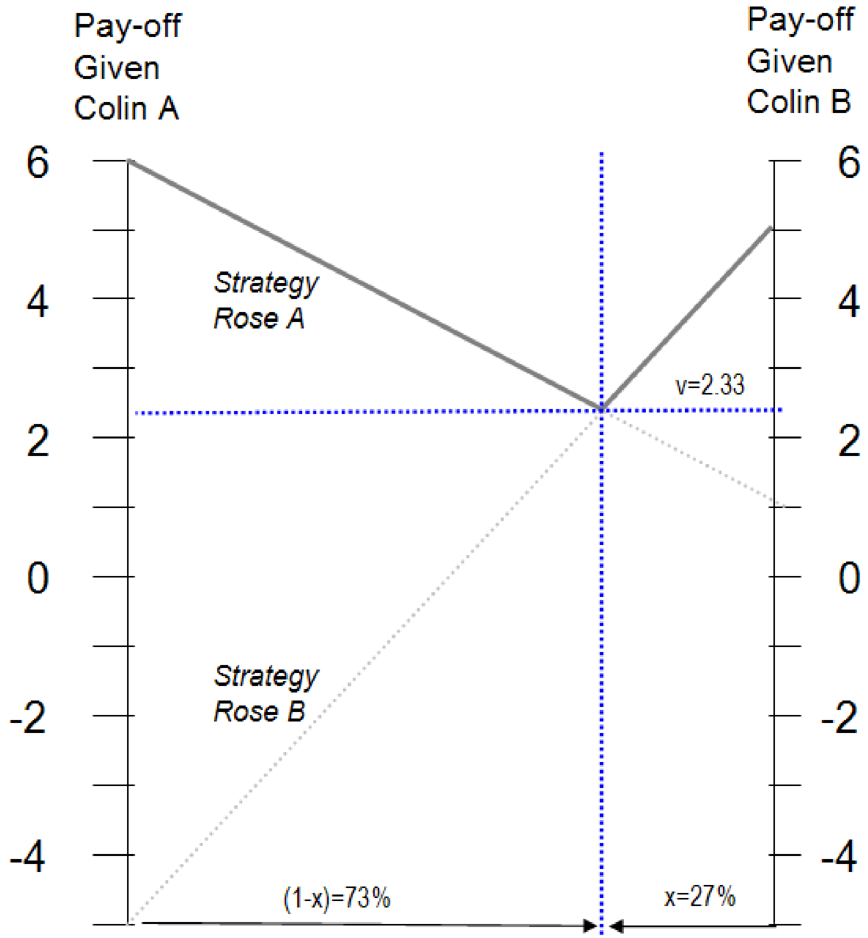
Example of Graphical Method

		Colin A	Colin B
Rose	A	6	1
	B	-5	5

Strategic Form Game



Graphical Solution of Game

Example Williams Oddments

		Colin A	Colin B
Rose	A	-9	-4
	B	-4	-10

Row Differences		Rose oddments		Rose probabilities
(-9 - -4) = -5		6		6/11
(-4 - -10) = -6		5		5/11
Calculate Row Differences	Take Absolute Value	Swap	Add oddments	Divide Through
Step #1	Step #2	Step #3	Step #4	Step #5

Guidelines for Solving Arbitrary Game Matrices by Hand

- Simplify the matrix using dominance
- Check for saddle points
- If there are no saddle points, then check for mixed strategies
- If mixed strategies fail then you must identify active strategies
- Use the graphical method on (2 x n) or (m x 2) strategies
- The graphical method eliminates all passive strategies

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