Chapter 33. The Value of Game Theory

- Précis of the Course
- The Value of Game Theory
The Course in Précis, Part One

- **Minimax** for zero sum offers clearcut answers, but
- **Non-zero sum** games are no longer Pareto optimal
- And there are often **multiple equilibria**
- and **games against nature** introduce the ambiguity
- Much effort has gone into breaking the **prisoner** out of their **dilemma**
- **Nash arbitration** offers a hard and fast rule . . .
- If only we knew how to set the **status quo**
The Course in Précis, Part Two

• **Cooperative game theory** seems to glory in multiple and conflicting approaches
• Can’t we keep it simple using only the **characteristic function**?
• **Shapley** offers a single and clear line of reasoning affording a **value** for payoff
• But **Banzhaf** offers a different alternative
• Cost allocation leads to different lines of reasoning involving **bargaining** and the **nucleolus**
What Do We Make of This

• . . . apparent failure to provide uniform predictions and prescriptions for a wealth of human endeavors?
• The mathematician revels in clear answers
• But also enjoys the processes of reasoning
• Are there exact and incontrovertible answers which game theory has failed to deliver?
Complexity of Underlying Processes

- Game theory reveals that the underlying processes themselves are “also complicated, rich and various”
- Coalitional bargaining is sometimes impossible, as revealed by an empty core
- The idea of power in political science is not logically simple
- Nor is idea of fairness from welfare economics
- Game theory is complex because it models an underlying complexity in society
Mathematical Analysis and Dogma

- “Mathematical analysis of the kind embodied in game theory is the enemy of dogma” – Straffin
- “Tyranny thrives on the canonization of nonsense on the perpetuation of falsehood” – Rapoport
- Be skeptical concerning “rationality” and “fairness”
- Draw upon a useful collection of concepts, models and lines of reasoning to support your skepticism
- Apply game theory to generation of insight not just simple answers
Exemplifying Theory

• Fisher (1989), Rasmusen ()
• Also known as “no fat modeling” or “modelling by example” or “MIT-style theory” or exemplary theory
• The simplest assumptions needed to generate an interesting conclusion
• The starkest, barest model that has the desired result
• This desired result is the answer to some relatively narrow question
The Process of Exemplifying Theory

- Start with a vague idea
- Model the idea formally in a simple way
- The idea might
  - survive intact,
  - be found to be meaningless or in need of qualification
  - or the reverse (of the idea) might be true
- Proofs, precise propositions, generation of ideas
- Return to thinking in words
Ockham’s Razor

• Good theory of any kind uses Ockham’s razor
• Restricts attention to one issue at a time
• A robust answer to fundamental uncertainties
• Modesty in the face of empirical uncertainty
• Creating parameters for a range of possible explanations within a common framework
Stories That Might be True

• “Exemplifying theory does not tell us what must happen. Rather it tells us what can happen.”
• Chicago-style “Stories That Might be True.”
• Not destructive criticism if the modeller is modest, since there are also a great many “Stories That Can’t Be True”
• Unfortunately these stories which can’t be true are often used as the basis for decisions in business and government
## Seven Possible Solution Concepts

<table>
<thead>
<tr>
<th>Triangle Diagram</th>
<th>Algebraic Work</th>
<th>Logical or Rhetorical Defense</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Core</strong></td>
<td>Map the associated inequalities. If there is a feasible region, identify the region. If the core is empty, say so.</td>
<td>Derive the associated inequalities</td>
</tr>
<tr>
<td><strong>Stable Set</strong></td>
<td>Display the associated stable sets.</td>
<td>Not needed.</td>
</tr>
<tr>
<td><strong>Shapley Value</strong></td>
<td>Not needed.</td>
<td>Provide the imputation. The complete solution including the permutations, marginal value calculations or the swing voting combinatorics.</td>
</tr>
<tr>
<td><strong>Banzhaf Value</strong></td>
<td>Not needed.</td>
<td>Provide the imputation. The complete solution including the permutations, marginal value calculations, or the swing voting combinatorics.</td>
</tr>
<tr>
<td><strong>Bargaining Set</strong></td>
<td>Show the set of possible bargaining solutions if there are multiple possible solutions.</td>
<td>Provide a list of the possible partitions. Provide the equalities resulting from the bargaining setting, and their solution. For each partition state whether the imputation is a single value or a set. If it is a single value, then provide this value. If a set, then describe the set in a series of inequalities.</td>
</tr>
<tr>
<td><strong>Nucleolus</strong></td>
<td>Identify the core on the diagram Geometrically identify the nucleolus point.</td>
<td>Provide the imputation. If you have not already identified the associated inequalities for the core, do so now. Calculate the excess for all players at this point.</td>
</tr>
<tr>
<td><strong>Gately Point</strong></td>
<td>Not needed.</td>
<td>A calculation of the marginal values associated with each of the players. A calculation of the appropriate ratios of division to minimize disruption. Calculate the associated disruption at this point.</td>
</tr>
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* The Banzhaf Value, unlike all other techniques, works only on weighted voting games