Chapter 7.
Game Trees

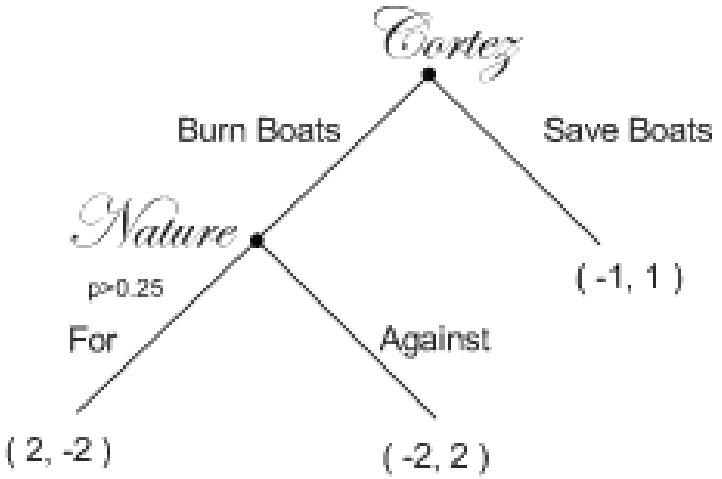
- Sequential Moves and Information Sets
- Games in Extensive Form
- Examples

Sequential Moves

- Many significant games involve sequential moves
- Previously we've studied only simultaneous move games
- The extensive form (or game tree) offers a natural way of representing these sorts of games
- The book offers two examples of sequential moves:
  - The Game of Poker
  - The Cuban Missile Crisis

Extensive Form

- Games in extensive form are represented as "game trees"
- Player choices are represented at the nodes.
- The consequences of the decisions are shown as branches
- Precommitment



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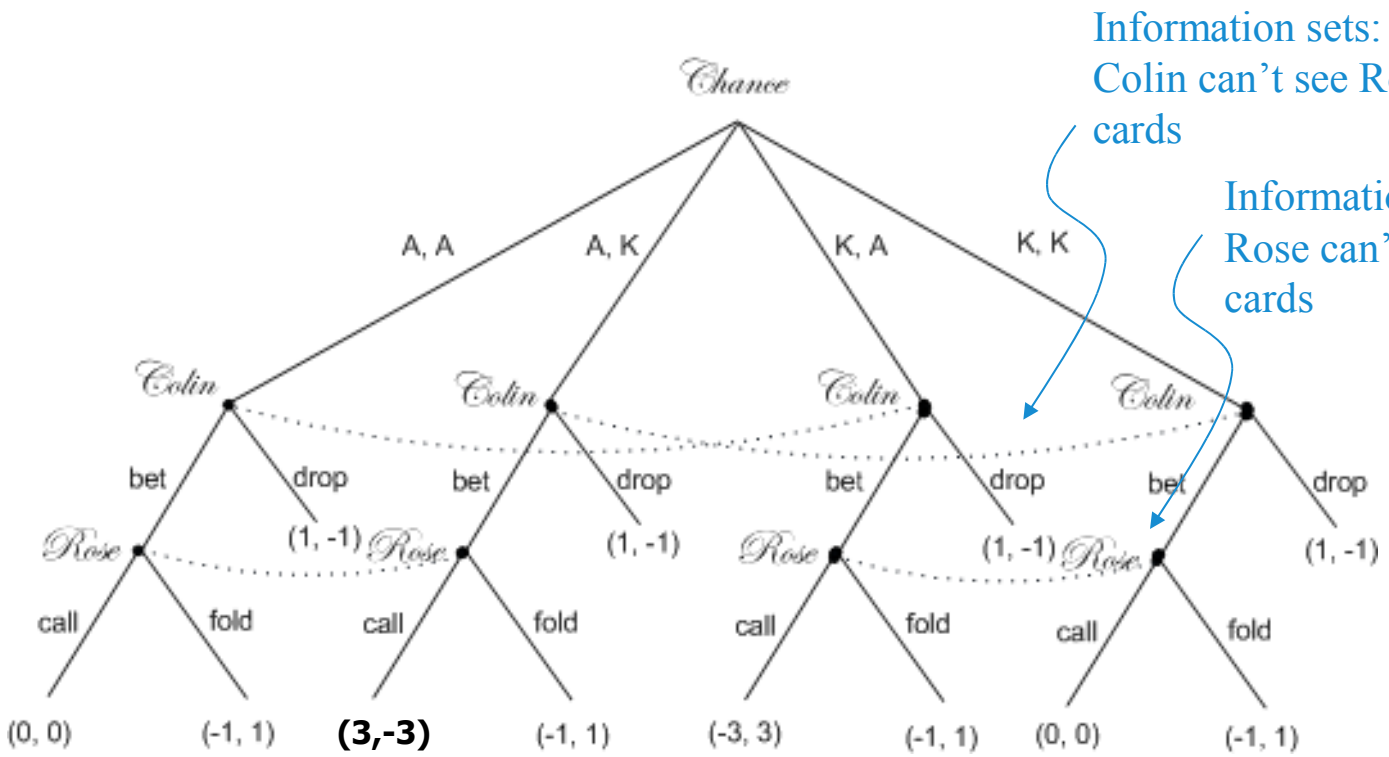
Information Sets

