

Chapter 5.

Application to Warfare: Guerrillas, Police, and Missiles

- Problem of Guerrillas vs Police
- Problem of the Missile Penetration System
- Limitations of Zero-Sum Games as Modeling

Guerrillas

- Suppose there are m guerrillas, n police and two arsenals
 - The guerillas take the arsenal if they are stronger than the police force at the arsenal.
 - Should the guerillas take an arsenal, they win.
 - Winning means gain one point. Loosing means gain no points.
- What should the attack plan of the guerillas be to maximize their win?
- How should the police respond to this plan?

Known Wins and Losses

- The guerrillas can clearly win if $m > n$
- They should attack either arsenal with full force.
- The police always win if $n \geq 2m$
- They should defend each arsenal with force m .

Straffin's Simplification

- Consider the case of 2 guerrillas and 3 police.
- Suppose the arsenals are "East" and "West"
- We could enumerate multiple possible strategies for the guerrillas and the police. On the guerilla side:
 - $2W - 0E$
 - $1W - 1E$
 - $0W - 2E$
- Straffin simplifies by saying the real decision is how to divide the force, not where to send it.
- This does make analysis easier, but we might doubt his insight.

Sample Model of $n=3$, $m=4$

- Not given in book
- I choose to fully enumerate strategies

		Police				
		4E - 0W	3E - 1W	2E - 2W	1E - 3W	0E - 4W
Guerrillas	3E - 0W	0	0	1	1	1
	2E - 1W	1	0	0	1	1
	1E - 2W	1	1	0	0	1
	0E - 3W	1	1	1	0	0

Sample Model of $n=3, m=4$

- 0E is dominated by 1E for the police. Why loose an arsenal by not trying?
- Remember, the police are trying to avoid losses

		Police				
		4E - 0W	3E - 1W	2E - 2W	1E - 3W	0E - 4W
Guerrillas	3E - 0W	0	0	1	1	1
	2E - 1W	1	0	0	1	1
	1E - 2W	1	1	0	0	1
	0E - 3W	1	1	1	0	0

- Also 4E dominated by 1E.
- The game begins to unravel

Further reductions possible

		Police		
		3E - 1W	2E - 2W	1E - 3W
Guerrillas	3E - 0W	0	1	1
	2E - 1W	0	0	1
	1E - 2W	1	0	0
	0E - 3W	1	1	0



		Police		
		3E - 1W	2E - 2W	1E - 3W
Guerrillas	3E - 0W	0	1	1
	0E - 3W	1	1	0



		Police	
		3E - 1W	1E - 3W
Guerrillas	3E - 0W	0	1
	0E - 3W	1	0

- Mixed strategy 50%/50% both players
- The value of the game is 0.50

Reflection on the Game

- Straffin is correct with his assumption of mixed strategies.
- Interpreting the strategies, we see that the guerrillas prefer an “all-in” strategy.
- This relates to the assumption of breaking ties in favor of the defender.
- This generalizes to larger force levels. There is no use for the solitary guerrilla.
- These features of the game ultimately stem from the assumptions of perfect knowledge. Do the opponents really know the force levels?

Missile Penetration Problem

- Based on missile defense problems.
- Suppose that there are two countries, Red and Blue.
- Red has four missiles: two missiles have warheads, two missiles are dummies. Red wishes to destroy blue bases.
- Blue has two anti-missiles. Each blue anti-missile can scan two incoming missiles, select one and destroy it.

Eliminating Dominated Strategies

		Red					
		WWDD	WDWD	WDDW	DWWD	DWDW	DDWW
Blue	12	1	1	0	1	0	0
	13	0	1	1	1	1	0
	14	0	0	1	0	1	0
	23	0	0	0	1	1	1
	24	0	0	0	0	1	1
34	0	0	0	0	0	1	

Diagram illustrating dominated strategies in a game between Blue and Red. The table shows the payoff matrix. Blue strategies are 12, 13, 14, 23, 24, 34. Red strategies are WWDD, WDWD, WDDW, DWWD, DWDW, DDWW.

Annotations:

- 13 dominates 14 (indicated by a blue arrow pointing from 13 to 14)
- 23 dominates 24, 34 (indicated by a blue arrow pointing from 23 to 24 and 34)
- WWDD dominates WDWD (indicated by a blue arrow pointing from WWDD to WDWD)
- WDDW dominates DWWD (indicated by a blue arrow pointing from WDDW to DWWD)
- DWDW dominates DDWW (indicated by a blue arrow pointing from DWDW to DDWW)

- Dominated strategies give up wins to dominated strategies (its more than counting wins)

Reduced Game Matrix

- Mixed strategy solution $1/3, 1/3, 1/3$ for both players

		Red		
		WWDD	WDDW	DDWW
Blue	12	1	0	0
	13	0	1	0
	34	0	0	1

- The basic insight: send the red missiles in volleys, hoping to overwhelm blue defenses

Small Scale Tactics and Zero-Sum Games

- Straffin defends the use of zero-sum games in military tactical planning
- However he acknowledges its limitations in larger-scale strategic questions of war
- Another explanation for Straffin's reservation
 - Some strategic choices in warfare open up entirely new theaters of war.
 - Thus the scale of win and loss is no longer constant
 - Thus, military planners have the capability of choosing tactics which open up a non-zero sum game.