- We use information sets to capture knowledge about the game
- Information sets are represented by
  - Dotted lines between sets of nodes, or
  - Encircling sets of nodes
- These nodes must all belong to one player.
- That player does not know their current position on the game tree

A Simplified Form of Poker

- Large deck containing only Aces and Kings in equal proportion
- All in – both players commit a chip (1 point)
- One card dealt to each of two players
- Betting player looks at his cards, and either raises (throwing in another 2 points) or drops (loosing his stake)
- The other player then calls (by throwing in another 2 points), or folds (thereby letting the better win)

A Simplified Version of Poker



cards to (Rose, Colin)
payoffs to (Rose, Colin)

Diagram from Game Theory and Strategy (Straffin 1993) p.38

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Enumerating Strategies

- Colin has two information sets
  - Receives a King
  - Receives an Ace
- He has two choices at each of two information sets
  - Bet
  - Drop
- Analyzing the game requires us to specify the strategy for every contingency
- This is 2^2 different possibilities

Spelling Out Colin's Strategies

- Colin may bet if he receives an A, or if he receives a K

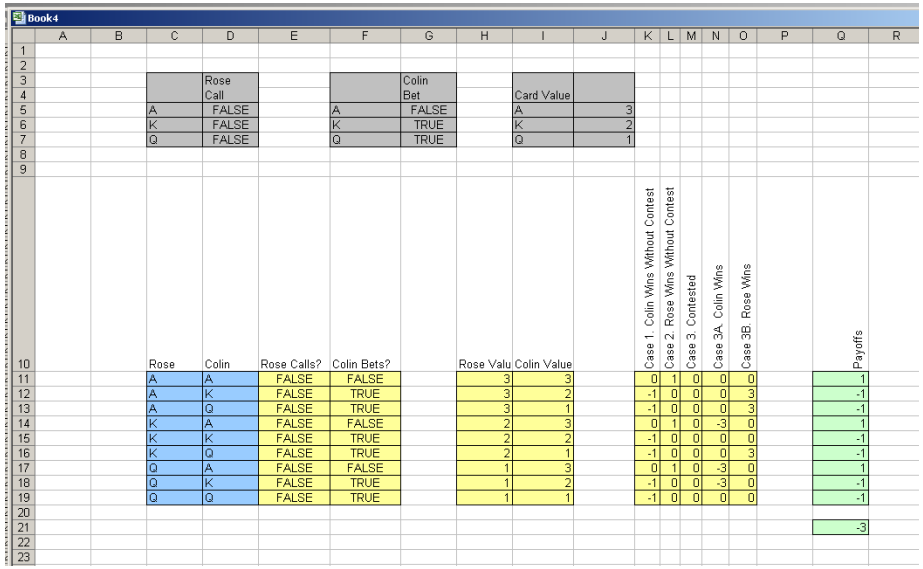
  "Bet Always"
- Colin may bet if he receives an A, but drop if he receives a K
- Colin may bet if he receives a K, but drop if he receives an A
- Colin may drop if receiving an A or a K

  "Drop Always"

Enumerating Strategies

- Modeling strategies under uncertainty is difficult!
- First, we identify available information coming into our information sets
- Examples in the book
  - This might be based on draws from a deck of cards
  - Or it might be based on the observation of the other player's strategies
- Then we specify what choices we will make once we receive that information

Implementation



Book4

	Rose Call
A	FALSE
K	FALSE
Q	FALSE

	Colin Bet
A	FALSE
K	TRUE
Q	TRUE

Card Value	
A	3
K	2
Q	1

Rose	Colin	Rose Calls?	Colin Bets?	Rose Valu	Colin Value	Case 1. Colin Wins Without Contest	Case 2. Rose Wins Without Contest	Case 3. Contested	Case 3A. Colin Wins	Case 3B. Rose Wins	Payoffs
A	A	FALSE	FALSE	3	3	0	1	0	0	0	1
A	K	FALSE	TRUE	3	2	-1	0	0	0	3	-1
A	Q	FALSE	TRUE	3	1	-1	0	0	0	3	-1
K	A	FALSE	FALSE	2	3	0	1	0	-3	0	1
K	K	FALSE	TRUE	2	2	-1	0	0	0	0	-1
K	Q	FALSE	TRUE	2	1	-1	0	0	0	3	-1
Q	A	FALSE	FALSE	1	3	0	1	0	-3	0	1
Q	K	FALSE	TRUE	1	2	-1	0	0	-3	0	-1
Q	Q	FALSE	TRUE	1	1	-1	0	0	0	0	-1

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- One homework problem asks that you figure out the payoffs for a three card version of Poker
- I used Excel
- Enter in the strategies, then read out the payoff
- Requires 64 checks 2^3 times 2^3

Screenshot of Microsoft Excel spreadsheet by Scott Cunningham

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Strategic Analysis of Poker

- Policy significance. If skill, then can it be legislated against? Is Poker a game of skill or chance? If skill, how much does a skillful player earn?
- Insights from analysis. Beginners bet too often, and bluff too little.

Cuban Missile Crisis

- A famous study of decision-making (Alison, 1972; Alison and Zelikow, 1999)
- First example in the book of setting arbitrary utilities and using a game tree as a thought experiment
- Real-world situation probably involved information sets
- Uncertainty: the ultimate source of conflict?

President Kennedy with General Curtis LeMay.
Image adapted from a public domain (usgov) image.